Multi-Criteria Decision Making and Applications
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Week 12
Lecture 59

Welcome back my dear friends, a very good morning, good afternoon and good evening to all of you. And as you know this is the NPTEL MOOC lecture series which is spread over 12 weeks, 60 lectures and the topic is multi criteria decision making. And we are in the last set of lectures and we are still two left to wrap up this course. And as you know in this course each week we had 5 lectures, each lecture being for half an hour and after each 5 lecture we had assignments of 10 questions each and they would be a final examination covering the whole set of topics for multi criteria decision making. My good name is Raghunandan Sengupta from the IME department at IIT Kanpur and if you remember I will just highlight the important points we have covered in depth multi objective methodologies, different methods, non parametric methods, different methods, univariate, utility theory, multivariate, but just very fittingly. And under non parametric methods, electro epsilon electre, topsis, vikor, AHP, DAA and in the last lecture we started the decision tree.

Now for the last lecture and this lecture as the diagrams cannot be made in excel sheet hence I am using the word document. So, the general caption for all the lectures which you used to see that we are in this lecture covering this set of points that coverage would definitely be there when the slides are shared with you, but for here to maintain continuity and without erasing whatever small calculation I am doing please excuse me I am continuing in the same format. Now if you remember in the problem for the decision tree we were at the point D2, D2 the value was 194 I am not reading the decimal 194 -150 which is 44 to be compared with 0. So, 44 was > 0 we will take that when we want to find out the value of C 1.

Now for C 1 remember this now the logic would have become clear to you the value for high demand probability was 0.7 the value was 44.2 as I said I would not be reading the decimal, but it is 0.2. So, 0.7 into 44.2 and for the low demand where you will stop the probability is 0.3. So, 0.3 multiplied by 0.8. So, total value the expected mean value at C 1 comes out to be 30.9. So, it is still positive. So, when I come to third. So, which is here which is C 1 now there was an investment of 20 million for the pilot plant and test marketing.

So, if I consider that value 30.9 consider is 31, 31 - 20 is 10.9 is still > 0. Hence the decision tree would be to set up the plant and the path to be taken are which I will highlight here are as follows. Which means the company I am putting a hash for the other diagram

to because as a complicated diagram the second example it will be colored.

So, for the spectrum company it will build up test marketing then there is a high demand obviously with 0.7 then it will build the actual plant at 150 and the expected value either is 30 million for 1 year for 20 years for a high demand of 60 % or low demand of 40 % whichever it is the thing would be. So, it will take the path as given. So, if you see we are solving from the right, but the overall diagram would be given. Now, the problem is and the question which I did pose to you is that can we consider any other set of criteria to evaluate? Yes, we can use the concept of variance and try to basically find out whether variability can be utilized in order to rank them or any other measures.

Now, immediately when I say any other measures even though not very evident the idea should immediately ring some bell in your mind. In the DA method if you remember what we did we were trying to find out the efficiency. And the word efficiency was the ratio of the bundle of output divided by the bundle of inputs depending on which model you are going to use. Remember we had the decreasing return to scale, increasing return to scale and constant return to scale. And in another case we for the efficiency we maximize the efficiency.

And in the case when you want to take the ratio of the input to the output we took minimization of that. So, whichever direction you are looking it is basically the efficiency we are considering. Now, obviously in the concept of expected value and variance no such term as efficiency can be coined, but we can also consider the ratio of expected value to variance and consider it as one of the main way or criteria of trying to basically rank the decisions or the paths accordingly such that we will take maximum value of the expected value to variance and rank them from the highest to lowest. Or else we can take the ratio of variance to expected value and rank them from the lowest to the highest. So, it is again I am saying it is not efficiency, but it is the way of trying to analyze the problem accordingly.

So, the problem which we did for expected value can also be looked from the point of view of variance, ratio of expected value to variance, ratio of variance to expected value and all the ways can be done. Now comes the second problem and here the second problem is a little bit more intuitive much more involved. And here till now for the first problem the probabilities are given very straight forward no complication. Here the probabilities will be given, but with the twist such that you will understand that depending on one process how the probability would be decided for the next stage. An oil company while evaluating the oil basin is considering three alternatives which are drill, conduct seismic test, before at a cost of 20000 and find the nature of the underlying oil basin and finally, do nothing.

So, the company can have three alternatives of course of action which are it can drill directly, it can conduct seismic test before which costs about 20000 and another thing can be do nothing. Now the question would immediately be when it drills obviously there is a cost. So, cannot this cost be incorporated in both alternative 1 and 2 yes it can be I have not considered that in order to make things simple. If the company drills then it is likely to find the oil basin of three types it is dry, wet or soaking. A dry will yields nothing because totally dry which means that probability or the chance or the quantity of finding oil is almost 0.

If it is wet it provides moderate quantity of oil and if it is soaking it will generate substantial quantity of oil. If the oil company had conducted seismic tests obviously seismic test would give a lot of information about the rock layers, how permeable they are, how hard they are, what type of rocks they are depending on that. So, elementary rocks permeability is there whether they have a long of history where oil can be found out at greater depths. So, those concepts of tests results would basically give a lot of information for the company. So, let me continue reading it if the oil company conducts seismic test then it can learn about the underlying structure of the oil basin before deciding whether to drill or not to drill.

The underlying oil missing structure may be depending on the test it can be found out there is no structure, there is an open structure and there is closed structure. If no structure is found then the prospect of finding oil is bleak, if an open structure is discovered then the prospect of finding oil is fair while finding finally, if the structure is closed then the prospect of finding oil is bright. So, we already know the structures are no open closed and we already know the wells are dry wet soaking. So, obviously it will mean that this combinations of doing some test and what is the actual drill results would give you different types of combination which is very interesting to analyze. The company also knows the following and this is what the information is given stage by stage and please mark the values because when we come to the solution all those values would be important.

By the way when you are solving the problems trying to read the slides would be all be all with with there with you. So, please look at the slides accordingly. The probabilities of various oil bearing states are it is dry, probability is 0.5 wet 0.2 soaking 0.2 if you sum it up the total value comes out to be 1 as it should be. Probabilities of various soil geological states are as follows no structure 0.4 open structure 0.3 closed structure 0.3 again the probability comes out to some comes to be 1.

So, these are two different things probabilities of various oil bearing states dry wet soaking and what is the structure open no structure open structure closed structure. So, one is for

the drilling part one is for the testing part. Also we have the following joint probability distribution based on this information which would be interesting to know. Also we have the following joint probability distribution between underlying geological structure and oil bearing state. Now by the way if you remember I said that all the values will be important.

So, please remember these values again I am repeating the p is basically probability which you should be aware by this time as we continue doing the problem. So, state of dry 0.5 state of wet 0.25 state of soaking 0.25 these are the probabilities. Again the probabilities for no structure 0.4 open structure 0.3 closed structure 0.3. So, note them down as we continue discussing the joint probability overall table and here is the table and this is interesting.

So, if we see I have made different coloring schemes the vertical ones which is basically denotes the underlying geological structure which are no structure open structure closed structure are blue in color the vertical part. And the oil bearing states which are horizontal and they are dry wet soaking are shown in pink in color. Now what is interesting if you do not know if we have noted down few minutes back I said note down the values of 0.4, 0.3, 0.3, 0.5, 0.2, 0.2 these are the marginal values. What are the margin values? If you see at the right in the table the right column and the bottom row they will give you the marginal values at the margins. So, the marginal probability of the states of the oil bearing states if you see they were given as 0.5, 0.25, 0.25 and marginal probabilities for the geological structure were given as 0.4, 0.3, 0.3 interestingly sum up of all these three values 0.5, 0.25, 0.25 is 1 sum of 0.4, 0.3, 0.3 is 1. Also interestingly the values which are given here like say for example 0.32 what it means? It means the oil bearing state is dry and there is no structure and the probability is 0.32. If I continue along the horizontal row it is dry with open structure 0.15 it is dry with closed structure is 0.03. If you add up 0.32, 0.15, 0.03 the total value comes out to be 0.5 as it is along the margin. Similarly, for the row weight the corresponding probabilities for no structure open structure closed structures are 0.04, 0.10, 0.11 and the total sum is 0.25. For the last row soaking oil bearing state the corresponding probabilities for no structure open structure closed structure the values are 0.04, 0.05, 0.16 the total value sum is 0.25. If I follow the same logic along the vertical for no structure with oil bearing state as dry weight soaking the probabilities are 0.32, 0.04, 0.04 the total sum is 0.4. Similarly for open structure oil bearing states being 3 as they are written the probabilities are 0.15, 0.10, 0.05 sum is 0.3. Finally for closed structure the probabilities are 0.03, 0.11, 0.16 the sum is 0.3. So, this would basically give you a total sum of 0.3 and all the information which is needed and we will utilize that for the problem. Finally the oil company has the following set of information apart from the conditional probability table which is regarding the net present value of the 3 states which is net present value for the dry state is given as minus 0.6 million which is negative.

Net present value of the wet state is plus 0.8 million and for the net present value for the soaking state is plus 2.4 million. Because as you know for dry state, wet state and soaking state the probabilities of finding oil was least then medium and the highest. Now let us go to the solution and before I go to the solution even though nothing is given here let us see that diagram.

Now this diagram I have to scroll up and down. So, please bear with me as I said the diagram which I tried drawing considering there are so many states decision forks, chance forks it was difficult, but I tried my level best to give the idea. Again the coloring scheme remains the same blue means positive input some value red means something going out negative. Again the same concept we start from the left hand side go on to the right hand side to draw the diagram whole set of gamut of all the options the fork and the chance and we calculate from the right to the left and follow the part which gives us the maximum value based on the criteria which you are trying to maximize or minimize. Now diagram is like follows.

So, we start at D1 and then consider the concept. So, D11 which is the arm which is going up is basically you drill without the test. D12 you conduct the test and if you remember there was a price of 20000 which is given as red color 0.02 and the other arm which is basically shown here is D 13 is that do nothing. So, obviously that was also an option. Now when you drill it what are the so obviously you do not have any idea about the structure you immediately drill it and find out the characteristics as dry wet soaking the chance fork is C 11 as you are starting from there for the first one which is dry then C12 for wet and C13 for soaking.

So, the chance fork which is at the top where the mouse is hovering is given by the number C 1. And what are the net present value if you remember net present value for the dry was minus 0.6 which is shown in red color plus 0.8 which is shown in blue color plus 2.4 which is shown in blue color. What are the probabilities dry has a probability of 1/2 which is 50% wet, has a probability of 1/4 which is 45 % and soaking has a probability 1/4 which is 45 %. So, these values are the marginal values. Now the question will come up why did I take the marginal value because here no test has been conducted. So, these probabilities are unconditional based on the drilling only.

So, there is no conditional fact added to this. Now if I consider the again I am saying the diagram I am scrolling up and down. So, when you see the video and also refer to the diagram it will be much easier for you. When you conduct the test what three things occur one is you find no structure which I am highlighting and this is given by C 21. Why C 21 because the arm which you reach the chance constant the point which you reach after

conduct under test is given by the point C 2. So, C 21 is basically no structure then you have C 22 which is given as open structure which is given here I am just made it here such that there is no confusion and the last point is basically C23 is closed structure.

Now very interestingly the probabilities are given probabilities are again the probabilities for the unconditional part for conducting at the test because you conduct the test and then only you drill in this case. So, conducting the test probabilities were 40%, 30 %, 30 % here they are 4/10 for no structure 310 for open structure, again 3/10 for the closed structure which is probabilities of C21 4/10 probability of C22 3/10 probability of C 23 3/10. Now when the no structure is found then you drill. Once you drill obviously you will find out as dry wet soaking similarly when you find out open structure again you will drill you will find at dry wet soaking and when finally the third case of closed structure again you will do it as drill and then dry wet soaking these have been incorporated where. Now if you see no structure and drill the decision fork or decision arm or decision point is D2 and the corresponding directions it can move is basically moves to C3 and the other arm goes to D22 that means you do not drill.

So, basically D21 is drilling and the outcomes are dry wet soaking and the net present value we have kept as fixed whether you conduct the test then drill or you only drill the net present values are for dry wet soaking are negative 0.6 positive 0.8 positive 2.4 and interestingly the corresponding arms are mentioned as C 31 C 32 C 33 which is for dry wet soaking and the probabilities are different. Now how they are different if you see these dry states have been encountered after the drilling and when was the drilling considered when there was no structure that means if I go to the no structure part no structure part and if I consider the concept of drilling.

So, what it would be with no structure if I consider this this column the total probability is 0.4 for no structure driving 0.32 and the hence the conditional probability would be 0.32 by 0.4. Similarly, when I go to no structure wet the conditional probability will be 0.04/0.4 and finally, for no structure with soaking oil varying state the case is 0.04/0.4. So, if I consider the values it is 0.32/0.4 0.04/0.4 0.04/0.4 and here the values are 0.04/0.4 comes out to 1/10, 0.04/0.4 for the third 0.04/0.4 which is soaking comes out to be 0.1/10 and for the case when is 0.32/0.4 it comes out to be 4/5. Similarly, there is a arm D22 which is do not drill which you have already mentioned and following this logic if I go to open structure which is D3 and then you drill D31 other part is D32 1/2 and then you drill D32 which you do not drill after you drill again you count at the same thing dry wet soaking net present value being -0.6, +0.8, +2.4 and the corresponding probabilities are now we go to the open structure. Open structure is the second column the corresponding probabilities are for dry wet soaking condition probabilities are 0.15/0.3 0.10/0.3 and the last one is 0.05/0.3. So, if I go to the open structure dry wet soaking values are here ½, 1/3, 1/6.

So, if I consider 1/2 is here 0.5 0.15/0.3 is 1/2 0.1/0.3 is 1/3. So, you see the value of 1/3 here and the last value 1/6. So, if I see the value of 0.05/0.3 is 1/6. So, obviously, for this case when there is open structure D32 would be do not drill.

Obviously, you have to have a do not drill in order to basically find it out and finally, for the closed structure with D4 there is D42 do not drill and with D41 where there is a drill again you have dry wet soaking net present value again I am assuming simplistically as -0.6 +0.8 +2.4. Hence, -0.6 is red in color +0.8 is in blue in color +2.4 is in blue in color and the corresponding probabilities are given which I will go back to verify is 1/10 11/30 16/30 from where do get we get it from the open structure did I do anything wrong no we get it from the closed structure sorry we have in the last term in the closed structure the probabilities are 0.03/0.3, 0.11/0.3, 0.16/0.3. So, if I go 0.03/0.3 comes out to be 1/10 and the corresponding values of 11/30, 16/30 are for the last two values. So, once you have there the total diagram is done now I need to do the calculations from the right to the left and here is where we will again try to find out for the first case we will basically for this arm we will multiply the probabilities with the net present value. So, probability of 4/5 into 0.6 and remember I am not I am showing it. So, obviously 0.6 will be < 0 because a negative value (+1/10) × 0.8, 0.8 is > 0, +10 /2.4, 2.4 > 0. So, based on this value you compare with D22 which is 0 then you calculate the values of this D31 root which is 1/2 into -0.6 (+1/10) × 0.8, 0.8 is > 0, 1/3 × +0.8 +1/6 × +2.4. You compare with D32 and finally, for the case where you have the closed structure when you want to find out D41 you find out 1/10 × -0.6 +11/30 × +0.8 +16/30 × +2.4 you compare with D42.

So, once you have compared then when you reach D2 D3 and D4 the values will be found out. So, if they are negative then obviously you will take the negative value or you will take the value 0. Why 0 if you remember in utility theory I did discuss we can consider the values if they are negative as 0. So, once you reach D2 D3 and D4 the corresponding values which you will multiply for D2 would be 4/10 because that is the no structure probability for D3 it will be 3/10 and D4 it will be 3/10 and correspondingly once the overall value is found out for C2 remember that 20000 value is there that has to be incorporated and the problem solved accordingly. Similarly, the variance can also be brought in the picture the ratio of variance to expected value can also be brought in with the picture the ratio of C for example, expected value to variance can also be brought in the picture and the problem solved accordingly. So, with this I will end this class we have discussed in detail two problems in decision trees I have solved in detail what expected value it can be extended for other criteria also the slides would be shared with you I would not say the slide they are the word document will be shared to with you when you see the lectures please refer to the diagram they can be variety of problems, but once you get the hang over of this this this flow of how the problem can be solved you can solve it

accordingly.

But remember from left to right the diagrams are drawn the chance folks c and d their c and d are not the concordance or the discordance probabilities are to be considered condition properties are to be considered net present values have to be considered provided is expected value in other case you can consider the variance also. So, with the again I am say I am ending this class in decision trees and discuss a simple problem for the last set of lecture for this whole course. Thank you very much and have a nice day. Thank you.