Multi-Criteria Decision Making and Applications
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Week 01
Lecture 06

A very good morning, good afternoon and good evening to all of you, all the participants and the students for this MOOC course, which is the theory and applications of Multicriteria Decision Making and in under the MCDM, this is the second week of lecture which will start. So, in the first week we have covered all the different above examples, they are not exhaustive, but some of the examples which people can encounter in multi criteria decision making, some are quantitative, some are qualitative, we will handle them as we proceed with the course. So, in this sixth lecture and consecutive in this week, we will consider some definitions of MCDM in more details. We will consider the concepts of utility theory, what is the concept of safety first principle and also understand the concept of stochastic dominance. The coverage for this sixth lecture which is the first lecture in the second week would be about the definitions of MCDM, the simple concept of theory of choice, what are the different axioms of MCDM. We will consider them in more theory and then later on as we proceed examples will highlight that and we will also consider the Condorcet paradox in the corresponding examples for that.

So, under MCDM there are four main definitions obviously there are many more quantitatively, but in the quality sense we will consider four main definitions. So first would be the goals. So what we mean by goal is basically an aim or a purpose like with synonyms like intentions, ambitions, aspirations, hope and dream or the intent formal or specialized objective or the target which the decision maker wants to achieve. So goal maybe I want to own a house, goal maybe I want to buy the car which has this horsepower, this is the price or this is the make of the car that can be the goal.

Another goal can be maybe I want to get admission in the top notch engineering colleges or top notch management colleges or top notch social science colleges in India or abroad. So that may be the goal and these goals are fully identifiable with the decision maker and his or her desires and it is decided beforehand by the decision maker such that it is unambiguous to him or her. So when I want to achieve something I have a goal, so that should be absolutely clear for me if I am considering myself as the decision maker. The second one is basically the criteria. So it is defined as a standard or principle by which something is judged.

So when I was discussing about buying the house it can be I want the house to be in this

locality, some locality in the city where I want to buy that house or I want to rent that house. So other criteria can be what is the safety features of that house, whether it is in a good locality, whether it is a gated community, other can be whether my friends relatives are nearby. So other criteria can be how far is the school, college, the shopping center, the bazaar, the hospital or whether they are recreation facilities, good park, good malls where we can visit or the family of the decision maker can visit to spend time. So as I was mentioning is defined as standard or principle by which something is judged or with the help of which a decision is made. So criteria need not be only one, it can be set of criteria.

So considering the buying the car, so I can have different set of companies cars in front of me, it can be a Volkswagen, it can be a Maruti, it can be Hyundai anything. So for the set of criteria can be what is the price, what is the maintenance cost, what is the mileage of the car, what is the safety features of the car, what is the color of the car and how stable the car is. So they can be different ways of trying to analyze when you are buying a car. Similarly, for the third example which was discussing about choosing a college, so if somebody wants to join a management institute in India, so obviously the reputation would matter, the quality of education would matter, what are the opportunities of summer internships would matter, what are the courses which are offered would matter because somebody wants to do a major in marketing, major in finance, major in HR, major in operation management, major in MIS. So obviously the course structure should be akin to the desires or the wants what the student wants to achieve.

A criteria can also be considered as a measure, a rule or a standard based on which the decision maker is guided so that he or she is able to utilize a criteria or a set of criteria to make relevant judgment about the particular decision such that he or she is able to achieve the goal. Now the goal is basically to buy a car as I said, so under that they can be different brand of cars which I mentioned, so that would basically the car would not come as a goal, buying a car would be but which car you want to buy would not be the goal, basically that would be different concepts of alternatives which I will come to that very soon. Moving to the third important point under MCDM would be attributes, characteristics is the quality of feature regarded as a character or an inherent part of someone or something. They are generally subjective in nature and specific to the decision maker as to how he or she is assigning the characteristics with respect to the particular situation, think he or she is analyzing. I am analyzing to buy a car. So the attributes which I mentioned actually would be more qualitative, safety features for a car would be more qualitative for me or colour of the car would be more qualitative for me or if I consider say for example buying a house, so the locality maybe it is quite close by to my office or to the school but the safety features if they are, safety means what is the crime rate, how well the gated community is that may be more qualitative in nature when

I make the decision or when the decision maker makes the decision.

Few synonyms for attributes are quality, features, trade, property, characteristics and so on and so forth. So obviously if you remember in the first week of examples they were both qualitative as well as quantitative, qualitative like the first example painting the house in some green colour or the second example would be somebody is making a decision to study as the example we considered in the first week and there the characteristics of how far the college is from the country where the lady stays in Poland or what are the loan facilities, what are the different type of coursework would be more attribute based, characteristic based. While if I consider the concept of mileage, price of the car then the maintenance cost they are quantitative in nature or for the example of studying, so what is the cost per year of studying, staying there in that other foreign country, so those would be quantitative in nature. Having considered the goal the objective is closely identifiable with the decision maker, it is a thing aimed or at or sought for in other words is basically what the person wants to buy. So goal would be basically to buy the car then objective would be classified accordingly based on which he or she can take the decision, this is the decision maker.

So generally as we proceed we will consider that the decision maker has in front of him or her different attributes, characteristics or criteria based on that he or she wants to achieve and get some alternatives. So there are different sets of alternatives in front of him or her and once the decision is made based on the criteria the person will try to take the best alternative which meets his or her actual demand. So demand as I said the liking or what the characteristics can both be quantitative as well as qualitative and based on that the goal is achieved. So according to Maslow the hierarchy of needs, so this was stated by Abraham Maslow, a human beings motivation, decision, objective can be analyzed by his or her innate characteristics to improve his or her level of existence by reaching a higher level of existence provided the lower level of satisfaction are fulfilled or met. So obviously you go from stage by stage from the lower one to the next higher level and so on and so forth and based on that the person will analyze what the goal is, what are the characteristics, what are the attributes and accordingly the goal.

So if for a person who has say for example a yearly salary of 3 lakhs with respect to a person who has a yearly salary of say for example 15 lakhs, so obviously the requirements and the goals would be different. Requirements and the needs for the attributes would be different, requirements and characteristics would be different like the alternatives would be different. Alternative for the second person who is earning 15 lakhs would be rent a house or buy a house. The person who is only earning 3 lakhs it will be more of sustenance how to survive. Based on that the problem can be analyzed and decision can be made accordingly.

According to Maslow, the levels are physiological, safety, belonging and love, social need or esteem, self-actualization and transitance and can be used to describe the patterns through which human needs and motivations generally move. So they move from level 1 to level 2, level 2 to level 3 that means they are going up the hierarchy and this concept of hierarchy the word which I am mentioning would be used in one of the problems when we solve and consider the simple nonparametric methods of MCDM of multiple criteria decision making. Of course the question which may come whether there is some relationship between Maslow's hierarchy of needs and the ideas of MCDM is a question. Self-actualization and realization is the ultimate need or aspiration any decision maker wants to achieve. However, in reality there is no apparent hierarchy of human needs as they pursue a wide variety of wants and goals, drives and desires all simultaneously depending on their own predilections and choices.

So the person which I mentioned 15 lakhs a year the person can have the motivation or the need to buy a house, to buy a car, to get his or her son and daughter's education being the best. So they can be many things at different times or the other one can be he or she wants to basically invest such that long term financial requirements are made. So they can be different ones and how he or she the person manages these goals, these motivations would be many of them may be conflicting. I will come to that example of conflicting in a way in the simple sense now. Say for example you have the person 15 lakhs per year he has say for example disposable income of 2 lakhs per year. So how it is that to be invested either in stocks or trying to buy a house or try to buy a car or put that into the savings account or recurring deposit account for the children's education. So those would be decided accordingly to the decision maker.

The decision maker concurrently wants to achieve a complex set of interacting needs because they are interacting. So if the total amount of money is fixed so how much to spend in under which category has to be decided by the decision maker. So let me continue reading it. The decision maker concordant wants to achieve a complex set of interacting needs, desires which can be dependent, involved, inconsistent and not complementary in all its components. So considering the buying of the car for me the color of the car matters too much or the style of the car matters too much. For other persons it can be other way round safety feature matters, maintenance cost matters or say for example for the other case children's education somebody wants to go into a good government college because the costs are low but somebody wants to send his or her son to a private college where he or she thinks the education would be better. So that better or worse would depend on the decision maker. There is no ultimate best solution for all: that is not possible.

In view of this the decision maker's task of deciding can be classified based on the ideas

of computation. How you compute? What are the weights you give? What are the priorities you assign? The judgmental values, what are the social background of the person? What is his or her predilection? What he or she wants to achieve? The physiological, psychological, societal needs that has to be analyzed. They would be compromised and intuitively this be decided by the decision maker in trying to achieve the goal based on different alternatives and their subsequent criteria under each alternatives. Now whenever I am discussing all these things I am only considering one decision maker in the picture. So obviously they can be examples when they are more than two decision makers.

So say for example for buying the car, so in the family the husband and the wife both decide so obviously two people would have two different ways of analyzing how to achieve the goal. The husband may want to basically reduce the cost and the wife may basically want to buy a car where the safety features are high. So obviously that may increase the cost or consider the example that when a company is hiring a senior position, so obviously in the interview process there are in the panelist who want to conduct the interview they may be more than two persons or more than three persons. So the person who is being interviewed has to be analyzed and the people who are trying to conduct the interview they if all of them are individually considered as a decision maker they would have different ways of trying to analyze the person's credentials. Somebody may be more interested in the interpersonal skill the person has, person means who is being interviewed. The other person who is taking the interview may be more interested in the technical knowledge, somebody may be more interested in what is the financial knowledge the person has. So they can be different ways of trying to analyze the same problem of recruitment by different decision makers. For the time being whenever I am discussing all these things is based on only one person and that can be expanded where there are more than one persons and how collectively the decision can be made by considering all the decision makers in a group would also be analyzed later on. To emphasize the role of multi-objective process in economic sciences let us consider what Milton Friedman, the winner or Nobel Memorial Prize in economic sciences in 1976 had to say. According to professor Milton Friedman. an economic problem exists whenever scare means are used to satisfy alternative ends.

So the overall quantity of goods overall quantity of amount of money of raw materials is limited it is not infinite. In that case if the means are not limited, they are abundant, there is no problem at all. So there is nirvana, everything is achieved. The means are not abundant but there is only in a single end, the problem is how to use that means is basically a technological challenge or a problem. So amount of steel is limited for a company which wants to manufacture car, amount of working hours per day is limited, amount of utilization of the factory machine is limited it cannot be infinite in all the

cases number of workers who are working the factory is limited.

So in that sense what I mean is the availability of resources is not infinite in all the cases. No value judgments enter into the solution and only knowledge of physical and technical relationship are analyzed in order to arrive at the decision. Let us explain the concept with an aid of an example and this example as you can see in the slide is taken from the multi criteria decision making by Milan Zelani which is an old book published by McGraw Hill in 1982. It is a quite a classic book but it analyzes the ideas of Multicriteria Decision Making, more from a psychological, physiological and societal perspective. Obviously it gives different interesting quantitative example but the analysis have been done very systematically.

Suppose the following set of inputs are there which is raw material, manpower, technical knowledge, energy, machineries etc. and are used by different manufacturers to produce cars. So machineries are there to manufacture, energy is required, electricity, water, technical know-how of the experts are needed, manpower mean technical people who are working the shop floor and raw materials can be of different types. For the ease of explanation let us consider, I will come to the diagram very soon, but first let me explain it. For ease of explanation let us consider the inputs as labor which is given in the y-axis this is a 2-D Cartesian Coordinate Representation and the raw materials are given in the x-axis.

I am only considering two of the inputs. So obviously the question may be can we consider more than two materials yes in the 3D case yes, and higher dimension it will be difficult for us to visualize. Other way can be to club the inputs in different ways depending on the units but all this concept of clubbing the inputs considering different non-commensurable units, all these things we will consider later on. So now this diagram the curve LM we will come to that depicts the production possibility frontier. So what is the possibility of production which can be achieved considering labor and raw materials only.

So obviously the production frontier here is being given in a 2D space and as I said few minutes few seconds back in the 3D space, it would be analyzed accordingly. Then the maximum availability of raw materials is denoted by the point M while for raw materials while for labor it is L. Now if we considered a decision to buy a car, then the customer will judge buying the best car and the present level of technology production possibility frontier based on a whole lot of economic factors which can be what is the engine power of the car, what is the safety features, what is the speed, what is the price, maintenance cost, what is the mileage of the car and so on and so forth. So there are only two important things I am considering labor and raw materials, Based on that different type

of factors are being considered in order to make the decision to buy the car. These are the multiple economic criteria, multiple economic criteria means this concept of engine power, safety features, speed, price, maintenance cost, mileage what I mentioned.

Multiple economic criteria as I mentioned are utilized based on which different cars will be judged. So in front of you that if you are making the decision there are different make of cars, different companies, different SUVs and they would have different levels of economic factors to be assigned to each car. Power for one car may be more than the second one like Volkswagen would be more than a Maruti but the price of the Volkswagen may be more. Safety features for a Volkswagen may be more than Hyundai and so on and so forth. So different cars will be judged rather than technology at the production frontier is already fixed, you cannot move shift the production possibility frontier and what I mean by the production possibility frontier which I mentioned LM we will consider that in the next slide.

So before coming there to the slide, for ease of understanding, assume families of two different in different curves. There are one for power and power I have denoted by I 1, I 2, I 3 while for other safety features I have denoted by J 1, J 2, J 3. So let me come to the diagram and then switch back to the slide again. If you consider the red line which you have, so it is already colored I would not be drawing much here because it is quite visible. So if I consider this is A to B what I considered what was the production possibility frontier and if you consider this whole, A to B is the optimum one but if I consider the whole set of points starting from L here, which is along the y-axis and M, which is there on the x-axis, this is the whole curve and if you remember I did mention point L is best for labor, point M is best for raw materials, and the different sets for the technology and the concept which I mentioned is power and safety and power. So those curves are given, I will just mark them one by one J 1, J 2, J 3, and these green lines which were indifference curves are all concentric to each other.

Similarly, I have drawn it, but they can be different variations of the drawing, but the ideas would be same. Similarly if I consider I1, I am putting a double tick to I1, double tick to I2, double tick to I3, which are the blue ones are again concentric. Now if you see, if I am moving the J curve 1, 2, 3 accordingly, it moves like this. So I will use a different color because it is green so it will make more sense. So I am moving this it goes on, moves parallel and similarly if I consider the I set of curves, they are also moving parallel.

And very interestingly at point A and at point B the blue and the green line intersect and are so called tangent just at that point like two circles touching at the circumference, that set of point which is A and B a part of this LM line would be the best set of combinations

for the raw material and labor considering the level of I and J the person wants to achieve. So obviously the production possibility frontier concept which I have discussed would be true for only one person. So if say for example decision maker is different then considering the level of I's and J's the overall diagram may be different but the concept would remain the same. So coming back to this slide for ease of understanding assume families of two different indifference curves, one for power given by I's and one for safety given by J's are considered. The point A and B, which I mentioned the endpoints of the red line correspond respectively to the maximum level of power and safety achievable for the car under the given conditions. So what the decision maker wants to buy.

The arc AB, which is the red one, represents the combination of power and safety, these are the two only points which I am considering which may be achieved if the decision maker is looking at a compromised solution at the same technology level which is the production possibility frontier. So A and B, at any point in the line A and B, the overall so called collective benefit the person would have would be same. Collective benefit I am using as basically a bundle of the benefits which the person is giving getting based on power and safety and based on the production frontier technology as it is. So A would give the same level of so-called satisfaction. I am using the word satisfaction in a very loose term. Similarly point B will give the same level of satisfaction but the levels of individual power and safety would be different. In microeconomics a production possibility frontier or production possibility curve or production possibility boundary which I have drawn is basically a graphical representation showing all the possible options of output for two goods.

Here there are two goods which we are considering that can be produced using all factors of production where the given resources are fully and efficiently utilized per unit time. So obviously, if the technology changes the question is would the production possibility frontier change yes it will change. There also you will basically have a production possibility frontier depending on how technology is affecting labor and raw materials, say for example labor utilization really decreases raw material utilization increases or vice versa. This orange line which you have which is LM can have different ways of representation. Then based on the sets of I curves and the set of J curves which we have considered, what were they I will again go back they were power and safety. Again you can find a set of points which are concentric tangent to this point of this LM curve production possibility frontier curve and you can again find out the best combinations which will give me a different set of curves which we will denote by A' B'.

Again any point along the A' B' curve, as it was given in AB would also give you the same level of satisfaction not at the same level of satisfaction as the earlier case because their level of satisfaction for AB would be different from A' B', but once you are there on

A' B', the whole curve will give you the same level of satisfaction and you can have different combinations of power and safety features being there such that it will satisfy the decision maker. So with this I will end the 6th lecture and continue the discussion more so for the MCDM techniques and their definitions. Thank you very much. Have a nice day. .