

Multi-Criteria Decision Making and Applications
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Lecture 53

Welcome back my dear friends a very good morning, good afternoon, good evening to all of you and as you know this course is multi criteria decision making and under the NPTEL MOOC series and this is a course which is spread over 12 weeks which consist of 60 lectures and we are in the 11th week of set of lectures and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. So, if you can recollect as for the plan of the course we had discussed a lot of examples first then utility theory, then details about multi criteria decision making, solved problems for linear programming, integer programming, quadratic programming. Then we very briefly considered multi attribute, then went into multi attribute utility concepts just the formulas, then we went into different type of non parametric methods. We have covered Electre method, Epsilon Electre, Topsis, Vikor and today we will start a new method which will be the coverage of this course. So, the broader set of topics is multi criteria decision making, we have already covered multi objective decision making, we have already covered multi attribute decision making main points, multi attribute utility theory very briefly concept which I am again repeating. And the fifth set of type of methodologies which we will consider under the non parametric method for multi criteria is analytical hierarchy process.

So the word analytical hierarchy process is basically a subset of the ideas of analytical network process. So we would not be considering the network process, we will only concentrate on the simple formulation of analytical hierarchy process. So as the word hierarchy says, so there the decisions are basically broken up into hierarchies of decision and then based on how the hierarchies are connected we would basically go from the bottom level to the top level, keep adding the scores how the scores are added we will come to that later and then rank the decisions accordingly. I will just keep it to the basics.

Analytical hierarchy process is a structured technique for organizing, analyzing complex decisions. So decisions can be to buy a car, decisions can be to buy an apartment, it can be related to choosing a program for higher studies, it can be all the different type of multi criteria decision. These three examples I keep repeating in the way they make sense corresponding to the concept of the methodologies we are studying. It was developed by Thomas Saaty in the 1970s and is applicable for group decision making. So where is analytical hierarchy process used? It is used in multi criteria decision making, they used in the case where both qualitative and quantitative decision makings are used.

It tries to mimics human decision process, is actually generally well accepted in many of

the decision making process and is widely used and it is easy to implement, easy to fathom, easy to understand and easy to convey the results to the other party and it is quite intuitive. Now I will try to draw the overall analytical hierarchy process and explain it in detail. So what I will do is that I will create a blank slide and explain it in step by step. So here how we will consider? So this is the slide where we left. So all the main points based on which why we are considering AHP as a method.

So how does the AHP schematically looks like? So consider the problem is before I draw the diagram, consider the problem is to buy a car and consider for our understanding there are four different cars to be analyzed that means four different alternatives from which we want to choose the car. So without going to the name let us mark it as like this. So the alternatives for this cars is A_1, A_2, A_3, A_4 . So these are the cars and I want to make the decision accordingly. Now what are the criteria? So the criteria I will mention so that it will make much more sense why the criteria are be considered.

So mark the criteria with different colors and as I do for each and every explanation in this slide as well as in lectures I use different colors so that the information and the ideas which I am trying to convey is easily passed on to the participants and the students. And there is a very specific reason why I am trying to utilize different colors which I will explain later on. Consider the first set of hierarchy. So as I mentioned the word hierarchy, hierarchy means there are levels like if you consider a tree and if you are taking the cross section of tree and considering the from the level where the tree trunk touches the soil and the root goes. So if you consider that it is like this.

So this is the level of the soil, this is the tree and the tree trunk goes like this. So even though it not exactly mimic the roots but the hierarchies would be considered accordingly. So this is what I will draw now. So let me erase this one because this was just drawn in order to make some pictorial imagination how the hierarchy would be there. Consider the hierarchies are, I will use the red color the dark red was for the alternatives, hierarchies are say for example, price.

So I will mark it as C_1 on the same level the hierarchy next one again it is red in color. So consider the style of the car, the seating space, arrangement, the dashboard how the car looks like. So I will mark it as C_2 . The third one is again I will continue using the same hierarchy color consider it is mileage C_3 consider the next one as safety C_4 . Now under $C_1 C_2 C_3 C_4$ which are the first hierarchy let us consider there are second level of hierarchy some may have some may not have.

So I will use now the blue green color. Price may have say for example, purchasing price which will be denoted because it is under C_1 . So it will be denoted as C_{12} and there is

resale price considered as RP. So it will be denoted by C_{11} . So C_{11} and C_{12} style considered does not have any categories next hierarchy.

So let it be as it is C_2 . Mileage say for example, I consider again green color. So mileage I will consider as city based on city consider it as C_{31} and in the highway. So better the mileage better the car I would consider this C_{32} . Safety features consider there are other levels of safety features consider safety features can be crash safety features and without naming it let me put it as C_4 that is C_{41} , C_{42} and consider more such levels of hierarchies can be there which I am not drawing.

So not to clutter the hierarchy as such. So if I consider so now I will only mark and use the nomenclature of the numbering. So C_1 C_2 C_3 C_4 first hierarchy C_1 has C_{11} and C_{12} , C_2 does not have any second hierarchy, C_3 has C_{31} , C_{32} , safety has C_{41} , C_{42} and based on this criteria what are you choosing you are trying to find out which car to buy and how many cars are there, there are 4 cars A_1 , A_2 , A_3 , A_4 and finally what we want to choose is the main objective the decision of buying the car. So let me mark C_1 to C_4 are there so I will mark them as follows I am not going to mark the price style mileage safety it will be only marked in the in the numbering scheme. So cars are marked as A_1 so now the hierarchy will be drawn considering this example and the coloring scheme would be there in order to mark it.

So the main decision is to choose the car which is decision of choosing this is the black box, black box not as this just the coloring scheme I am doing. So this is the final decision which I will take and in the next table there was C_1 to C_4 so mark the connectivity with the black color and the this is C_1 the first one this is C_2 this is C_3 so they are in the same horizontal level of hierarchy C_3 , C_4 done. Now the next coloring was green so again I will use the black arrows connectivity and they were so there is no connectivity for C_2 in the next table. So I will not join it immediately C_3 had 2, C_4 had 2 so let me mark it as with the green color so this is C_{11} , C_{12} , C_{31} , C_{32} this is C_{41} , C_{42} so green color and the second hierarchy same horizontal level. Now what are the alternatives A_1 to A_4 so we will mark them so these are alternatives so A_1 , A_2 and how the connection is done I will come to that A_3 , A_4 how the connections are done I will use the black color arrow that is why the coloring scheme I mentioned a little more details.

So my decision is to buy the car so it can be one of the scores is here. Now A_1 connectivity is there with C_2 also so now as A_1 for choosing A_1 I can take the route A_1 , C_{11} , C_1 decision A_1 , C_{12} , C_1 decision A_1 , C_2 the decision decision is the black final decision A_1 , A_{31} , A_3 decision A_1 , A_{32} , C_3 , C_{32} , C_3 decision A_1 , C_{41} , C_4 decision A_1 , C_{42} , C_4 decision. Similarly for A_2 all connections would be there which is I will repeat for A_2 A_2 , C_{11} , C_1 decision A_2 , C_{12} , C_1 decision A_2 , C_2 decision A_2 , C_{31} , C_3 decision

A2, C32, C3 decision A2, C41, C4 decision A2, C42, C4 decision. Similarly connection is there with A4 also. So let us not get confused with this whole set of black lines, but I will again read it for A3 and A4 without drawing it.

So A3, A3, C11, C1 decision A3, C12, C1 decision A3, C2 decision A3, C31, C3 decision A3, C32, C3 decision A3, C41, C4 decision A3, C42, C4 decision. Finally for A4, A4, C11, C1 decision A4, C12, C1 decision A4, C2 decision A4, C31, C3 decision A4, C32, C3 decision A4, C41, C4 decision and finally, A4, C42, C4 decision. So all the connection routes have been analyzed. So what we will do is actually follow the paths from A1 to the decision with all the routes similarly for A2, next is A3, finally is A4. So there are 4 alternatives we will find out the scores corresponding to A1 till the decision then A2 till the decision A3 till the decision A4 till the decision and rank them accordingly.

So when I tell or mention the word from the alternative which is A1, A2, A3, A4 till the decision I am trying to find out all the combinations of path for A1 and collectively finding out the combined score. Similarly when I talk about A2 and the decision I find out all the paths from A2 to the decision and combine all the scores accordingly. Similarly from A3 to the decision and finally, from A4 to the decision. Now the question which may crop up in the mind of the participant and the students is that can there be more hierarchies? Yes they can be. So if you see this set of slide which is in front of you there are technically two such hierarchy one being C1, C2, C3, C4 at one hierarchy and then C11, C12, C31, C32, C41, C42 in the next hierarchy.

Correspondingly the hierarchies would increase so the connection would be there connection means the paths would be there you have to calculate it accordingly. The next question is that can there be more alternatives? Yes they can be. The final point is that how do we give the scores in trying to find out the corrective scores for each and every alternatives. So, that would be analyzed in the process of the discussion AHP and remember the scores are given in such a way that we keep multiplying and then adding the scores in order to reach the final ranking system with respect to each and all alternatives and then rank them accordingly.

So, this is the basic idea. So, in AHP analytical hierarchy process there are two fundamental approach in solving the problem one is the deductive approach and one is the inductive systems approach. Basically the deductive approach focuses on the paths whereas the system approach concentrates on the working of the whole and one can basically find out more details about it by in the book written by Thomas Satty and the title of the book is decision making for leaders. So, quite a comprehensive nice book. AHP combines these two concepts which I just mentioned in one integrated logical framework. AHP is that it is designed to handle situation in which the subjective judgment of the

individuals constitute an important part of the decision process and this subjective decisions is basically analyzed giving scores and if you remember I have mentioned that we basically multiply and add this scores or points accordingly and how this scores are given would be based on the analysis with the decision maker is making when he or she is trying to analyze the alternative based on the criteria.

So, if you remember in Electre process epsilon-Electre, Vikor, top-sis method in all these methods we have considered the alternatives are being analyzed based on the relevance of the criteria and this is exactly what we are doing. take the alternatives try to compare the alternatives based on each and every criteria and then give scores. In the other four methods which I just mentioned Electre method, epsilon-Electre, top-sis, Vikor, the concept of NIS, PIS, discordant set, concordant set, indifferent set all these was considered based on the positive and the negative net worth and in those methods we have already considered the comparison was being done between two alternatives based on each and every criteria. We will follow the exactly the same concept, but here some scores would be given based on the liking and the disliking which we will see within few more minutes. So, how does the step for the AHP work? Exactly the same way if you remember in all these four methods I will keep repeating because there is a logical flow in the different type of nonparametric methods we have considered.

So, they were steps given and if you remember in the four methods we have already considered Electre method, epsilon-Electre, top-sis, Vikor. We considered that we had decision matrix based on the value and then found out the normalized decision multiplied by the weights found out the weighted decision matrix and based on the weighted decision matrix we proceeded to find out the ranking and give the scores accordingly. So, whether it was AIS, PIS, concordance, discordance, indifference concept whatever. Here is we first develop the hierarchy of the components here means AHP. We establish priorities for the criteria and priorities are based on the criteria when we are trying to compare the alternatives.

We check the consistency and consistency point is important because in many of the cases ranking system may not be consistent and this point I have mentioned in the sense I will just briefly again revisit that point or that idea. Consider you have in the objective scale if you had numbers 3, $3 > 2$ and if $2 > 1$ then you can say according to the basic properties of mathematics 3 would be greater than 1. But if I consider these different MCDM techniques then it may be possible that in one case A are the alternatives see for example, A_i is better than A_j and A_j is better than A_k , but it may be possible that A_i is not better than A_k it is not obvious that A_i is better than A_k . So, those corresponding inconsistencies have to be looked into and taken care of. So, that is why the third step says check for inconsistency, establish the priorities for the decision alternatives based on each criteria as I just

mentioned few minutes back and compute the overall priority ranking of the alternatives.

So, if you remember the graph or the hierarchy and I mentioned about A1 till the decision, A2 till the decision. So, what we are doing when you are following any set of paths and when I use the word set of paths is means I am starting at A1 and reaching my decision. So, they can be different paths to reach the decision. So, I am basically calculating the points for each paths and keep adding them. So, that is what basically it says compute the overall priority and then rank the alternatives accordingly.

Decision analysis process involving finite number alternatives arise frequently practical situations. One must remember that the type of data available for analysis based on which one has to draw some conclusions can be deterministic probabilistic or uncertain. When the data is uncertain then one of the many tools which is very convincingly used successfully used is basically the analytical hierarchy process about which we are talking. In AHP subjective judgment is quantified in a logical manner and then utilized to reach some meaningful conclusion. So if you remember in all the four methods again I am repeating, Electre, epsilon-Electre, Topsis, Vikor we had considered the normalization along the columns and all the criteria's were mentioned along the columns.

So if it was a decision matrix which was the first one non-normalized case it was of a size m cross n where m was the number of alternatives, n was the number of criteria. So the normalization was done along the columns and based on that we multiplied that normalized matrix with the weights, weights were based on the criteria and found out the normalized weighted matrix. Now the concept of trying to normalize was it was very intuitive because many of the criteria's may have different units, some may be objective, some may be subjective and even if units are there for each of these criteria they can be different as I just mentioned like one of the criteria can be based on weight, another criteria can be based on length, another can be based on amount of money, rupees, another can be based on volume. So in order to compare these criteria where the units may be different not commensurate, so we normalize them and basically bring it on a score of 0 to 1 that concept would also be used in some way. Now here when we give the scores based on the criteria's for the alternatives they based on the ranking system of points and then these points are utilized to find out the consistency concept and then do the ranking if consistency is met.

One must remember that the decision maker's assessment towards the risk and his or her attitude toward return or average benefit, so the person is trying to balance the ideas of return and risk and this point we have seen was very evident in the concept of epsilon Electre. Epsilon Electre had that idea that you were basically trying to divide the overall liking disliking for any criteria for the alternatives based on the concordance index, the discordant index and indifferent index was also brought into the picture such that we had

the concordance set, concordance matrix, discordant set, discordant matrix, indifferent set, indifferent matrix. This was not the case for the simple Electre. So again coming back to AHP this would also be considered accordingly such that it would reflect such that means when the decision maker considers the risk and return he or she basically wants to convey the decision maker's overall outlook about any decision and take that decision accordingly. So with this I will end this lecture and continue more discussion about the AHP in the later lecture. Thank you very much and have a nice day. .