

Multi-Criteria Decision Making and Applications
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Lecture 08

Welcome back my dear students, a very good morning, good afternoon, good evening to all of you. And as you know this is the course on the MOOC with the title and the main idea being Multicriteria Decision Making and Applications and I am Raghunandan Sengupta from the IME Department at IIT Kanpur in India. So, this is the eighth lecture under the MOOC MCDM concepts and ideas course and this is the third lecture for the second week. What would be the coverage? Coverage would be we will try to finish the MCDM axioms, which we are discussing preference theory, choice theory, different axioms four of them and I did mention in decent clarity about the relevance of them. Then we will come to the Condorcet paradox with an example and if as the time permits, we will start the utility theory, if not we will start the utility theory in the ninth lecture. So, considering the choice theory a very simple example, let us consider as an example Mr. Zomo Nibarde wants to buy microwave oven and a water heater, which cost in Kenyan shilling 11000 and 60000 respectively. So, Mr. Zomo stays in Mombasa in Kenya and for just for information for people who are interested one Kenyan shilling would be 116 Kenyan shilling would be 1 USD, roughly or 1 Euro would be about 122 Kenyan shilling. Mr. Zomo wants to visit the nearest departmental store, let us consider it as shop number 1 or store number1 near his house in Mombasa, which I said is where he stays, and he wants to buy both the white good items, which is microwave oven and the water heater. Mr. Zomo visits shop number 1 and faces three different choices and situations. What are these? Choice 1, as the water heater is not available in the store 1, which I mentioned, he can get it at shop or store number 2, which is about 15 minutes away from the store number1, a little bit far. And there the water heater costs about 57000 Kenyan shilling and in the first place in the shop 1 it was 60000 and that store is basically as I mentioned is store number 2 and is 15 minutes away. But would he made a trip? The answer is obviously. It will may become apparent that 60000 is more than 50000 that is true, but that 15 minutes extra drive may cost time, may cost fuel, may take a long time because Mr. Zomo is busy with his work. So, whether he will make that choice to visit is a question.

Choice number 2 is similarly, the microwave oven he is interested in, is available at not 11000 Kenyan shilling, but at 8000 Kenyan shilling and a at third store shop number 3, which is approximately 15 meters away, but in the opposite direction. So, would he make the trip? So, shop number 2 from 1 or 3 from 1 they are 15 minutes, but they are in two different directions. Traffic may be different, time taken may be different, time taken for billing, for choosing, may be different. Would he make the trip. In choice 1 and choice 2 we have only considered one difference of the prices; number 1, for choice 1 and choice

2; choice 1 with which is basically store 1 and store 2 was only the difference in the water heater price, which was 60000 versus 57000, 60000 in store 1, 57 in store 2. When in choice 2 it is the difference between store 1 and store 3 where we are only considering the water heater prices, which are 11000 in store 1 and 8000 in store 3. So, considering these three options would he go to store 1 or store 2 or store 3 or a combination of that. So, that would depend on Mr. Zomo and choice 3 can be that both the items are not available in either store 2 or store 3 he has to go to store 1. So, would he go there.

So, these type of decisions would be coming under the choice 3 depending on the price, price is a factor here, but they can be other factors also: time yes, traffic jam yes, whether the quality is desirable at both store 1, 2, 3 of the same level, yes. So, they can be different ways of trying to analyze the problem. We will consider the concept of Condorcet paradox with example first the statement. Under this paradox collective preference can be cyclic collective by a group they are making a decision.

So, till now we have considered decision maker is only one, but now we will basically expand that for a group and we will see that. So, collective preference can be cyclic even if the preference of individual decision maker are not cyclic, straight forward and there is no ambiguity, but we will see as we collectively bring the decision makers into the picture it becomes a cyclic one, and there the paradox basically appears. Consider this as simple example suppose that in the final round of an international beauty pageant show we have four contestants from four different countries and they are from Miss India or from India the second lady is Miss Germany third one is Miss China and fourth one is Miss Cameroon who will be judged by four former beauty pageant winners who are namely Miss Brazil, Miss Poland, Miss Japan and Miss Sweden. When these judges who are from, I will only mention the countries, Brazil, Poland, Japan, Sweden, when they do the ranking, we have the following table of preferences and it is given here as you can see and I will mark it. So, for the judge from Brazil her preference one, two, three, four in the ranking order, the lady from Germany is first, lady from China second, Cameroon third, Miss India fourth. When it comes to the judge from Poland her ranking is, I will just read the names from one, two, three, four: is China, Cameroon, India, Germany. For the judge from Japan, her ranking is India, Germany, China, Cameroon and from Sweden it is Cameroon, India, Germany, China. Now, see individually for Brazil it is very clear cut; Poland clear cut, Japan clear cut, Sweden clear cut individually. So, if I consider the third one Japan and if there is only one decision maker, India would be first, the German lady would be second, China would be third, Cameroon would be fourth. But if I combine them, interestingly see it. I only consider, say for example, Germany, the lady from Germany, who is being judged, not the judge, but who is being judged. The lady from Germany, I will highlight in the first, by the judge from Brazil she is first, by the judge from Poland she is fourth, judge from Japan, the German lady is second and by the judge from Sweden she is third. Now, let us see that example for China. I will only highlight it, basically the judges are in

this order: Brazil, Poland, Japan, Sweden. I would not need the name of the judges, I will only mention the position for the corresponding contestant. Again to start with, for China first, then the second position, then first position, then third position, and then fourth position. If I check for India, fourth, third, first, second and finally, for the contestant from Cameroon; third, second; I should use a different color. Sorry for that. Third, second, fourth, first, and if you see that the ranking for all this four contents; all of them are in one of this positions either one two three four. So, combined if I combine all the four judges all of them are now on the equal standing. Individually for the judges it is clear cut, no ambiguity. Collectively there is ambiguity. So, how do we remove it?

A quick look at the preference gives rise to a paradoxical situation where a collective preference scheme does not give us any clear cut winner. It is possible to estimate the probability of the paradox by using cardinal scoring system. This scoring system we will consider or mathematical models to quantify the decision maker, in this case they are the judge or the set of judges, which you have, their preference, though the results depend on which model is being used. An important implication of the possible existence of this paradox is a practical situation in the fact that in two stage ranking process the eventual winner may depend on the way the two stage has been done and this is all basically known as the voting paradox also.

If you see in many of the countries there are stage-wise voting: stage one and stage two. So, stage one winners are taken to stage two depending on the cut-off. If the losers are less than cut-off, they are taken out and then it proceeds accordingly. Now assume each judge asks the four finalists the same set of three questions. So, which are: question one is, and there are points for questions. Consider the points, if I change, if you see in the example, you can have different ranking of the finalists. We are following, first the questions and their points which are given. As you change the points, again I am mentioning, the ranking may change. Question one is according, and they ask the finalists that according to you how can poverty be tackled and the four contestants answer, and the points assigned is six. Question two is, as a brand ambassador of change, how can you motivate the young generation. So, that is the same question asked to all the four finalists by the four judges separately. The points is ten and each judge gives each contestant a point out of ten. Similarly, for question one each judge gives point out of six for the four finalists.

Question three again how what do you think country should do to accomplish equality among its citizens that is third question and point is four and each judge again gives points individually, each judge gives points out of four for these four finalists. And now the scoring system has been done. This is the scoring which I have done hypothetically. As you change the scores, you will see the ranking will change. But here with the points now, it becomes much more clear cut how to do the ranking. According to Brazil, if you see, if

you go back to the slide, so according to the judge from Brazil, the ranking was Germany, China, Cameroon, India. So, it is basically you see in this slide Germany, China, Cameroon, India for all other three judges also the names are given as it was in the ranking system, but now the points are being analyzed. By the judge one which is Brazil Germany, China, Cameroon, India the finalists and the points based on the question one two three are and if you remember the points the judge gives 483 to German lady, 463 to the Chinese lady, 462 to the Cameroon lady and 353 to the Indian lady and if you see these are the points given and very interestingly if you add up $8 + 4$ is 12, $12 + 3$ is 15.

If you add up here China is 13, Cameroon is 12 and India is 11. So, here also 15, 13, 12, 11 they go in decreasing order. So, obviously the Brazilian lady's concept of giving scores on these three points are tenable, logical and hence she had made that judgment of giving the scores to the participants or the four finalists. When I come to the second judge the ranking as it was given in the table one is China, Cameroon, India, Germany and the scores are given if you see for the three questions $7 + 4$ 11, $11 + 3$ 14, 13 for Cameroon finalists India is 12 and Germany is 11 again 14, 13, 12, 11. So, in the first for the first lady judge it was 15, 13, 12, 11 here it is 14, 13, 12, 11 there is no sacrosanct scoring here this is based on the given. For the third judge it is 20 for India, 17 for Germany, 16 for China, 15 for Cameroon and for the fourth judge it is 18 for the first ranking given by Swedish judge, Cameroon which is the finalist, this is 17 for India, 16 for Germany and the Chinese finalist gets 14. So, if you see, there is, for each judge, no ambiguity. 18, 17, 16, 14 goes in decreasing order, perfect. 20, 17, 16, 15 perfect. 14, 13, 12, 11 perfect. 15, 13, 12, 11 perfect.

Now, if I want to basically combine the scores what I will do? From the four judges, the main, the collection center where they collect the scores and then mark. For the German lady, it will be the score of 15, then 11. They are being added up. Simple addition. 17 and 16. Similarly, if for the Chinese finalist one wants to add up all the scores from the four judges, it is $13 + 14 + 16 + 14$. Similarly, for India and Cameroon you can do the scoring by adding up the scores.

Based on that, if I have the final scores, using the information gives out the four finalist scores. So, India gets 60 German finalist gets 59, China 57, Cameroon 58. So, let us see for Germany for simple example. So, it was basically $15 + 11 + 17 + 16$, 15, 11, 17, 16, 15, 11, 17 and the last one was 16.

So $5 + 16$, $6 + 12$, $12 + 7$ is 19, 9 are 1. So, 2 3 4 5 so 59 and this is the score 59 which we have. Similarly, if you add up for India it will be 60 for China 57 58. Now, there is no ambiguity. If you I want to rank by the scores the final ranking for the beauty pageant shows can be inferred which is miss India is the first place winner, second one is basically the lady from Germany, third one is from Cameroon and the fourth one is basically from

China. So, it can be inferred that Miss India is the winner, Germany is second, the first runner-up, Cameroon is the third spot and the fourth position goes to Miss China. This example clearly demonstrates that some sort of scoring system for the questions enables us to eliminate this paradox though it is not certain whether the collective preferences ranking system will be sacrosanct for different scoring systems as allowed. Now, the question is that would the scoring system if given for this question: 1, 2, 3 result in different answers, the answer is yes. We can formulate different problems with the same set of questions, questions 1, 2, and 3 with different sets of points, but it will give a different ranking. So, based on some ideas or more detailed analysis we will try to consider very simple problems in this respect. Now, with this I will start a big portion which will be considered for this MCDM is basically the concept of utility theory.

The word utility the noun Latin being *utilitas* or French being *utilite* denotes the usefulness of something specially in a practical way. In economics it implies worth or value or the preference which are expressed using the concept of utility function; it is a function. And in our case we will consider the utility function is mapped into the real line; one dimension. So, obviously the question would be, can it be in two dimensions? Yes it can be, but we will simply keep it to the logical level where you can basically have the real line mark or give you the value of utility. Because if you go back the axioms of 1, 2, 3, 4 we said, would be become much more apparent if I consider this simple assumption to be true, where X would denote the different alternatives which are available and we later see that how X can be basically quantified using the concept of wealth also, money. The utility function that represents the preference relationship is not unique, it can change, and is only the ranking system of the alternatives which will be important based on the utility system. Like say for example, relative marking versus absolute marking. There can be two different ways of trying to give the marks, but the overall idea of ranking system would should prevail. It is also important to note that if there is a utility function that represents $>$, then $>$ must be complete and transitive based on which your unique system or ranking would be true. So, the first two axioms if A decision is better it can be better than B , $A = B$ or $A < B$ and in a other case if $A > B$ and $B > C$ then obviously $A > C$.

And we will try to remove this ambiguity later on. It is apparent that for ambiguity in the sense that if in this case. in the second axiom, which I am now marking with the star. So, if $A > B$ and $B > C$, it may turn out, based on the decision maker that A is not $> C$, A is worse than C . So, these type of confusions can be removed. It is apparent that for any discussion there are uncertainties involved with the alternative choices which leads us to the concept of risks and the word risk can have different notations different connotations.

We will see later risk can be analyzed as standard deviation, variance, concept of beta, conditional value at risk, expected regret all all these things which are very much used in

finance. They can be other concepts also. If considering the rational choice theory, if rational choice is true then can the individual utility functions preference be analyzed. The other side of the question is also relevant what which is if we can recover the individuals utility functions can we model the rational choice theory by simply adding the utility functions. So, both ways utility function given can the this if and if rational theory is true can individuals be able to model the utility theory be analyzed and if utility theory is there can we go back and try to basically understand the concept of rational choice theory and use some mathematical implications to add up or find out the collective utility theory.

A simplistic assumption can be stated here as follows. In the standard view point rational choice as I mentioned, is defined to mean the process of determining what options are available in front of the decision maker. Then choosing the most preferred one, according to some consistent criteria or consistent set of rules. So, that rule cannot be changed in the sense if the decision maker is making a decision based on say for example, combination of price, safety features of the car that being constant then it has to be defined as a determining process how well the decisions process is made. In a certain sense this rational choice model is already an optimization based approach.

So, you want to optimize maximize minimize based on some criteria. We will find out that by adding one empirical and restrictive assumption the problem or rational choice can be represented as one of maximizing a real valued utility function which we will see later on. Let us consider a very simple example and there can be different answers for that even though we will consider few of the rule based concept based on which ranking would be made. Here we are only considering two sets of decisions. It can be expanded as I said, that number of alternatives can be 1 to M and number of criteria can be 1 to N. So, those would come up later on.

So, here example is that the different decision maker have different attributes or characteristics which is a basically an opportunity set which is front of him or her and opportunity set we are going to consider it is finite not infinite. The decision maker can love risk what we mean by risk I mentioned that quantitatively, but it has a different notions and different concept how we can analyze risk. The decision maker can love risk the first point the decision maker can be averse to risk and the decision maker can be neutral to risk. How they are analyzed mathematically using utility functions or the first derivative second derivative we will come to that. In this example I am simply going to consider two different decisions and they are as given.

In decision one you have three outcomes and what are the outcomes I am not going to I am going to only mention the quantitative value nothing to do with the units and all these things will come later on. The outcomes are 15, 10, 15 probabilities are $1/3$, $1/3$, $1/3$. For the second one it is 20, 12, 8 the outcomes values probabilities are also one-third, one-

third, one-third. Only one important thing you may ask that why I have written 15 and 15 differently and not clubbed them. The outcomes are based on some initial decision which is taken.

So, the initial decisions can be different, but the outcomes are 15, 15. So, basically consider that you toss a coin or roll a dice and the value on the head, the value which comes when you roll the dice, is say for example 2 and another case the value comes out to be 6. So, obviously the probabilities in both these cases is $1/6$, $1/6$, but the actual outcome based on the rolling that the dice which is 2 and rolling the dice which is 6 gives you the same output which is what is given here as 15, 15. So, the initial game which is being played here considering this game are rolling the dice with the values coming out to be 2 or 6 with probabilities $1/6$, $1/6$ and the final outcome which you see on the other side are same. Here it is 15, 15 with probabilities of $1/3$, $1/3$. Now, if I consider the situation has changed in the sense that the outcomes are again 15, 15, 15, but the probabilities are half, one-fourth, one-fourth for the second one it remains the same.

What happens for this case when for the second example here if I want to find out very simply the overall average value of output for decision A and B. I just simply multiply the outcome with this corresponding values probabilities and for decision A if I am using the expected value here which is the average value and W I am using the concept of wealth for the first time. So, it will become relevant later on.

So, it is $(15 \times \frac{1}{2} + 10 \times \frac{1}{4} + 15 \times \frac{1}{4})$ gives me 13.75 and if I find out for B it is $(20 \times \frac{1}{3} + 12 \times \frac{1}{3} + 8 \times \frac{1}{3})$ comes out to be 13.33. So, obviously, if I am looking the idea is not sacrosanct is not ultimate. So, if I look at the idea then obviously 13.75 is more than 13.33 and I will basically take decision A for our case. But what would happen for the previous example if, let us go back. If I find out the previous one the answer would be for A which I have not done I am going to do it that will bring out the stark reality into the picture.

For the first case the expected value would be 15 + because I am trying to multiply the outcome with the probability, probability being fixed I can take it outside. So, it will be $15 + 15 = 30$, $30 + 10 = 40$. So, the probability is $40/3$ for A and I will use a different colour for B. So, it is 20, where $12 + 8 = 20$, $20, 20$ is $40/3$. So, in this case very interestingly for the first example there was equality because the probabilities in this case was same and obviously, the outcomes were different. So, that does not matter much, but in the same example with some changed values you see immediately decision A is preferred with decision B. You can basically have a different probabilities and values also where decision B would be preferred then decision A. So, with this I will end this lecture which is the 8th one and continue for the analysis for the utility which should cover about the second week and the third week and I am sure the

ideas as we proceed will become clear to you. Thank you very much and have a nice day.
Thank you.