Multi-Criteria Decision Making and Applications
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Week 12
Lecture 56

Now welcome back my dear friends and participants for this multi criteria decision making course under NPTEL MOOC series. And as you know this course is spread over 12 weeks with 60 lectures and each week we have 5 lectures each lecture being for half an hour. And after each week we have one set of assignments of 10 questions and after that total course is over we will have a final examination. Once again a very good morning, good afternoon, good evening to all set of people who are watching this lectures. My good name is Raghunandan Sengupta from the IME department at IIT, Kanpur. And if you remember in the last few weeks we have been going through the non parametric methods and we had covered a lot and as the methodologies of trying to solve the problem were almost similar.

So, I just highlighted the main context, main idea and the different methodologies we have considered was the Elektre method, Epsilon Elektre, TOPSIS, VIKOR and the last part we discussed the AHP. So, AHP is basically the analytical hierarchy process. One another method we will discuss in the first lecture of the 12th week which is the 56th lecture is the idea of data and development analysis and this is very heavily used in economics and econometrics in production economics where you want to basically rank different decision making units or organizations. Now, all that other methods of Elektre method, Epsilon Elektre, TOPSIS, VIKOR, AHP we have discussed is basically the it was leave aside AHP.

The other four methods we discussed was basically the closeness to the best solution, for this it is from the worst solution we use the concept of concordance matrix, discordance matrix in some case we use the indifference concept in epsilon Elektre. We have used the concept of PIS, NIS and then we found out the ranking method for all the alternatives based on the criteria. First we compared them taking one at a time and then collectively. In HP it was a little bit different where obviously the comparison was between alternatives based on the criteria. So, for the example where we considered two students Ram and Shyam were selected for the MBA programs and they are different set of criteria to select the institute or else consider the example where you are buying a car.

So, in all these methods including AHP there was a combination of quantitative as well as qualitative different criteria in order to judge how you will rank. DA is also in the similar line, but it is more from the point of view of operation research and I will come to that later on because operation research and the simple concept of linear programming and all these things is considered to be a part and parcel as a prerequisite for this course. So, I will just give you the highlighting points of this methodologies and then you can solve the problem accordingly. I will only concentrate on the formulation other things are very straightforward. So, this is the 56th lecture out of the 60 set of lectures the idea is multi criteria decision making.

We have already covered multi objective decision making, multi attribute utility theory. Now we are considering multi attribute decision making. Even though the methodology of DA you may find out some mathematical formulation, but still I am considering it as an part of nonparametric methods and why I will just highlight few of the points as we proceed. So, the coverage is data and development analysis and we will consider it accordingly. Now where is data and development analysis used? So, for the examples we consider if you remember I was always trying to harp on three examples.

Trying to buy a car where there can be different criteria some which are quantitative, some which are qualitative, some are objective, some are subjective. Trying to buy an apartment in a particular city with different sets of criteria, some may be price, some may be locality, some may be closeness to the school or how safe it is and other case was trying to choose the institute where you want to go and study. In the similar line you can use DA, but DA consider that I want to analyze and rank or I want to find out ranking methodologies or ranking of different type of alternatives. Here the alternatives are not mentioned or named as alternatives in DA method they are known as the decision making units or DMU as written in the slide. So, what are the decision making units? Consider that the same example we want to choose the best college, best university.

So, when you make a list of the different type of universities be it engineering college list from where you want to take or the management school or social sciences you have different sets of priorities set. Somebody wants to basically go to the best college where the quality of teaching is excellent, somebody wants to look at the job opportunities, somebody wants to have a look at what are the internship opportunities and so on and so forth. Or else consider a totally a different example that you want to basically analyze how are the different type of companies under the same sector are doing. Consider the car industry. So, there are many different car industries companies in India it can be Hyundai, it can be Honda, it can be Volkswagen, it can be Tata, it can be Mahindra.

So, you want to basically analyze how does this set of companies compare amongst themselves considering there are different sets of inputs and there are sets of outputs. Now, the word of sets of inputs and outputs you are hearing for the first time in context of DA and obviously we have not encountered these two words in other non parametric methods. So, we consider a DMU in a very simple sense is a black box or a unit where different inputs are utilized in different quantum's, different units and the units of input can be electricity, can be manpower, can be water, can be raw materials anything. And outputs can be different type of outputs considering the case of car. So, it can be what type of car the companies are making is it a passenger car of certain cc or is it a heavy vehicle or a SUV and so on and so forth.

Now, if I consider the case of the choosing the educational institute consider for our example education institutes are all the IITs. So, in the IITs obviously the input would be the students, input would be the infrastructure, input would be the faculty members, input would may be the location and what would be the output. Output can be the type of jobs with the students get, type of research output on the front of paper, books, different

type of projects which are sold in the market, so based on the set of inputs and set of outputs you want to basically rank this decision making units. So, basically this is the background based on which we are going to study the DA method. So, let me continue the discussion further on by reading the slides.

Consider you have a system or so called decision making unit or units which takes some inputs in different quantum, different units process them and produces some outputs. Now, when I am using the word processing, processing can be different it can be a chemical process, it can be a mechanical process for an organization which is producing some goods and for the educational institute you are basically processing the students to be well acquainted with the job market, well acquainted to for the academic world. Now, in another important point is if you remember in I did mention and I did mention it few times when we are solving the problem that if you remember the concept of discordance set or matrix, if you remember the concept of how far it is from the most positive point, how far it is or how close it is from the most negative point, so those theoretical points. So, we are considering the negative value of for taking a decision for choosing an alternative. So, in this case when you considering a DMU the question may crop up is that can we have some outputs which are negative in nature, yes we can have.

Say for example, when producing some steel or making some cards the pollution can be considered as an output which has negative impact. So, in the same way as we are analyzed you can consider the negative discordance concept or the how close it is to the negative theoretical point, we can also analyze the concept of negative outputs considering the DML methodology. Examples of systems or DMUs can be a machine, a factory, a hospital, an airport and a university. One example of the university I have just mentioned and factories for example consider that I am considering all the different type of small car manufacturing companies which I mentioned Tata's, Mahindra's, Hyundai, Honda, then Volkswagen and all this list. Invariably there may be many such similar systems or DMUs which are taking this inputs in some combinations or sets of inputs in some combinations or in different quantum processes them and produces the outputs also in different quantum.

Example consider two different the old IIT's which was built in India just after independence and consider the new IIT's. May be the amount of land the old IIT's have with respect to the new IIT's may be different, may be the total infrastructure may be different. So, we will consider the outputs and inputs here its inputs and outputs would also change accordingly. So, basically what the situation is. Another point is that when you are considering the factory example it may be possible a certain cc of car is not being manufactured by Tata Motors.

So, in that case the output for that company or for that DMU which is Tata Motors would be considered nil, but still we will try to basically fathom it how it can be considered in trying to model the DA optimization model. I am using the word optimization for the first time and as I did mention it is a different version of optimization and you will see it is a very simple in nature. In such cases we are required to find out what is the rate of processing of this inputs or outputs or what is the

efficiency of the system. So, that we can compare them amongst themselves and also find out to what degree they are deficient when compared to so called ideal system or a full efficient system or an efficient DMU. Maybe you are comparing two companies and the first company is operating at 100 percent efficiency.

So, obviously you try to find out that what is the efficiency of company 2 and how it can be improved. Now here is one important point to mention and we will revisit that later on. Consider I will give you two examples. Consider that you as a teacher you are trying to basically give scores for one examination which you have conducted and the grading system can be based on two factors. One is relative grading, one is absolute grading.

In the case of relative grading and absolute grading what you do is that in the absolute grading you consider the maximum marks as it is and then basically give the grades or the marks of the other stress system exactly as they the student he or she would get. In the relative concept the marking of the highest student is normalized to the maximum scale and corresponding to that scaling system the other students marks are also analyzed. Now the question is that why did I mention this example. Consider you are trying to compare all the steel industries or steel companies in India. So, consider the companies steel authority of India sale, it can be other private companies which are JSW steel and so on and so forth which are operating in India and when you compare them it may come out say for example, that out of the 10 different steel companies which you are considering JSW steel and steel authority of India have an efficiency of 1 and correspondingly Tata steel has an efficiency of say for example, 0.95. Now the question is does that mean that the first two companies which I mentioned JSW steel and sale, steel authority of India are operating at their highest level of efficiency answer may not be emphatic yes it may not be possible why because you are comparing the companies of the sample set which is in India. So, if I basically increase the sample set of trying to compare like say for example, bring other companies into the picture Nippon steel, POSCO and other companies in Germany it may be possible sale and JSW steel then their efficiency may be less than 1. That means when the comparison basically is much more based on the population then actually we will be able to get the overall efficiency of those set of companies. So, this would also be remembered as we proceed with this DA formulation. Now what are the nomenclature? So, I will basically mention the nomenclature they have similar set of analysis or the ideas which have already mentioned.

So, here M is the number of inputs if you remember there M was number of alternatives, N is the number of outputs, there N was the number of criteria, here the number of alternatives which is the DMUs we are considering as K and I mark with the different color K as in blue. So, we want to find out based on the set of inputs M in number for each DMU outputs being N in number for each DMU we are going to find out the efficiency of the K number of DMUs. And what are the variables? Variables are if you read it is XIK is the ith input variable for the Kth DMU, YJK is the jth output variable for the Kth DMU, UIK is the input weight for the ith input weight for the Kth DMU and finally, VJK is the jth output weight for the Kth DMU and what is IJK? Let me verify once again, small i changes from 1 to M, small i changes which I am highlighting with

blue color here, small j changes from j is equal to 1 to N which I am highlighting in green here till capital N and finally, the value of K which is for the DMU changes from k is equal to 1 to capital K. So, you have basically IJK, i changes from 1 to M, j changes from 1 to N and k changes from 1 to capital K. Now, before we consider the simple model we are going to consider the very simple model.

There can be three different types of productions and returns concept. Now, this basically comes from the concept of how the economy is doing. So, we will consider that is in a increasing return to scale market or economy, one is a decreasing return to scale and one is a constant return to scale market. Increasing return to scale means the more you produce more the profit is, more the output is, more the revenues are. So, per unit increase would basically mean your total profit function is on an upward trend.

Decreasing return to scale means more you produce the overall output, if it increases it gives a negative return in the sense it is decreasing revenues are decreasing and constant means the more you produce the rate of change of the revenue remains constant. Now, these three graphs of increasing, decreasing and constant we have already seen, but not in this context we have already seen in the concept of the risk. If you consider the concept of risk there was marginal utility was increasing at an increasing rate, were increasing at a decreasing rate and was increasing as a constant rate. So, the graph which we had in the context of risk was very simply given like this. If you have the wealth W along the X-axis, utility I am only utilizing the symbol U which is along the Y-axis, the graphs would be the blue one would be where the rate of increase is constant, the green one would be where the rate of increase is increasing at increasing rate.

And finally, the red one would mean that is increasing at a decreasing rate, because the slopes are changing with the blue one constant slope d Y/d X or d U /d W and the green one is increasing, rate is increasing and the red one it is decreasing that is d U/d W is decreasing. Now, let us consider the diagram in more details and if you consider the decreasing return to scale diagram in details other things would come out very easily. Consider I am plotting the bundle of inputs all the inputs which are there along the X-axis which I am just marking for I will erase the color, but I am marking for simplicity for all the viewers and the readers to see this IP which is input I am marking along the X-axis. The output I am again marking in red is along the Y-axis and the graph which is here which I will highlight using the violet color which I will again erase is this one is increasing, but the slope is slowly falling and this is what I want to now highlight. Now, notice here notice here there are DMUs are on this side of the curve depending on all the DMUs to be maximum efficient they should lie on the curve of the production possibility frontier because it is decreasing return to scale.

So, if a DMU is with suffix 1 and considered is inefficient because it is not a sufficient not on this the efficient frontier line. So, it can reach the efficient frontier line in many ways it can go either vertically up which is the blue line, it can go horizontally to the left which is the pink one and apart from the infinite ways it can go I will only consider one and why I will do that I will come to that later is the one which is shown in green which is moving radially in the direction till it touches the curve. Now what I mean by radially

is basically if I consider the green line the angle which is there the angle which is there which I am drawing here is 90° because that would be considered as the shortest possible way in trying to reach the point of the curve such that the DMU_1 which was inefficient can be made efficient. Now when I say the word efficient and I have marked the blue green and the pink one what does that mean now that means the following. If I consider the pink one and if you notice the slide the pink one means the input was at IP_{12} which I am now highlighting here the output remains same which is OP_{11} and the input decreases from IP_{12} till IP, see for example here that means at the same level of output the inputs are being decreased that means I want to maintain the same output but my total input is being decreased.

 Example I may hire less number of people all the contractual labors I am just going to the other example of say for example the steel company which I mentioned or the car companies which I mentioned I want to produce the same amount of output but the more I produce the revenues would decrease at the at would start decreasing at at at a increasing rate and I want to basically maintain my output but use less input. Less input may be one example can be I recruit less number of people do not hire the temporary workers give some extra over time to the permanent worker and do that work. Another way can be I want to basically maintain my input and increase the output till it reaches the vertical height which is basically DMU_1 efficient which is blue in color this DMU_1 efficient input based is the horizontal movement on to the left hand side if you are seeing the slides yourself. And in the case when you when I am considering the blue one which I just mentioned inputs are being fixed outputs are being increased and in the case of the DMU efficient which is the green one where it moves radially there what we are doing is we are trying to based on DMU_1 which was inefficient we are trying to decrease the input as well as increase the output that means doing a balancing act on based on that. Now which means that the DMU_1 which was inefficient can reach the efficient frontier in different ways and these three are the three different scenarios based on which DMU_1 which was inefficient can reach.

 Now if you have noticed one thing in the graph the blue one is marked as DMU_1 efficient and then in the bracket in the suffix I have written OP which is output oriented because their input is not being changed output is being increased because it is vertically up. In the case when I am going horizontally on to the left it is written in the pink one DMU_1 efficient and in the suffix is written I/P which is input oriented which means that output is being kept fixed and input is being changed. So, this input oriented model and output oriented model are the ideas of input oriented formulation of the problem and output oriented formulation of the problem will be coming up within few minutes. Now similarly if I consider the decreasing return to scale I have spent quite a lot of time in trying to explain the decreasing return to scale, but there is one small change in the graph. If you consider the graph here which we considered there the output were all considered along the Y-axis and the inputs were all considered along the X-axis.

 So, in this graph they are just interchanged that means output is along the X-axis and input is along the Y-axis actually it is the first diagram is actually I spend time because that would make much better sense. So, all the detailed discussions which I mentioned

that how the DMU which is inefficient can go vertically up go horizontally out to the left or go radially would all be repeated in this diagram also. Now consider the increasing return to scale, now in the increasing return to scale again I am considering the idea of input along the X-axis output along the Y-axis ideas are same. The curve as it is increasing return to scale I will highlight it once first and then do the detailed discussion. So, this is the curve which is increasing return to scale and all the DMUs are on this side of the curve and they want to reach the efficiency which is along the curve which is marked in blue.

Now here is the analysis same thing, but I will just repeat and highlight the important facts here. I consider the DMU is inefficient DMU_1 inefficient and in order that this inefficient DMU which is 1 it can be efficient and can reach the efficient frontier again there are innumerable ways, but I will consider only these three different ways of trying to move to the efficient frontier. One is moving vertically up which is the pink line going horizontally left with the blue line and going radially which is the green one. And if you notice the pink one is mentioned as DMU_1 efficient IP which is input oriented one moving on to the left which is the blue one horizontal left is in the as I mentioned the color is blue DMU_1 efficient OP which is output oriented and if I consider the green one again it is tangent $90°$ on to the curve this angle is $90°$, this angle is $90°$. If I consider this the movements are the green one would be trying to balance both of them.

Now let us see how the balance comes up for the pink blue and the green. If I consider the pink one it means that I am keeping the input fixed and I am increasing the output exactly the same way we would have considered in the first diagram which is decreasing return to scale keeping input fixed increasing the output. Now when I consider the output being fixed input being decreased it is moving along the horizontal left and when I consider the radial direction it is basically a balance of trying to adjust the input and adjust the output. Now the reason why I have drawn a DMU_1 efficient input based DMU_1 efficient output based in different colors pink and blue here in the increasing return to scale and also in the similar way DMU_1 efficient input oriented and DMU_1 efficient output oriented which are marked in pink and blue color respectively will come to the model formulation when we pay specific attention to the decreasing return to scale model and the increasing return to scale model where input oriented model or output oriented model which is important and which is more possible and one more practical will be analyzed. With this I will end this class and we will continue discussion of the DMU model later on.

Thank you very much and have a nice day. Thank you.