Multi-Criteria Decision Making and Applications
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Lecture 52

Welcome back my dear friends and students who are taking this course of Multicriteria Decision Making a very good morning, good afternoon, good evening to all of you wherever you are and whatever time you are listening to this lectures and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur and the course name as you know I keep repeating the course name and the lecture number and do go into the last set of slides or the last lecture so there is a continuity. So the course name is Multicriteria Decision Making and we are going to start the 52nd lecture out of the 60 lectures and the broader outline is Multicriteria Decision Making and here we are considering the non-parametric methods. What is the non-parametric methods? We are considering the concept of VIKOR method, we have already done the TOPSIS method, we have already done the electro method, we have already done the epsilon electro method. For the initial part of the top-syst method everything is same like there is an X matrix of the criteria and the alternatives, the value which is coming then we normalize it, there are different methods of normalization and by the way in few of the lectures I did mention the values which are accruing from the criteria for the alternative based on the utility or the decision makers concept of utility. After normalization we also pay attention to the weights which are based only on the criteria depending on the size of the matrix for the normalized matrix and the weights we find out the weighted normalized matrix and then we proceed. So that is what we are going to do for the VIKOR method.

So the coverage is we are continuing with the VIKOR concepts and I added this slide in order because the values which we will be working with for the problem is are here. This is the set of values which we will be working with which is weighted normalized matrix. Now this is the slide where we ended the last class and we need to determine what we said was basically and where I highlighted is basically find out the maximum best benefit of that and what is this. For each of this criteria which is marked in J considering for each of these criteria which is marked in J why because there are N number of criteria we find out F_{ij} and trying to find out the maximum for all of the alternatives considered.

So if I consider that we will basically have how many criteria we have, we have N number of criteria so how many such maximum varies would be it will be F_1^* to F_2^* where the star concept basically means the maximum. So this is F_1^* for the first one till F_n^* for the nth one and what we do is basically we find out the maximum. Now here it is very simple we have 1,…,M number of criteria alternatives and for the first criteria we find out the maximum. So for the first criteria which we are considering along the columns we try to

find out the maximum and note it down as F_1^* then go to the second column for the second J = 2 for all the alternatives 1,.., M we find out the maximum which is denoted I am putting a tick mark here F_2^*. Similarly we go to the last criteria which is the nth one and for the nth one considering the criteria for all this M number of alternatives 1,…,M we find out F_n^* and this is what we do and these are based on the values which we already have.

So we want to find out the maximum along the first column this is marked in green. So if you see it here the maximum values of 0.0 I would not read the decimal I will just read the numbers 536, 590, 765, 233, 916. So these values are given which are marked in green. Similarly for F_2 which is the second criteria marking in blue we need to find out the maximum here. So they would be maximum of again I am not reading the decimal just reading the values 1718, 1793, 1682, 2214, 1526 let us see they are the values here see. Similarly for J is equal to 3 third criteria maximum value found out again I am using the same color of blue and for the final one for the fourth criteria I am marking in violet the maximum value found out. So the maximum values found out are as this I will use the same color now just to highlight the values I am using the green color. So is 0.0916, 0.2214, 0.0016, 0.0161. Now I find out the minimum also if you remember max and minimum is basically based on this criteria of the nearest and the furthest. As max has been found minimum would be found out for each criteria consequently for all alternatives at each go.

So here the calculations are there I will use the red color F_1 - is for the first criteria all the alternatives F_2 - for the second criteria all the alternatives. F_n - is basically for the nth criteria all the alternatives and it is given here first, second alternative, last alternative. Again if I find out there are the minimum I am just marking the minimum word. So there is no confusion and when the minimum is found out they are based on the same weighted normalized matrix along each column for the criteria. So mark the minimum values and then I will read them so there is no confusion 0.0233, 0.1526, 0.00040, 0.0046. Now I plot them and this is an interesting one why I plot them is to bring the overall concept in a much more start and pictorial manner. What I am marking along the x-axis is basically the criteria if you see it when you have the slides mark it so I will mark it with the blue color and let me not use the blue color with the blue graph is there let me use the violet one.

So this is C_1 criteria 1, C_2 criteria 2, C_3 criteria 3, C_4 criteria 4 and along the y-axis is the value or the maximum and the minimum. So for C_1 the maximum and the minimum are given I will use the green for the maximum this is the point for the maximum and the minimum is given by this. So if I want to find out the distance of the measure of the maximum minimum in case required we will come to that this is this distance for the first criteria only. Similarly if I want to find out for the second criteria which are marked in green and red the distance is given by this. Similarly and interestingly if I want to mark it

for the third criteria I am marking at the same point the green and the red which is maximum minimum on the same so the distance is 0.

And finally when I mark it for the fourth criteria which is very nearby between each other the distance measure is given by again using the black color is given by this. So distance would basically mean that on the maximum and the minimum scale for criteria 1 and criteria 2 that what is the overall liking disliking concept which you can garner or understand. For the third one there is not much of distance 0 and similarly for the fourth one is very minimal but there is some significance for the second and the first criteria as you can see. So this is the graphical representation of the maximum based on the minimum words for each criteria which I have done. Now once this is done we need to compute the ratios and if you remember the computing the ratios which you have found out is based on the fact that I want to find out if you which I have done is to find out the difference within the maximum minimum and which is being normalized and then find out the difference of that value with the minimum or that value with the maximum also.

Now important thing if you remember in the case of the TOPSIS method we were only concentrating even though I did discuss we were only concentrating about the distance measure the ratios based on the negative impact PIS and NIS were there. So we are considering not the positive one, one the negative distance. Here we will be considering the positive distance because in the TOPSIS method the main emphasis is basically to find out the negative impact and here we want to find out the positive impact that is why here is what I mentioned and let us make it very clear. The ratio and here the denominator I will use the blue color is basically the difference within max and min and everywhere if you see this slide when you go to the slides which you receive you see everywhere it is basically in the ratio the denominator is the distance within the max and min. Here I will just mark few of them max and min corresponding to that criteria which you have similarly for all of them I am not marking all, but I am just highlighting where the denominator is.

Now in the numerator is basically the difference within the positive and that value. So now I will plot and try to compare what we did in the TOPSIS and what we are doing in the VIKOR method. Here is the straight line and the green one is the positive, the red one is the negative and the black one is say for example, the point which you have. So, in the TOPSIS method we were concentrating on the negative distance. So, it was basically I will use in the numerator is always this distance and in the denominator is always the difference with the max and min as it is there in VIKOR.

So, what is the max and min which here I will use the black color. So, in order to avoid confusion is this which is in there in the denominator. So, red divided by the black was the concept which was utilized in the TOPSIS method. What is here and there is some other

change. Here we are going to use the ratio of the green versus black, but the green will be weighted which will be given and if you remember where are the weights.

Weights are not corresponding to the fact which we have already utilized. Weights would be basically decided on the concept of what is the weightages you are going to give later on to fine tune your ranking system. May be the weighted system based on which you have found out the weighted normalized matrix need to be changed. So, here some of the weights can be done and where are the weights if you check they are the weights I will use the dark red color these are Wj's and j is what j = 1,…,n. So, that the weights are corresponding to the number of criteria's which we have.

So, for the first case which is I am considering the L_{11} based on the fact that I would basically have the L_1 norm for the first alternative the weights I am basically multiplying is W_1 for the first one then W_2 for the second one similarly W_n for the nth one. If I go and correspondingly do that I am using a different green color just to differentiate there is no concept wise W_1 for the first and similarly Wn for the last one and here remember the weights are accordingly considered. So, if I consider this here and to bring the notion more into reality I stick to the weights as it was decided what was the weights decided for each criteria it was 0.25, 0.25, 0.25, 0.5 why 4 number of times because there were 4 criteria and these I will highlight W_1, W_2, W_3 and W_4. So, based on that when I find out and with the ratios, ratios are remember again I may be repeating please bear with me the blue part which I am highlighting are the ratios when I am considering the first criteria. When I consider the second one they are marked in violet max minus min, when I consider the third one I am using say for example, the dark yellow. So, they should when the coloring scheme on this video lecture which you see and listen should make clear and the last one I am using the black. The other denominators and the numerators are based on the difference between the max and min which I just highlighted in the last slide.

Weights are given and once these weights are given which is fixed as one-fourth for each and erase them these are not required anymore for discussion because I have already done. So, here comes the weightage is based on the L_1. So, this is L_{11} is 0.29, 0.64 then is L_{13} is 0.68 I am only doing the first two decimal L_{14} is 0.71 and L_{15} is 0.68 and they are on the L_1 distance now. Now if you remember the graph which we have drawn L_1 was the maximum $L\infty$ was the minimum. So, I am trying to take the whole spectrum when I take the spectrum what I do is that I find out the $L\infty$ also for each and every criteria considering all the alternatives.

So here it is the formula I have $L\infty 1$ till $L\infty M$ and there I am trying to find out the corresponding length based on the alternatives. So, it is the same thing for each alternative for each criteria I find out the maximum which I found out and I will do it the same way.

So, I am trying to find out the max and again same concept persists the denominator is difference between the max and min everywhere. So, actually the coloring scheme would make it much clear. So, if I want to use this is for the first criteria the green one in the denominator.

If I use the red one these are for the second criteria. Similarly if I go to the nth one I am using the violet one this is for the violet last one and the numerator remains the same again difference between the maximum and that value. And also they are being multiplied by the weights which I will highlight using the light blue color these are the weights for the first criteria. This is for the second criteria similarly for the nth criteria and the weights are as usual given and considered as 0.25 for each of the criteria. This is what I do the weights I will basically first highlight using red color then I will erase them so there is no confusion. So, for this four criteria there and they are given also here. We can remove them to remove any confusion of coloring system later on as I discuss and then I consider the values of the maximum minimum I am considering which is for the first L1 for the L∞1 which is for the first alternative I am highlighting. Why 4? Because there are four criteria for the second one I am marking in red this is for the first criteria, second criteria, third criteria, fourth criteria and the red color means for the second alternative. Similarly the green one I am marking is for the third alternative.

Similarly I can mark for the fourth one and the fifth one and finally once done multiply the weights I get the values and what are the values and mark in green 0.1553, 0.2500, 0.2500, 0.2500, 0.2500. So, I have the L_1 distance measure for all alternatives corresponding to the criteria taken together similarly for L∞ and then I need to find out the max of them minimum of them. Now this max and minimum are not based on the fact of L∞ now they are the maximum value which we have considered like the ratios. So, based on that we find out the max and this max are given for all of these alternatives considered because max for all I, I = 1,…,m. So, once I have that I find out L_1 max and I find out the L minimum also based on L_1 values which I found out. So, I found out L_1, L_{11} till L_{1m} then L_{11} to L_{m1n} and find out the maximum and minimum in the same way as I showed you the graph based on the criteria.

So, once I have that the values of what are the values if you see these values they are for the L1. So, where as the L1 these values which were there I erase them and just highlight 0.2943, 0.6418. So, 0.2943, 0.6418 what are the other values 6804, 6807, 7194. First one is 6822, I find out the maximum here I will mark in red. Similarly the values for the minimum they are the same values from them I find out the minimum which is 0.2943. Similarly as I have done it for L1 I do it for the L infinity also I find out the max and mark it here is the max I find out the minimum I will mark in other color and I do it for all the L ∞ values which I found out from 1 to m, m is the number of alternatives.

So, these are the values I will mark them and again go back to that slide 2. So, these are 0.1553, 0.25 4 times. So, these are the values 0.1553 and 0.2500 4 times. So, once I have that maximum value is here I am marking in red again I take the same values I am marking in green now same values and I find out the minimum. So, I found out the maximum minimum for L_1 maximum minimum for $L\infty$. Now I can do a comparison. The comparison has nothing to do with the weights which was given for the criteria nothing. I tried to do it for the based on the fact if you remember the graph which I have what was the graph I am trying to find out plot P here and I am trying to plot L_P here. The graph which I will do in a different color was like this. The maximum was for P = 1 and the minimum was basically P = ∞ . So, in between I want to compare and find out the distances.

These are the extreme distances. So, what is the weightages I will give or some sort of values which I will give would be given by V_1 and V_2 or V and 1 - V. So, if I give V value of 1,…,L_1 that means I am putting all the distance measure based on L_1 which means if they are if you remember they were not unique L_2 was unique and in the case when I am considering the L infinity the distance concept was basically it was a max. So, there is only one max value based on that we can find out. So, only in L_1 where it was basically the mod value and you can mod value wise is same, but you can take different ways of trying to find out that as I showed in the graph. So, based on that I give the weights which I am going to highlight using the green color this is V, V_H which I will put for the case again same idea continues I put in the denominator which I will put in red color as the difference between the max and min for the L_1 and max and min which I am using now the blue color for the L∞ and in the corresponding numerators we will have the distance between L_1, $L_{\min}$ 1 and L∞, L∞ min here it is I put the tick mark for the L∞ - L∞ minimum for each and every alternative and in the denominator and in the numerator for the L1 norm. So, based on that I have basically the weights and weights if you remember here they are V and 1 - V. So, put a V of 1 which means the second half which is I am just putting a tick mark this part will be vanishing if I put a weight of V0 the first part which I am putting a double tick mark will vary. So, I am basically trying to balance the weights such that I have the corresponding lengths multiplied accordingly. So, considering these values which I have is now for there are m number of alternatives. So, basically I would have the value of Q from Q1 to Qm based on this, but once decided on the weights V, you will stick to that for all the alternatives.

So, there is no change now these are based on the distance for the alternatives now the criteria's are not there already been subsumed and considered. In the initial case W1 to Wn were the weights for the criteria's which we consider. So, once we have that V is introduced as the weight of the strategy of the majority of the criteria of the maximum group utility based on that. Let us consider V as 0.5 and based on that we find out these values and rank

them accordingly.

So, if I want to rank them basically when you have that you sort them based on actually it can be done on Q1 where you are giving the weights, but it can also be done on the minimum one L1 if you remember. Another was the case of L infinity which I am marking in red and the third one which I have used the blue color, but I should change the color is the green one is basically the Q. So, L1 is basically the concept based on only L1 distance norm the maximum which is not unique L infinity is based on the infinity norm. So, two extremes and Q is basically the balancing part which I do based on the weights which I am going to. These results as I continue reading it there are three rankings propose a compromise solution such that the alternatives can be obtained accordingly.

So, with this I will close I gave the steps and the values can be calculated accordingly and the values were also there. Now if I want to continue let me consider this I will take two minutes and then find out. So, consider these values which I have. So, if I have just for discussion.

So, I will add an extra slide make it blank and continue. So, if I have because with this discussion I should be able to finish the VIKOR method. So, if I have the distances. So, the distances for the L infinity L1 was given L1 max and L1 min which is 7194.2943. Let me check 7194.2943. This was the case for L1 L1 max and min if I go to the L infinity it is 0.2511.1553. Now, let us consider V as 0.5 1 minus V as 0.5. Now when I do it what I need to find out is this. So, let me take some extra minutes and then try to at least give a good idea about the end part of the VIKOR. So, once I have the maximum minimum based on the L infinity and L1 in the numerator I have the L1 minus L min. So, I consider the L1 is basically for the first one.

So, the first one is L1 is given by 0.2943. So, if I consider 0.2943. So, it was basically for L1 is 0.5 into 0.2943 minus the minimum value 0.2943 this is for L1 remember and in the denominator it is max minus for the L1 is 0.7194 minus 0.2943. So, this is the first part plus 1 minus V which is 0.5 for the infinity norm. So, in the infinity it will be.

So, the denominator we can immediately write 0.2500 minus 0.1553 and in the numerator what it would be it is basically the value of minimum being is basically when I am considering let me check it will be minimum. So, when I have considered the minimum value it will be 0.1553 -0.1553. So, the first case the numerator is 0. So, this vacuum is 0 this is also it is 0 this becomes 0. So, it is 0 if I consider L2 I will only do 2 because lest I exceed the set of time, but I will try to. So, if I consider the second one again 0.5 the numerator is 0.17194 minus 0.2943 this part I will only fill up the numerator. So, I will go back to the slide 0.2500 minus 0.1553 and the minus value always remain same here the

negative minimum value remains the same.

So, if I consider for L the second one 0.6418 and it is 0.255 6418 0.255 0.6418 0.2500. So, I find out this value. So, obviously, that will be greater than here because both are positive it will be greater than 0.

So, considering if you have that you will rank them based on this value. So, obviously, this is positive value. So, if I am comparing only A2 and A1 A2 is definitely better than A1 and in this way we can find out the points for A3 A4 and correspondingly do that ranking accordingly. So, with this I will end the concepts of the VIKOR method and continue discussing we have finished 3 different methods we will consider discussing other methods accordingly have a nice day and thank you very much. .