Multi-Criteria Decision Making and Applications
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Week 12
Lecture 58

Welcome back my dear friends, a very good morning, good afternoon, good evening to all those who are taking this course in multi criteria decision making. This is the last week of the next classes which are there for this NPTEL MOOC course which is spread over 12 weeks which is 60 lectures. Each week we have 5 lectures, each lecture being for half an hour and after 5 lectures we have an assignment. So, this is being the 12th week which means you will take the 12th assignment and then they would be the final examination. My good name is Raghunandan Sengupta from the IME department at IIT Kanpur in India. Now, as you can see the slides are in word document the things which we will be discussing today and there is a reason for that because the diagrams and all these things which we will show as we solve the problems it was difficult to make in ppt file that is why I converted into I made it into word document and we will discuss in that.

I will share it along with the slides for the last 2 lectures. So, including this we have another 2 lectures to go before we wrap up this course. Now, till now we have discussed the concepts of utility theory in details univariate case, consider different properties, stochastic dominance first order, second order, safety first principle, what are the different type of utility functions. Then we consider different type of multi-objective formulation problems, such as linear programming, non-linear programming, integer programming.

Then we went into multi attribute concept, multivariate utility theory and another multi attribute theory we considered in definitely in some detail. The concepts of electre method, the epsilon electre in electre method and epsilon in specifically I mentioned that how you can bring a one to one correspondence considering the concept of risk aversion, risk indifferent person and the property of risk loving. And there we had the concordance set, discordance set and the indifferent set in the epsilon electre. Then we considered vikor method, topsis method, the concept of NIS, PIS, different distance measure using the LP norm, P being from one to infinity and how you can analyze different ranking system of the alternatives considering both positive and negative set of criteria. Then in the last class we formulated the concept of DA data and well up analysis and I did mention that DA was a little bit different in the sense that it was based on optimization for concept using ideas of efficiency.

We formulated the problem for the output oriented model, input oriented model and how you can utilize simple linear programming to solve the problem. The idea which we are going to discuss here is decision tree analysis is also in a different line and why we are

considering that under multi criteria decision making that will become clear to you. Now, whenever we make a decision tree and we talk about decision tree. Decision tree is a process mathematical logical process based on which we take the decision that which objective needs to optimize and even if we optimize how we rank them or which objective function needs to be minimized, how we rank them and take that decision. We have seen that when we are considering in multi objective case, multi attribute case the semblance and or the one to one correspondence in the qualitative sense when we are talking about different objective functions and the alternatives, criteria's, constraints, decision variables all these things were discussed as we proceeded. Now, for all the examples in the nonparametric method the analysis of the idea was always to consider three different examples, one was about how to buy a house apartment, then second was to buying a vehicle a car and third one is choosing your university where you want to study.

The problem which we will consider here the two different problems are about different projects and they would be given in detail. Now, before we go into decision tree analysis the key steps in trying to delineate or draw a decision tree are as follows. So, is basically first you have to identify the problem and what are the alternatives, different alternatives we can utilize to achieve the goal you want to have set in mind. Goal can be trying to maximize your profit and minimize the loss can be throughput has to be maximized, loss has to be loss means not monetary loss, loss can be per day loss in electricity. If you are using electricity has to be minimized or number of trucks you want to basically use to send forth goods from city one to city two that has to be maximized.

So, they can be different ways of trying to analyze decision tree problems. So, first as let me read it identify the problem what is to be achieved and what are the alternatives you draw the decision tree and the drawing of the decision tree is done from left to right. So, if I am looking at the problem, looking in the problem in the sense when I draw the problem is basically starting from the initial stage which will very simply mention as the starting point and we will have sets of ending point on to the right. Now, the decision tree diagram is not exactly equal, but it basically resembles the PERT CPM diagram which is a big part which is used in project management and project analysis. You will see that how the similar concepts in the sense how the logical sequence are kept in mind and how the diagram is drawn from the left to the right.

You specify the probabilities and monetary outcomes. Now, in PERT CPM in one of the method it was deterministic time and in another case stochastic time. Here we will consider the set of probabilities are there depending on which decision is to be taken and for our case we will consider the monetary outcome based on which the probabilities are assigned and based on the probability based on the monetary outcome we will take the route which will maximize or minimize the objective function which is there in mind. As I said

maximize the profit, minimize the loss. So, we will evaluate the various decision alternatives and take the one which basically best suits the set of assumptions based on which the problem has been formulated.

Remember the decision points in decision tree I will because as I would not be able to utilize the marker here let me check. So, I will basically try to highlight it using my mouse. So, remember the decision tree also has decision forks. So, decision forks are basically as we proceed they would be denoted by D and the and from the decision forks you will have alternative actions which are available for experimentation or moving from the direction from left to right and actions are possible at this points are the decision forks while the chance points called chance forks which I am just now highlighted would be denoted by C. So, do not confuse this D and this C which you see in decision tree analysis as the discordance set and the concordance set no or not as the criteria also.

So, this chance forks are the points and there would be many points in the decision tree as you go from the left to the right where the outcomes are dependent on a chance process and the likely outcomes at these points would be decided accordingly. Now, let us see a problem. So, rather than explaining for maximum the cases what we have done is that we have solved the problems in order to understand how somebody can apply these methodologies in some of the theoretical problems and in practical sense. It is basically gives you an idea. So, we will consider two examples.

First example will be where probability would not be as involved as the second one and when I mean by probability it will be conditional probability and we will come to that. So, if all of you have done the concept of condition probability based theorem. So, it will be easy for you to understand problem 2 and that is a very fundamental concept in probability and statistics. The problem says as follows. The scientist at a company spectrum have a come up with an electric moped.

The firm is ready for pilot production and test marketing. So, obviously, before the product is launched. So, the company will make the product, test market it, get the customers feedback about the quality, this if as it moped they will check the quality factor from the customers, the safety features from the customers, what is the maintenance cost, how are the braking system, how is the power of the engine, whether the car or the moped can be navigated in a big city, crowded city, whether more than two persons can sit and so on and so forth. So, they will get the feedback from the customers and obviously, the prices also factor and then make changes accordingly as it is required. For the pilot production and test marketing it will cost 20 millions and take 6 months.

Now, let me tell you few important facts. There would be some very simplistic

assumption, we can definitely go ahead by making the assumption more realistic. Only thing which will be involved in the problem, the problem would get much more in depth. So, you have to do much more stepwise calculation, but the idea would remain the same. So, management believes there is a 70% chance that the pilot production which has been made for test marketing, it will be successful and obviously, 30 % chance it would not be successful.

So, I am saying that 30 % chance it is not successful. So, they can be more than two arms also. So, as it is mentioned 70 % successful, it can be 20 % not successful, 10 is lukewarm medium. So, those issues I will keep mentioning them, but we will consider the simplest form of trying to solve the problem which I just mentioned few minutes back. In the case of a success spectrum can build a plant costing about rupees 150 million.

The plant will generate an annual cash flow of 30 million for 20 years, if the demand is high or an annual cash flow of rupees 20 million if the demand is low. Now, after the pilot production test marketing success is there, the company builds the plant at 150 million there are two ways of trying to handle the demand. One is highly successful world for 20 years, annual cash flow 30 and another case is where demand is low, annual cash flow 20 millions and I am considering simplistically again the time frame is 20 years also. Now, the question may immediately come up that cannot there be more than two outcomes like high demand, medium demand, low demand yes they can be I have not considered here. The next question can be that let us consider for high demand 30 million cash flow is 20 years or for low demand 20 million cash flow per year, but the time duration is not 20 years is much smaller like say for example, 15 years can it be yes it can be.

Again I am saying I am not considering that I am considering the simplistic case. High demand has a probability of 0.6 hence low demand has a probability of 0.4 again the question can there be three types of demand high demand, medium demand, low demand. Here we have considered again I am mentioning high and low if you consider high, medium, low it can be say for example, 0.

5 high, 0.3 medium and 0.2 low. So, obviously if you note down the probabilities would all add up to 1. Now, the question is that what is the optimum course of action using the decision tree analysis. So, let us first make the diagram and this is what the diagram would look like. Details for the diagram will come up later, but first the schematic sketch.

Now, if you see the problem from the company, for the company what it wants is this and I will keep highlighting there. So, these D's are the decision forks and C are the chance forks, D are the decision forks, C are the chance forks. If you remember as I had mentioned when the nomenclature thing was being done. Number 1, number 2 the numbering as I

mentioned the diagram will start from the left hand side. So, numbering has been made accordingly.

So, first you have D 1 then obviously you have C 1 I have not marked anything inside the circle C 1. Then from C 1 obviously you will have 2 arms, why 2 arms I will come to that later. Then you have D 2 as you going from left to right along with that you have D 3, then you go to the right top corner which is C 2 and so on and so forth. Now, why there are 2 arms from each of them. Now, if you remember the problem says the company would invest 20 million for the pilot production and test marketing.

So, this D 11 arm which I am mentioning and which I am highlighting has the negative value of 20 million that is why is marked in red. So, the first path D 11 is invest in the pilot plant and the path D 12 is do nothing that means do not invest anything that can be also be an option for the company. Once this 20 million is invested the test marketing is done if you remember I had said that this 70 % chance it is success 30 % chance is not. So, this C 12 for the which the probability is 0.3 means that the company has to stop production in spite of the fact there was a pilot production and test marketing was there, but as the demand was very low it was stopped.

For the case C 11 where the probability is 0.7 there the success was high the company will build the plant or else the company can also build the plant and do nothing. Now, obviously the question will come up if the company has a success of 70 % then the obviously the question would be build the plant that is true. And this building the plant if you see from D 21 you have invested 150 million over a number of the 20 million which you have already spent for building up the pilot production plant and test marketing. So, this D 21 is basically the overall factory investment which you need to do to start the actual production.

And in spite of the fact if you see the arm D 22 in spite of the fact that it was a success of 70 % the company still did not want to do it may be possible due to many circumstances like say for example, the company was bought over by some other company or the 150 millions it was planned to invest suddenly did not come up from different type of lenders or it could not basically make the money and meet the demands. There can be many other practical situations. And once the production starts we know that there is 60 % chance is the high demand 40 % chance there is a low demand. So, this C 21 with a probability of 0.6 gives you 30 million per year and this is blue in color because this is a cash flow coming back.

For the low demand which is C 22 with a probability of 0.4 with a low demand we know there is 20 million will come back to the company and the time duration you remember

was given by 20 years for both the cases even though 20 million 20 years for case 1 and case 2; case 1 being 30 million per year high demand and case 2 being 20 million per year for low demand the time duration of 20 years may not be true for both the cases. Now if you see there are some vertical lines also green one which I am now pointing out. So, these green lines are the stage of evaluation you as a decision maker will do and try to consider which path to take. Now remember the path were or the decision tree was drawn from the right left to the right, but when you do the analysis you will be doing the analysis from the right to the left.

So, obviously your analysis would start being taken from the right side which is trying to find out the overall net value or the revenues or the cost whatever it is starting from the right most part. So, obviously you are seeing only one set of arrows which is C 21 and C 22 will start from there and proceed backwards till the left. Now this is what we are going to do. Now if you see for the part which is I will just I think I have to write. So, I will see if I consider now there are different ways of trying to analyze and this I will spend some time here consider 30 million.

Now if it is 20 years consider. So, now to give a background I would not be writing much I will be talking because these ideas definitely many of you who have taken finance courses would be aware of that. Generally we know there are two types of rates and this rates we have discussed in utility theory which was capital R and small r total rate and the value of rate of return. So, one was capital R and one was small r. So, we will only consider small r and we also know that when you are considering interest rate we can utilize the concept of simple interest and the compound interest. And the compounding factor if it is run infinite number of times in a particular cycle or in a particular time for above 1 year you have the continuous compounding interest rate CCIR I am using the short form continuous compounding interest rate and based on that we can find out.

So, I will first utilize the concept of continuous compounding interest rate and then proceed backwards. Backwards means try to basically highlight that in case of continuous compounding interest rate was not there how we can use the concept of continuous compounding rate of the simple interest rate and do the calculations, but the idea would remain the same. Now, there were many assumptions which I said. So, another assumption is that for this 20 years for the 30 million and for this 20 million for this 20 years. So, for high demand, low demand probability being 0.

6, 0.4 we are considering the continuous compounding interest rate to be fixed even though that is not true. We will consider that interest rate continuous compounding interest rate as say for example, r_c. So, if I consider the values. So, consider the time frame and this time frame is basically I am considering from the value when C 2 to C 2 as the production

has started has finished and then the demand is on the higher side for 20 years. So, if I consider the fact that if I want to basically find out the value of money if this is P and the value is P 1 and in the continuous compounding interest rate and consider this time frame is 1 then P, e to the power because continuous compounding interest rate means continuous compounding it becomes exponential, e^{rc}, time frame is 1 = P1.

So, if I want to find out the value P1 is basically the per year rate which is given. So, P can be found out as P1, e^{-rc1}. So, this is for 1 year. Now if it is for 2 years. So, obviously in this case it will be P = P2, e^{-rc2} and that can be considered accordingly.

Now based on the fact that we would have basically such 20 such payments being made. So, obviously if I have the time frame. So, this is 0th year 1 year 2 year then dot dot it goes on 18th year 19th year and 20th year. So, if I consider the first year the value at time frame 0, that is, when C2 to that chance node has just been made.

So, this would be for a time frame of 1 year. Similarly when I consider the second payment of 30 million after the second year this will be considered for time period of 2 years. Similarly this one from the 18th year it will be for 18 years 19 years and 20 years. So, you basically if I have I need to basically sum up all the values of P, P means the present value based on the fact the continuous compounding interest rate is given as rc which is fixed which is an assumption and the time frame is changing from 1 year then 2 years 3 years 4 years till the 20th year and add up these values. That would be the net present value given at the case when I am considering the value for the high demand. Now similarly I will find out the low demand only in the low demand case remember this value of P1 which is given here for the high demand case was 30 million because for per year and this low demand is 20 million per year.

So, P1 basically changes from 30 to 20 that is point 1 time frame is same. Point number 2 when I want to find out the expected value at this stage as I am going from the right to the left I need to find out the expected value of the chance node considering how many arms are there and this type of problem you have already done. If you remember in neutral theory we found out the expected value is exactly the same. Now the question immediately which may come up in many of the candidates and the students mind is that can we use variance also in place of expected value yes you can. Now the moment the answer is yes you can immediately this should give you all of you the information that see here we can use multi objective case functions in order to analyze the problem and what are the multi objectives? Multi objectives the first case is try to take the expected value only, second case can be tried to take the variance only.

So, depending on the problem formulation you can take different objectives and formulate

the problem and rank them accordingly. So, that is why when I started discussing the decision tree analysis I did mention is a multi criteria problem which can be seen it is not the classical type of multi objective problems which you have already solved, but they can be fine tuned as a multi objective problems and the problems can be solved accordingly. Now this was at stage C22, so if I consider the problem as C22 is if we consider so obviously the expected value expected value I am using EV at C2 would be 0.6 is the probability for the high demand which would be multiplied by the net present value for 30 million plus 0.4 multiplied by net present value for 20 million and where is this NPV 20 and NPV 30 coming from? This NPV 20 and NPV 30 are coming on from trying to find out the net value for the high demand separately multiplying it by 0.6 and finding a net present value for the low demand then multiplying by 0.6. So, once you have that the net value of C22 is given and the net value of C22 is based on the formulation of the case when you have the simple interest rate. Now if you remember I did mention that you can use the concept of continuous compounding interest rate, compounding interest rate, simple interest rate. So, if I consider the simple interest rate only for the 30 million per year arm high demand, so it will be basically given by the case of 30 this is for the first year 1 +r1, r1 is the simple interest for 1 year, to the power 1 because you are doing it for 1 year trying to find out the net present value for 1 year and what is this 30? 30 is that per year payback this is 30 for the second year it is r12, this is r1 sorry because you are considering the interest rate as fixed square and it goes to $31 + r1^{20}$. Why 20? This 20 is not 20 million 20 is the year.

So, once you find out this and multiply this by 0.6 + 0.4 multiplied for the case when you have 20 million coming up per year again interest rate is fixed r1 this is for the low demand that is why it is 0.4. So, this value calculated comes out to be approximately 194.2. Now, why there is a 150- because if you go back to the slide or the word document my apologies 194 is the income, but 150 is the outcome.

So, based on that if I want to find out 44.2 is the net value which is positive for the arm which we have here for that means for D2 arm total combined input and output and obviously that value 194 minus 150 which is basically 44.2 has to be compared with the arm which is D22 which is you do not do anything. So, obviously that being positive that will be subsumed that you should go through that process. So, with this lecture I will stop here and continue the discussion in the next class and try to wrap up the decision tree problems accordingly. Thank you very much and have a nice day. .