Fuzzy Logic and Neural Networks
Prof. Dilip Kumar Pratihar
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 01
Introduction to Fuzzy Sets

I welcome you all to the course on Fuzzy Logic and Neural Networks. Now, in this particular course, we are trying to model human brain in the artificial way and in other words, we are also going to discuss the principle of soft computing, in details. Now, let us start with the first topic, that is, Introduction to Fuzzy Sets.

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Now, here so, this is actually the topics, which I am going to covered in this lecture. So, at first, we will give a brief introduction to the classical set or the crisp set and after that, the properties of classical set or the crisp set will be discussed in details. Now, before I start with the concept of fuzzy sets, we will try to explain the reason behind going for this particular the fuzzy sets, we will also discuss, how to represent the fuzzy set.

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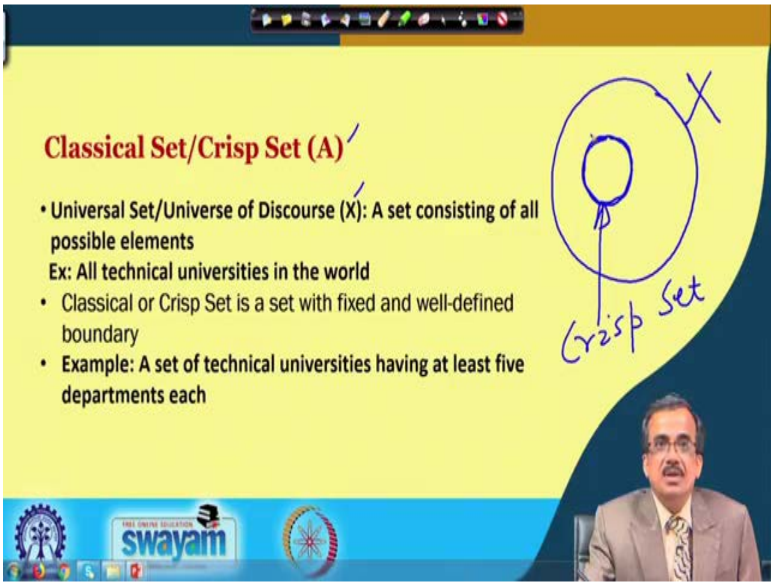

Now, introduction to the classical set or the crisp set, which is denoted by A; now, before I just go for discussing like what do we mean by the fuzzy set, let me discuss, what do we mean by classical set or the crisp set, first. Now, to define the concept of the crisp set, what we do is, we try to explain the terms: the universal set or the universe of discourse. Now, let me take one example, now supposing that we are going to form a set of all technical universities in this world, now all the technical university universities of this world will constitute one big set and that is nothing, but the universe of discourse or the universal set, that is denoted by capital X.

Now, let me draw one universal set; now supposing that this is nothing, but the universal set denoted by capital X. Now, next I am just going to ask another question, like can you not form a set of technical universities having at least five departments each? Now, if you just investigate a number of universities throughout the world, we will have at least five departments each and there is a possibility that inside the universal set, I will be getting a subset and this is nothing, but this particular subset and this subset is nothing, but the crisp set. So, this is nothing, but the crisp set because, this set is having the well-defined boundary.

So, by definition, the classical set or a crisp set, we mean the set with fixed and well-defined boundary. So, this is nothing, but the classical set or the crisp set.

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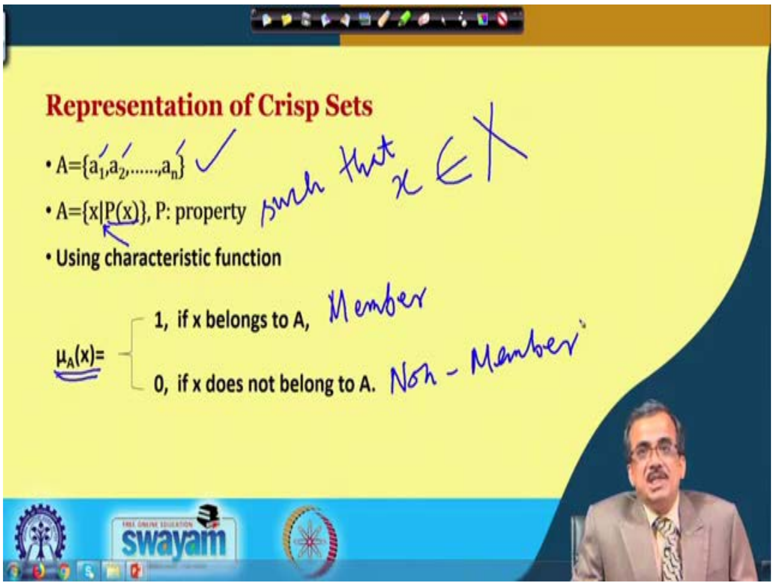

The next, I am just going to discuss, how to represent a crisp set, that is denoted by A. Now, if you see the literature, the crisp set has been defined in 3 different ways, the first method. So, A is nothing, but a collection of all the elements like you're a_1, a-2 up to a_n. So, the classical set or the crisp set, this is written as A equals to a collection of a_1, a_2, up to a_n. So, this is one way of representing the crisp set, the second method of representing the crisp set is as follows.

So, A is nothing, but a crisp set which is nothing, but the collection of x, such that it has got the property P(x). So, P(x) is nothing, but the property and these particular symbol indicates that A is actually a crisp set, which is having the properties P(x) and this particular small x of course, it belongs to your the universal set or universe of discourse. Now, the third method of representing the crisp set is as follows, we take the help of some characteristic function to represent the crisp set.

Now, this mu_A(x) represents the characteristic function of the crisp set A. Now, this mu_A(x) is equals to 1, if x belongs to A, and if it does not belongs to A, then this mu_A(x) is nothing but 0. Now, this is almost similar to the situation, whether it is a member or a non-member. So, if it is member, then the characteristic function, that is, mu_A(x) is nothing, but is equal to 1, and if it is a non-member, then the characteristic function is equal to 0.

So, this is either 1 or 0. So, these are the 3 ways actually, we can represent the crisp set or the classical set.

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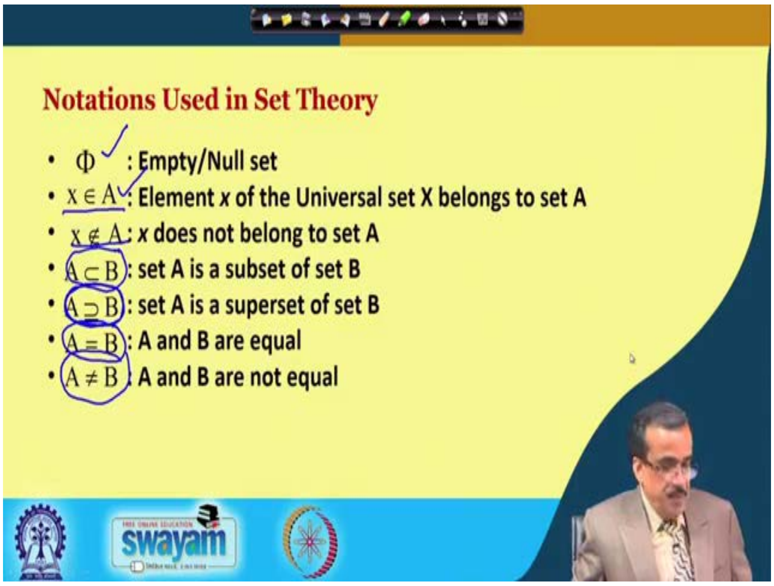

Now, let us see the some of the properties of this particular crisp set, but before that, let me try to concentrate on the notations, which are generally used in set theory. Now, the set theory, you have already studied might be during your school days or in the first year of your under graduation. So, whatever I am discussing related to the crisp set or the classical set are nothing, but recapitulation for all of you. Now, this particular symbol, this indicates actually the empty set or the null set, then this symbol x belongs to A. So, x belongs to A so, this is represented by this particular symbol, then x does not belongs to A is this particular symbol, then comes your A is a subset of B. So, this is the symbol, which is generally used to represent A is a subset of B. The next is A is a superset of B.

So, this is represented by this particular symbol, and if you write A is equal to B. So, this is the symbol; that means, your set A is equal to set B, and if set A and set B are not equal, we use this particular symbol. So, these are the different symbols used and there are a few other symbols also, which are generally used in the classical set or the crisp set.

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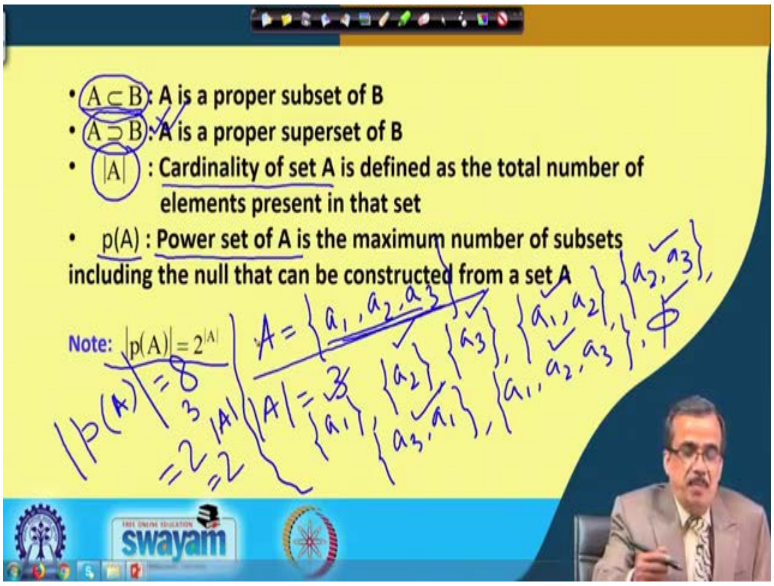

Now, here, this particular symbol, A is a proper subset of B. So, to represent A is a proper subset of B, we use this particular symbol, then the next symbol indicates A is a proper superset of B. So, this is nothing, but A is a proper superset of B. Now, I am just going to concentrate on another symbol. So, this is actually the symbol of the mod sign or the mod value. Now, this symbol indicates actually the cardinality of a set A and by cardinality actually, we mean the total number of elements present in a particular set. Now, let me take a very simple example, now supposing that I have got A, the set, say, this particular set, that is a crisp set and it has got 3 elements like your a_1, a_2 and a_3.

Now, what is the cardinality of this particular set? It is very simple, the cardinality of this particular set is nothing but 3, because it has got 3 elements a_1, a_2 and a_3. So, it is so simple. Now, the next symbol is p(A), now this p(A) indicates the power of set A. Now, by power of set A, we mean the maximum number of subsets that can be constructed from a particular set. Now, let me try to concentrate on the same crisp set, that is, A is a collection of 3 elements a_1 comma a_2 comma a_3. Now, let us see, how many such subsets can be constructed from this particular set a including the null. Now, if you see, if I take one element set, that is called the singleton.

So, I can from here construct the subset like, a_1 next the subset like a_2, next I can construct a_3, next I can construct a_1 a_2, next I can construct a_2 a_3, next I can construct you’re a_3 a_1, I can also construct the same that is you’re a_1 a_2 a_3 and I can also construct the null set. Now, let me count how many subsets, we have constructed; so 1, 2, 3, 4, 5, 6, 7, 8.

So, 8 such subsets, I have constructed. Now, if I just try to find out, what is the cardinality of this particular power set of A; that means, what is this cardinality of power set of A and that is nothing, but 8 because, I am able to construct 8 such subsets. Now, this particular 8 can be written as 2 raised to the power 3, and that is nothing, but 2 raised to the power cardinality of A; that means, I can find out. So, this particular relationship that is the cardinality of A is nothing, but 2 raised to the power cardinality of A. So, this is the way actually, we can define the power set of A and it is cardinality.

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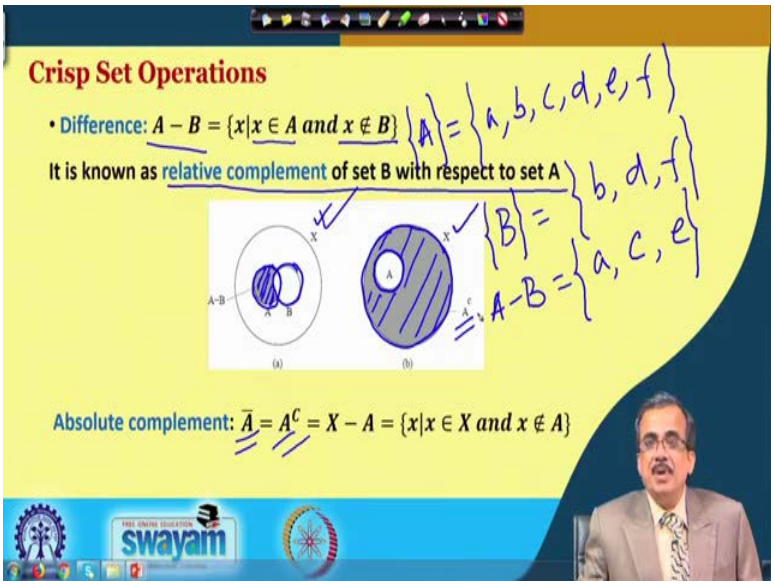

Now, I am just going to concentrate on some crisp set operations. Now here, I am just going to define like, what do we mean by the difference between the two classical sets that is $A-B$. Now, supposing that I have got one universal set or the universe of discourse like this, and I have got two such classical sets or the crisp set 1 is nothing, but A and another is nothing but B. So, my aim is to find out so, $A-B$, that is, the difference between A and B.

Now, this $A-B$ is also known as the relative compliment of set B with respect to set A, and mathematically, $A-B$ is nothing, but x, such that x belongs to A and x does not belongs to B. Now, if we just follow that so, x belongs to A and x does not belongs to B. So, I will be getting, $A-B$ is nothing, but is this. So, this black portion or the shaded portion and that is nothing, but the relative complement of B with respect to A. Now, let me take a very simple example, now supposing that I have got a classical set or a crisp set that is nothing, but A and it has got a few elements like your a, b, c, d, e, say f. So, I have got say 6 elements.

Now, I have got another classical set or the crisp set say, it is denoted by B and supposing that this is nothing, but b, d, f. So, there are 3 elements. Now, if I find out; so, this $A-B$, how to find out the $A-B$? It is very simple. So, I will be getting here a then comes your c and then I will be getting e. So, this is nothing, but the difference between A and B or the relative complement of B with respect to A. Now, there is another concept that is called the absolute compliment. So, this absolute compliment, that is, represented by $\bar{A}$ or A^C now; here let us try to understand from here.

So, supposing that we have got. So, this is nothing, but the universal set denoted by X and I am got a crisp set that is denoted by A here. So, $X-A$ that is nothing, but the absolute complement of A and this is nothing, but this black region or the shaded region. So, that is nothing, but A^C or the complement of your A. Now, this is the way actually, we can define the concept of the difference between two fuzzy sets.

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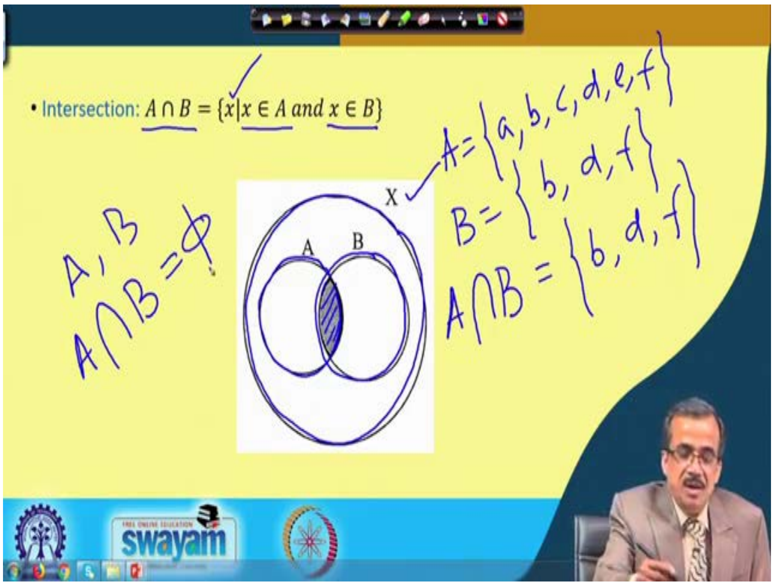

Now, I am just going to discuss another, that is called the intersection between two crisp sets. Now, let me try to concentrate. So, this is nothing but X, is nothing but my universal set or universe of discourse denoted by capital X, I have got two crisp sets, one is called A and another is called B.

So, by definition the intersection of A and B, that is $A \cap B = \{x \mid x \in A and x \in B\}$; so, this is a common region between A and B, and if you see. So, this particular black portion or the shaded portion is nothing, but a common to both A and B, and that indicates actually A intersection B. Now, if I take the same example like say the crisp set, which is having the elements like your a, b, c, d, e f. So, I am taking the same example and supposing that I have got another crisp set and which is nothing but b, d, f.

Now, if I try to find out the common region between A and B and that is nothing, but A intersection B and that will be nothing, but so, this should be common to both. So, this is nothing, but is your b, d, f. So, this is what we mean by the intersection and here, let me mention supposing that I have got two sets like set A and set B and supposing that the two sets are disjoint; by disjoint we mean there is no common element between these two classical sets or the crisp sets. Now, if they are disjoint. So, this particular A intersection B will be nothing, but a null set, so if A and B are disjoint the intersection between these two sets are nothing, but the null set.

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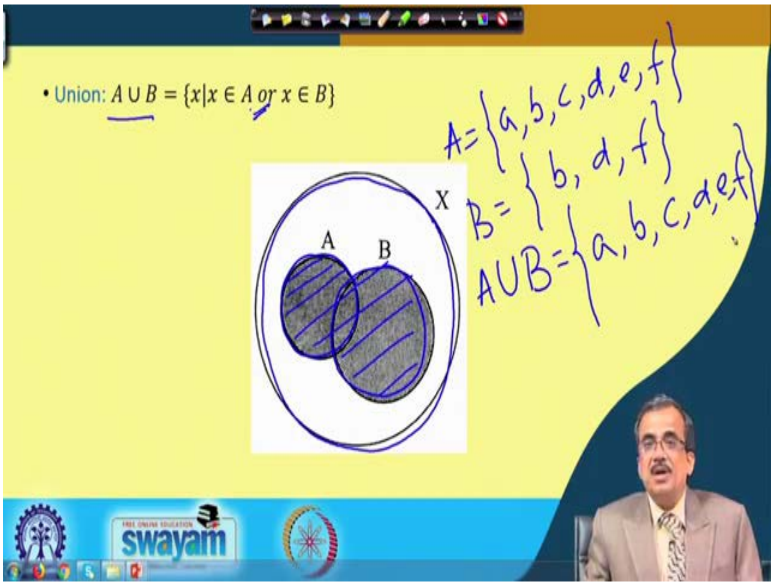

Now, these are all fundamentals of the classical set and all of us we know, and as I told this is some sort of recapitulation. Now, then comes the concept of the union. So, once again, I have got the universal set that is your capital X and I have got two sets, two crisp sets: one is called the A and another is nothing, but B, and I am trying to find out the union, that is, $A \cup B = \{x \mid x \in A or x \in B\}$; that means, we consider the maximum area, and this is $A \cup B$.

So, this particular shaded portion or the black portion will be your $A \cup B$. Now, once again, let me concentrate on the same example like A is nothing, but your a, b, c, d, e, f and B is another crisp set, which is nothing, but is your b, d, f. Now, if I try to find out what should be $A \cup B$ it is very simple. So, we consider the maximum; that means, your $A \cup B$ will be nothing, but a, b, c, d, e, f. So, this is nothing, but is your $A \cup B$.

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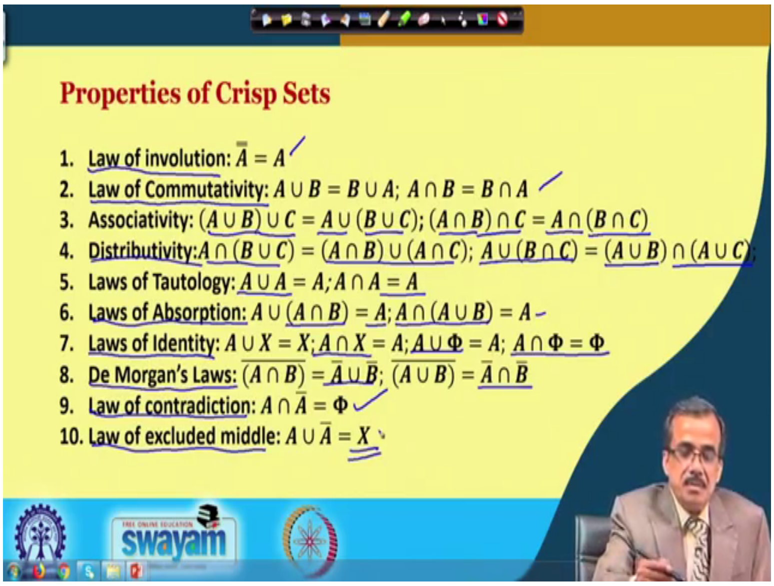

So, this is the way actually, we can define your the union of two crisp sets. Now, I am just going to concentrate on the properties of the crisp set and these are very important because, the crisp set follows all ten laws. Now, I am just going to state the laws one after another and as I told this you people have already studied. So, this is some sort of recapitulation, the first law, now this is known as law of involution. So, it states that the complement of a complement of a crisp set is nothing, but the original set, ok. So, this is nothing, but law of involution. The next is the law of commutativity, now it is states that $A \cup B = B \cup A; A \cap B = B \cap A$.

So, this is nothing, but law of commutativity, next comes your law of associativity. It states that $(A \cup B) \cup C = A \cup (B \cup C); (A \cap B) \cap C = A \cap (B \cap C)$. Now, then comes your law of distributivity, it states that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C); A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$.

Next is your law of tautology, that is $A \cup A = A; A \cap A = A$. Next, come laws of absorption, which states that $A \cup (A \cap B) = A; A \cap (A \cup B) = A$, then comes laws of identity $A \cup X = X; A \cap X = A; A \cup 0 = A; A \cap 0 = 0$, now then comes very famous De Morgan's laws.

Now, it states that A intersection B complement of that is nothing, but complement of A union complement of B, another statement A union B complement is equal to A complement intersection B complement. So, these are very famous, as I told, De Morgan's laws. The next is your law of contradiction. Now, it states that $A \cap \bar{A} = 0$ and the last law that is known as law of excluded middle and it states that $A \cup \bar{A} = X$. Now, here actually all such ten rules are followed by the crisp set or the classical set.

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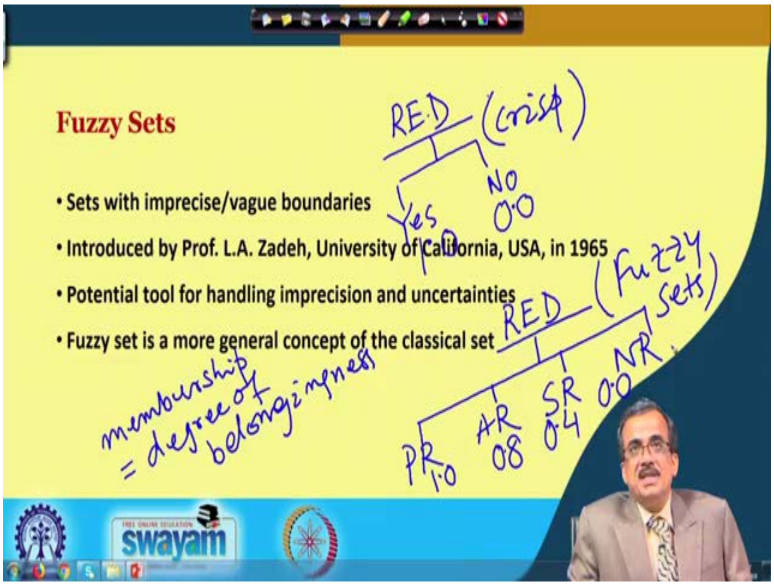

Now, then comes the concept of fuzzy set, now before I start with the concept of fuzzy set, let me try to spend some time just to understand why do we need the concept of this particular the fuzzy sets. Now, if you see the real-world problems, real-world problems are very complex very difficult and these are associated with some sort of imprecisions and uncertainties.

Now, prior to the year 1965, people used to believe that it is the probability theory, which can tackle the different types of uncertainties in this world, but in the year 1965, Professor L.A. Zadeh of the University of California, USA, told that there are many uncertainties in this particular world and the probability theory can handle only one out of different uncertainties. And, there are a few uncertainties, which cannot be tackled using the principle of probability theory, which works based on the classical set or the crisp set and Professor Zadeh argued that we need something, which is at higher level compared to the classical set, if we want to represent the different types of uncertainties or the imprecision, which we face in real-world problems.

Now, let me take a very simple example, very practical example, now supposing that one of your friends is going to market and you are requesting him please bring 1 kg red apple for me and supposing that your friend has brought that particular apple from the market and there is a probability associated with the availability of apple and it depends on the season. Now, there is a an uncertainty of regarding the availability of the apple and this particular uncertainty can be handled using the principle of probability theory and the probability of getting apple varies from say 0 to 1 supposing that it is 0.6.

So, with the probability of 0.6, your friend has got some apples. Now, my next query, what is the guarantee that this particular apple is red, how can you define the colour red and the definition of this particular colour red will vary from person to person. Now, the problem is how to tackle this particular uncertainty regarding the guarantee that the colour of the apple is red. So, this particular uncertainty cannot be answered using the probability theory or using the crisp set and to answer that, we need to have another set and that is nothing, but is the fuzzy sets.

Now, the same example which I took let me just write it here. Now, the colour of this particular apple is red, now according to this classical set or the crisp set, there are only two possibilities, one is your the apple will be red or it will not be red. So, there are two answers, one is yes and another is no. So, if the apple is found to be red, it characteristic value is 1.0 and if it is not, its characteristic value is 0.0; in the classical set or crisp set.

So, there are only two answers, one is the member, another is non-member; that means, 1.0 and or 0.0, but the same colour red will be defined in a slightly different way in fuzzy sets. For example, the colour red can be perfectly read and the same colour, some other person may say this is almost red AR, some other person will say this is slightly red and there could be a possibility that it may not be called read also, ok. Now, if it is perfectly red, then we declare it is read with membership function value.

So, this defines a term that is called the membership function value and that is nothing, but the degree of belongingness, the degree of belongingness and that is nothing, but the similarity of an element to a particular the class. Now, this particular membership function value will vary from 0 to 1, now as I told, if it is perfectly red, then we say that it is red with membership function value 1.0. And, if it is almost red, then also it is called red with some membership function value 0.8, if it is slightly red then also it is called as a red with membership function value 0.4 and if it is not red, then also it is red with membership function value 0.0.

So, this is the way actually we define the colour red in the fuzzy sets, and now you can see that this is actually a very practical way of representing this type of uncertainties, and that is why actually this fuzzy set has gained popularity.

Thank you.