Multi-Criteria Decision Making and Applications
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Lecture 54

Welcome back, my dear friends, a very good morning, good afternoon, good evening to all of you and this is the course title multi criteria decision making which is under the NPTEL MOOC series and the total number of classes is 60 which is spread over 12 weeks and we are in the 11th week considering the different concepts of multi criteria decision making, but from the non parametric concepts. And as you remember each week we have 5 classes, each class being for half an hour and after each week we have one assignment. So, in totality there are 12 assignments and in the end of the course they would be one final examinations based on the total coverage of the course. And as you can see my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. So, this is the 54th lecture which is the last but one lecture in the 11th week and the total idea is of the concept is multi criteria decision making, we have covered multi objective, we would also consider multi attribute theory, utility theory very briefly again I am mentioning and we are discussing multi attribute and MCDM techniques in the non parametric cells. So, in the last class we have started discussing about analytical hierarchy process, we drew the hierarchy, explained the hierarchy, what were the steps we have discussed and how the linkage was being done from the alternatives to the final decision through the hierarchies, hierarchy 1, 2, 3 accordingly and we left where the scoring system were to be analyzed and this is what we will consider and I will solve a very simple problem accordingly.

So, that coverage is analytical hierarchy process for this 54th lecture. Now consider very simple example, consider Ram has received the final call from three IIMs which are IIM A, B, C, Ahmedabad, Bangalore, Calcutta. His main criteria based on which you will take the decision in joining one of the IIMs would be there are two points, one is academic reputation and one is placement position. Obviously, when Mr. Ram is trying to take the decision of joining one of the management institutes or consider any student is trying to join any education institute, the reputation, the type of faculty members and obviously, what is the infrastructure, what type of peer groups are there, facilities everything matters, but I am only keeping it very simple for this example. So, reputation of the academic and placement potential is important for Mr. Ram. For Mr. Ram reputation is twice or two times more important than placement potential, thus the placement potential is given a score of 1 by 3 by him while the academic reputation is given by a score of 2 by 3.

So, the total score adds up to 1 and there is a background I will come how the scoring system is done. So, thus when the ranking system is made for Mr. Ram, the criteria and

the ranks for the alternatives, what are the alternatives? R3, IIM A, B, C. So, I will mark the alternatives with a different color which I always do in order to highlight. So, I will use the color red.

So, alternative one is A1 which is IIMA, A2, IIMB, A3, IIMC and what are the criteria I will use a different color consider blue. So, this academic reputation C1, placement potential C2 and the scores for when you compare the criteria comparison that has already done by Mr. Ram 1/3, 2/3. So, that was in the last slide. Now, consider based on the alternatives for each and criteria are being analyzed and what are the scores according to Mr. Ram are as follows and put a black tick mark in order to mention that. So, for academic reputation for Mr. Ram IIM Ahmedabad is the first position which is 0.4, second is IIM Calcutta which is 0.35 and third is basically Bangalore which is 0.25. Interestingly if you add up $0.4 + 0.25$ which is 0.65, $0.65 + 0.35 = 1$. So, overall score based on academic reputation across all the alternatives when added up is 1. When I come to placement potential according to Mr. Ram they are as follows for IIM Bangalore is 0.4 which is the highest and the next in line of the same ranking is Ahmedabad 0.3 Calcutta 0.3. So, adds up to 1 also for all the three alternatives for placement potential also. Now, when the scoring system is being done by Mr. Ram he would basically multiply or as per the concept which I drew in the hierarchies it would be the scores multiplied by the corresponding points which are being given for the criteria.

So, if you remember the corresponding points which were being given for the criteria were as follows. So, it was placement potential carried a weight of 1/3 and academic reputation carried a weight of 2/3. So, again here also $1/3 + 2/3 = 1$. So, for Ahmedabad it will be from where do we get 0.3, 0.3 is coming from placement potential $\times 1/3 + 0.4$ coming from academic reputation $\times 0.23$ into two third which is 0.67 would be the total score for Ahmadabad. Similarly when I consider Bangalore it will be 0.4 which is placement potential corresponding to Bangalore multiplied by the score which is for placement potential which was 1/3. So, $0.4 \times 1/3 + 0.25 \times 2/3$ and finally, for Calcutta it would be 0.3 for the placement multiplied by its corresponding weight which is $1/3 + 0.35 \times 2/3$ and we find out the scores accordingly and then we can rank them. So, the work here which we did looks very simple, but now slowly we will bring one concept at a time. Consider Ram has a brother by the name of Shyam till now we did not bring him into the picture Mr. Shyam. Suddenly after few days Mr. Shyam also gets calls from the same three sets of institute Ahmadabad, Calcutta and Bangalore and due to some reason either both of them want to go to the same institute because they want to be closer to each other or some other reasons I will talk about that or they would mainly try to be at different places. Why they want to go to the same places? Maybe they want to be there in Bangalore because all their relatives stay there. So, it is easy or maybe Bangalore has better connectivity with their hometown or consider it would be easier for their parents to travel to Bangalore if

they want to visit them. So, they can be different reasons for that. So, now consider the corresponding points which are being assigned by Mr. Shyam the second brother. He gives a placement potential a score of 1/4 it is not 1/3 because 1/3, 2/3 was the decision based on Ram's thought process how he wants to assign the scores and Shyam gives a higher points for academic potential maybe he wants to be going to academics and based on that he wants to pursue a career to be a faculty member. So, for him the academic reputation is 3/4 which is 75 % and placement potential is 1/4 which is 25 %. Again remember here 1/4 + 3/4 add up to 1 and for him Mr. Shyam the placement potential corresponding points are as follows. He thinks I am Calcutta is very good in academics. So, he gives a score for placement potential sorry my mistake I am talking about placement potential. So, placement potential comes out for Calcutta as for according to Mr. Shyam is 0.5 for Bangalore is 0.25 for Amadhar is 0.25 again 0.5 + 0.25 + 0.25 add up to 1. This is totally different sets of scores for placement potential for the three institutes or the alternatives based on Shyam's analysis. Mr. Shyam's analysis for academic reputation is also different with respect to his brother Mr. Ram. Academic reputation for Mr. Shyam for these three institutes which I will mention as I am A, B, C are correspondingly 0.35, 0.35 and 0.3. So, if I want to find out the corresponding points for Shyam only I will follow the exactly the same procedure of trying to multiply and get the scores accordingly.

So, if I want to find out the scores from Ahmedabad I will do one by one. So, placement potential is 1/4. So, it will be 1/4. So, this is multiplication I am using a * of 0.25 + academic reputation is basically 3/4 × 0.35. So, if I want to find out for I am Bangalore it will be 1/4 × 0.25 + 3/4 multiplied by 0.35. And finally, for I am Calcutta this is only for Shyam remember this is 1/4 × 0.5 + 3/4 × 0.30. So, this scoring of Shyam would be for Ahmedabad then he will also do for I am Bangalore and will do it for Calcutta and similarly Mr. Shyam has also done. So, they are individually trying to maximize the benefit, but now the situation gets more interesting. So, this was for Ram this is for Shyam and as I said it is basically scores are correspondingly done.

So, if you consider one-fourth as 0.25, so I find out the scores accordingly. So, 0.25 × 1/4, 3/4 × 0.35. So, this was for Ahmedabad, this was for Bangalore and the three tick marks which I put is for Calcutta.

So, Ram's and Shyam's collectively hierarchy is given and here I would basically mention the hierarchy points which I am giving and this hierarchy is being giving the corresponding scores. So, if you remember if I go back to the last class set of hierarchy which I have done. So, I will use a small structure in order to denote the hierarchy. So, this was the decision. So, there may be hierarchy like the first one, the second one and the third one.

Similarly, the hierarchies would be ending and the second hierarchies are given by see for

example, green color and the alternatives are given by the blue color violet. Let us consider violet. So, consider there are two alternatives in order to minimize cluttering. So, the connectivity is like this and here for the second alternative, so I will mark A 1, A 2 here as we have done. Now, according to for each connectivity which is there, there are points given.

So, this is the points which I am going to consider. Now, very interestingly the decision is to be considered collectively by both of them in consultation with their parents. Now, if we consider equanimity, so that means both these siblings are given equal weightages by their parents. So, the scores being given to Ram and Sham in the collective decision is 0.5 here and 0.5 here, which we are considering it as an example. It can be different for many decisions it can be different and when I consider the points for hierarchy 2 for Ram for placement potential and academic. If you remember they were I will use a different color red, it was 1/3 for placement 2/3 for academic for Ram and in the same hierarchy even though I will use a different color dark in color, it was 1/4 for placement for Shyam and three fourth for academic reputation. So, 1/3, 2/3 are given by the points P 1, P 2 and remember P 1 + P 2 adds up to 1, 1/4 and 3/4 are given the scores of Q 1, Q 2 which also add up to 1. Now, let the graph be there on the right hand side I will come back to this within   few   minutes.

If I consider the corresponding points and the alternatives were given again I will use a different color consider the little bit darkish yellow. The points based on alternatives for Ahmedabad, Bangalore and Calcutta were if you remember 0.3, 0.4, 0.3. So, it was basically 0.4, 0.3 and 0.3 for the placement. So, it is Ahmedabad 0.3, Bangalore 0.4 and Calcutta 0.3. So, if I come here they are given as this Ahmedabad 0.3, Bangalore 0.4, Calcutta 0.3 add up to 1 remember. If I consider these are mentioned as P 1 1, P 1 2, P 1 3 because   they   are   following   from   the   node   P.

So, they were basically if you consider node P 1, P 2, Q 1, Q 2 1 was there. So, obviously any arrow or any sub heading or later hierarchy which is coming out from P 1 would be named accordingly like P 1 1, P 1 2, P 1 3 accordingly. Any hierarchy coming out from P 2 would be mentioned as P 2 1, P 2 2, P 2 3 and accordingly mentioned. Similarly, for Q 1 and Q 2 they will be mentioned as Q 1 1, Q 1 2, Q 1 3. Similarly, for Q 2 it would be Q 2   1,   Q   2   2,   Q   2   3   that   is   what   we   are   doing.

So, if I see this table, so say for example, if this is I am just mentioning if this is P 1 and this is P 2, then the corresponding points here would be P 1 1, P 1 2. I am not considering the middle hierarchy red one because or it will it will get cluttered and I consider the extreme red one as P 2. So, that would be mentioned there again two arms coming out. So, it will be 2 P 2 1 and P 2 2 and I will consider it accordingly and go down. So, if I consider

say for example, from P11 accordingly there are other two.

So, it will named accordingly based on the nomenclature. So, I do not want to write it here. So, let us it get too much cluttered. So, similarly when I consider the arms or the arrows coming out from P2, they are mentioned I will use a different colour violet. So, there is P 2 1, P 2 2, P 2 3 are mentioned as 0.3, 0.4, 0.3 and if I correspondingly consider the values for Q 1 1, Q 1 2, Q 1 2, they are given as 0.25, 0.25, 0.5 and finally, Q 2 1, Q22, Q 2 3 they are mentioned as 0.35, 0.35, 0.3 and the hierarchy goes. So, finally, when I find out the total scores of Ahmedabad, Bangalore, Calcutta which are the three alternatives and if I go back here to the diagram. So, what I will be doing consider there are two alternatives again I am mentioning it can be extended I am only considering two alternatives.

So, I will basically find out the total scores and calculate them accordingly. So, the total scores if I consider from A1 and I will mark the arrows with let me see which colour can I use. I will use the blue one. So, path 1 from A1 is basically marked here. So, I will basically multiply all the points along path 1 where I put only one tick mark of blue.

Similarly, if I consider path 2 where I will put a double tick mark are these. So, accordingly I will basically find out all the paths from A1 to the decision keep multiplying the points for each of the path and then considering there are 10 different paths I will add up all the 10 different points and for each path they are being multiplied as I move from the lower most hierarchy to the top most that means from the alternative to the decision. This is what is being done. So, if I consider the corresponding Ahmedabad. So, what are these scores? So, this P and if you remember P we have considered that with they are being giving equal weightages by their parents to both their sons Ram and Cham that is why P is 0.5, 0.5 for both of them is 0.5 here, 0.5 here, 0.5 here and I follow the hierarchy. So, the first route, the second route, third route, fourth route all the routes are being added up. So, this will be the total score. Similarly, for Bangalore first route, second route, third route, fourth route finally, for Calcutta again first route, second route, third route, fourth route and they are multiplied and the values of P, P 1, P 2, P 11, P 12, Q 11, Q 12 all these Q's with the suffix P's with the suffix are already given. Now, we have understood how the scores are to be calculated total score, but how the scores were given for each path is not yet dealt with not yet discussed.

There are wide ranging on applications exist like selecting a car, deciding upon a place to visit, deciding about the MBA program and so on and so forth. The AHP algorithm is basically composed of two steps, we need to determine the relative weights of the decision criteria and we also need to determine the relative ranking or the priorities of the alternatives. Both qualitative and quantitative informations can be compared by using

informed judgments to derive the weights and the priorities. Derive the weights if you remember Ram and Sham were given different weightages for placement with respect to academic, it was 1/3 with respect to 2/3. In the second case for Sham it was 1/4 with respect to 3/4.

So, how were these scores given that is what we are discussing now. So, let us consider the example of where the objective is to select a car criteria is a style, cost, fuel economy which I mentioned, alternatives of civic, I-20, Escort and Alto very simple example. And here is the structure which I have drawn and now it should be absolutely clear. The alternatives I am circling civic which is A-1, I-20, A-2, Escort, A-3, Alto, A-4 and the hierarchy goes I am only drawing one hierarchy for ease of explanation. The hierarchy is style, cost, fuel economy if you can see one horizontal and the final decision is to select a car and all the alternatives are attached to all the hierarchies and finally, to the decision process. So, what we were mentioning few minutes back the scores is basically giving along the arrows and we keep multiplying for that particular route and then add it up for corresponding all the routes basically which start from that alternative to the final decision.

So, that alternative means if I am considering civic and only consider all the connection between select to the selecting the car which is the decision. Now, comes the interesting part how the points are given. Now, to talk about the background if you see in this table intensity of importance is given from score from 1 to 9. So, it is given 1, 3, 5, 7, 9, 2, 4, 6, 8 definitions are given which is equal importance as I am reading the second column somewhat more importance much more important very much more important absolutely more important and intermediate values and explanations are given. So, let me read the explanation equal importance means two factors contribute equally for the explanation to be made.

Three means experience and judgment slightly favors one with respect to other and as I go down the table with the scores increasing 5, 7, 9 the explanations are experience and judgment strongly favors one for score of 5, experience and judgment very strongly favors one is importance is demonstrated in practice very important and 9 means the evidence favoring one over the other is of the highest possible validity. For 2, 4, 6, 8 which are in between they basically are compromise in between a score of 1 to 3 there is 2, 3 to 5 there is 4, 5 to 7 there is 6 and 7 to 9 there is 8. So, technically SATI gave initially the scores or odds which were 1, 3, 5, 7, 9 and the scoring was given and here I will basically try to explain few things in details. So, this lecture may be a little bit more that we will try to basically try to wrap up AHP accordingly.

Now, when I am giving a score of 1. So, what I am doing I am trying to basically compare for any criteria the alternative 1 with respect to 2, A 1 with respect to 2. So, if I give a

score of A 1 to A 1 to A 1 when we are comparing A 1 to A 2 it means I am equally balancing the decision of taking A 1 or A 2. So, the corresponding score to A 2 would be given is the ratio or the inverse of 1 which will be 1 is to 1. If I give a score of 2 to A 1 in the score given to A 2 will be half 1/2.

If I give a score of 3 to A 1 it will be 1/3 for A 2. Similarly, for 5 it will be 1/5, 7 it will be 1/7, 9 it will be 1/9. So, that means higher the score for 1 the corresponding score which should go for the other one would be just the inverse which will much lower. So, if I consider the HP ranking for the criteria and alternatives. So, if I consider style the first hierarchy style, style cost, fuel economy, style fuel economy, cost and fuel economy. So, the principle diagonal if you see when I am comparing the criteria with itself alternatives would also come, but first I am only considering the criteria that is the hierarchy.

Style to style is 1, cost to cost is 1, fuel economy to fuel economy is 1 of the diagonal in element is important that is what the decision maker thinks cost for him is more important than style. So, he gives a score of 2. So, the corresponding value when he or she tries to analyze style will cost is half. So, if you see this 2 and this is half when I am comparing style and cost that means, the principle diagonal is 1 of that diagonal elements are not symmetric exactly the same idea we have conveyed when we are doing top says epsilon, epsilon, epsilon and VIKOR. Similarly when I consider style to fuel economy for him or her the value for style was 3 when compared to fuel economy.

So, fuel economy is one third and finally, when he or she is considering cost to fuel economy the score is for cost is 4 and hence fuel economy is one fourth again if you see they are not symmetric. So, we will consider the concept of considering the ranking based on the criteria and the alternatives. So, we will consider the concept of eigenvalues and eigenvectors. So, we will consider the maximum eigenvalue and corresponding the eigenvector would be of size and we are considering there are n number of such comparisons to be made.

So, A would be the comparison matrix of size n cross n for n criteria. So, that is what we are doing 3×3 and obviously, when you are comparing the alternatives that same idea of the size of the matrix will come. So, this will be called the priority matrix and X would be the basically the eigenvector for the size n cross n which will be called the priority vector. We will find it out using the simple mathematics and lambda max would be the eigenvalue. To find the ranking of the odd priorities we namely the eigenvector for X which is basically comparison matrix will first normalize the column entries by dividing each by the sum of the column exactly what we have done for the normalization scheme for electro epsilon electro topsis and vicodin you will take the overall row averages and that it which is being done. If you remember the principle diagonal as 1 the values I am only marking 1 here 1

was 2 and another was 1/2 which is 0.5 and I basically normalize the columns. When the columns are normalized sum is 1 and the row averages are found out which gives us the priority vector. So, I am not reading the values because the slice will be there with you can easily find out the normalized sum columns and then the row averages accordingly. So, which this I will end the class and continue discussing about the AHP which went over little bit more, but it is nice that we are discussing and we will try to conclude AHP in the next class and start of the next method accordingly. Thank you very much and have a nice day. Thank you.