Multi-Criteria Decision Making and Applications
Prof. Raghu Nandan Sengupta
Industrial Engineering and Management Department
Indian Institute of Technology, Kanpur
Week 11
Lecture 51

Very warm welcome to all the participants and the students for this course multi criteria decision making under NPTEL MOOC series. And as you know this is a 12 week course spread over 60 lectures and we are going to start the 11th week. So, we have 2 more weeks to go and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. Again warm welcome a good morning, good evening, good afternoon to all of you at whichever time and place you are listening to this lectures. So, as you know we are we were discussing for the last few days and last few lectures about non parametric methods. We have gone into details about the electro method and the TOPSIS method.

And as you know that we have few more methods to consider which are non parametric multi criteria decision they are more subjective not as objective as a multi objective decision methods. So, this is the 51th lecture out of the 60 and the broader outline is multi criteria decision making and main focus is this non parametric method just to repeat it. What is the overall coverage for this lecture which is starting out as 11th week? It will be the third method which is VIKOR method and the full form of VIKOR method is given this these all these methods which I discussed the TOPSIS, electro and these methods were all developed in Europe. So, what does the concept of VIKOR method mean? It is basically and when you convert it in English its multi criteria optimization and compromise solution.

So, mark the word compromise. Now, if you remember the concept of TOPSIS in electre in TOPSIS it was more to do with the distance concept to the best possible solution theoretically best and theoretically worst possible solution or PIS and NIS as we have discussed and we always wanted to find out the ratio and based on that we rank them. And when we did we had the indices the matrix is calculated based on that the distance measures if you remember we have considered the distance measures as L1 norm, L2 norm, LP norm, L2 infinity norm we will be again visiting this under the VIKOR method. In the electre method there were two broad different concepts the first one where we had the concordance, discordance they were based on the idea and all of these methods are in the TOPSIS also electre also and also in the VIKOR. Initially you have a matrix X which is based on the overall value which accrues to each and every alternative based on the criteria then we normalized it after normalization we multiplied with W found out Y and utilized Y based on which we considered again I am repeating in the concept of TOPSIS the NIS, PIS concepts and rank them found out the ratios.

In the concept of electre method we found out the dominance and the concordance and discordance indices first after finding on the indices we found out the this concordance, discordance matrices and then we rank them. And then means found out the corresponding ranking of the alternatives based on all the conglomerate criteria both for the first method TOPSIS and electre. In epsilon-electre the added part which was there and which was discussed in details was basically rather than have the concordance and discordance we have the indifference concept also and that is coming from the ideas of indifference under the risk analysis a person can love risk, person can hate risk and person can be indifferent. And I have given this example of losing 100 rupees rather than gaining 100 rupees and they can be unequally liked disliked and that was considered in the epsilon-electre. And in all these methods so the concordance, discordance both for the electre and epsilon-electre apart from the indifference matrices the other two matrices for the concordance and discordance were asymmetric.

And then we at the end in both the methods of TOPSIS and electre we found out the overall matrix E which was basically balancing the liking and disliking for the TOPSIS which NIS-PIS in the case of electre it was balancing the concordance discordance in the case of epsilon it was balancing the concordance indifferent and discordance and then found out the ranking correspondingly which I did discuss in details saying how they can be compared in the last lecture which was the last lecture of the tenth week. So this is an MCDM technique which was developed by Serafim Opricovic during his page work as I said in this was Europe. VIKOR method was introduced as one applicable technique to be implemented within the MCDM problem and it is developed as an multi attribute. So they are all characteristics if you remember they are all characteristic based less on quantitative objective concepts. They are used to solve discrete decision making problems with non-commensurate world or different units of measurements for the criteria and that is true for all the methods I did mention that when we were discussing the multi objective one time and again I mentioned about multi attribute concepts this idea would be coming up and that what we are doing.

In this method the decision maker likes a solution that is closest to the ideal hence the decision alternatives are evaluated compared and ranked accordingly. So if you remember in the case of the methodology of TOPSIS it was based on the disliking. So how close or how far it is from the negative solution based on that it was the main emphasis was. Now in this method it is how close or far it is from the positive solution based on that. So even though I did discuss in the TOPSIS method that how can rank based only on the concept of closeness to the ideal solution, but technically closest to the ideal solution is basically the under the purview of VIKOR method.

So while ranking the decisions alternatives, alternatives you remember are given by the set A. While ranking the decision alternatives rather than the best solution our target is to find out the ideal what you want to achieve theoretically how close you are which may not always be feasible in the practical sense, but rather we want to find out how closest is this to the ideal and rank them accordingly. So it is basically like this you are as a teacher, as a faculty, as a professor you give marks. So obviously consider the marks are given out of 100 and the highest marks you assign to one student is say for example 80. So obviously when you rank the students it will be based on the highest marks 80 and the corresponding students will be marked accordingly like first, second, third, fourth, fifth accordingly.

Now the best solution, best marks the student could have obtained is 100. So what you want to find out is that how close the best student is to the ideal solution and based on that you want to find out that how close he could have been or she could have been or how close he or she is there to the ideal solution based on that you to the ranking. Even though not exactly the same the concept of relative marking. So when the student who has got 80 his or her marks is pegged on to 100 and based on that again you find out the other marks of the other students it can be considered a very rudimentary way of trying to basically rank the students based on the ideal solution where the highest mark has been pegged up to 100 and accordingly all the marks of the other students are recalculated accordingly. These two of the methods Y-code which we are going to do and TOPSIS we have already done the first one before ever I mean ELECTRE and EPSILON ELECTRE.

It is they are based on aggregate functions which represent the concept of closeness of the solution to the ideal solution. Now if you remember in the concept of TOPSIS I did mention and I will write the functions accordingly I will use the dark red color. So if you remember we use the function GJ AK, AL. So what is this? You have basically K and L are sets of the alternatives I will write ALT alternatives and J is basically one of the criteria. So I will write it CRI criteria.

So based on each criteria I want to find out what is the overall function value when you compare AK with AL. Now if you remember we have also discussed and I will use a different color here green that when you find out GJ for the same criteria when I am comparing AL with AK. Now why this comparison comes? The comparison comes is that say for example due to some reason in the first case I took the kth alternative and I am comparing with the Lth one and in case two I have taken the lth alternative decision has been taken and I am comparing with the kth one. So obviously these two would not be equal that is why the corresponding different type of matrices which you have apart from the indifference one all were asymmetric. Now a very interesting thing if I consider and which I did mention few times, but I will make it much more evident here say for example I have a Cartesian coordinate and I have x_1 as the this axis on the x in the along the x direction and x_2 along the y direction and consider there are two points I will mark one with red which is A and I will mark the other one with blue which is B and I mark the distance between them A to B as black.

So, if I take the Cartesian coordinates the distance measure between A and B and B and A would be same, but very interestingly when I am comparing AK to AL based on the Jth criteria when I am comparing AL to AK based on the jth criteria they need not be equal that means very rudimentary in a very very important terms. So, fundamentally in a very layman terms the distance or the type of functional value which you get comparing AK with respect to AL and comparing AL with respect to AK are not the same. So, that is why the asymmetry and the comparison in many of the cases may become difficult. So, if I continue reading it, so TOPSIS and VIKOR are aggregate functions which represent the concept of closeness of the solution to the ideal solution. In VIKOR we follow linear normalization and while in topis we follow the vector normalization and that will become true as we consider the examples.

Why normalization? The use of normalization which you have done in VIKOR which will do in VIKOR which we have done in topis which we did also in epsilon, electron, electro method is used to eliminate the units of criteria functions. Now if you remember one word which I mentioned is non-compensated units may be measurement in liters, may be profit in rupees, may be measurement in say for example, joules like these are some of the criteria. So, consider joules means like say for example, I am trying to basically purchase a boiler. So, I will try to basically find out the heat output, I am trying to basically buy a car then I will try to basically compare the output of the car using the horsepower and I am considering the price of the car it will be in rupees. So, these units are different, but once you normalize them the ratio basically is between 0 and 1 and the units are not there that is why they would be eliminated.

So, it eliminates the units of the criteria functions and thus ensures a level playing field for different criteria that can be compared. In VIKOR method we determine a maximum group utility for the majority and a minimum for the individual and regret for the opponent and based on that we proceed. In TOPSIS the first method we consider a solution with the shortest distance to the ideal solution and the greatest distance from the negative solution is required. If you remember considering the shortest distance and the longest distance, so if I draw the line means where you have the higher values on to the right hand side like a real line and the lower values on the left hand side and if I have the negative solution marked in red which you see and if I have the positive solution marked in green. So, obviously if I have two different ranking to make consider the first one is marked as alternative 1 and say for example, the second one is marked by alternative 2 then obviously if I consider the closeness of A_2 to the ideal solution which is green in color and the closeness of the idles to the ideal solution for A_1 again I am marking in green and on the other hand if I want to find out how far it is from the non ideal solution for A_1 this is here which I first mark and for A_2 it is there.

So, obviously red and green color comparison of the ideal solution non ideal solution will be able to do. So, this is what is basically the third point while doing this we do not consider the relative importance of these distances units are not there because they have been eliminated and if you remember they have been eliminated in the sense we are trying to find out the ratios of the distances and then compare. Assume for the method by by code again the same set of M number of alternatives M as in mango A_1 to A_{MC} number of C i J number of of criteria. So, A 1 A 2 A 3 A 4 till A M are the alternatives C_1 C_2 C_3 till C_n n as in Nagpur are the criteria. We will consider C_j A_i as the value of the jth attribute decision for the ith alternative.

So, each criteria will give some weightages some value to the alternatives and based on that we are trying to compare. So, if I consider this we will consider the for the ith solution the distance measure would be given by this. So, what we are considering is if you look at the denominator which is being normalized in the sense I will use a different color. This basically gives the distance from the maximum that between the maximum and minimum and based on that we would try to basically weightage or find out the weightages of the distance between the positive and that point which is there. So, if I remove W_j which is the weights which will be given the weights depending on some sets of criteria.

I need to find out in the numerator which where I will use the blue color is the distance from the ideal solution for that particular solution which is there. So, if I draw it along the again this line the negative solution which I mark in red here the positive solution I mark in I was using green. So, it is green so in the numerator is the difference between the red and the green. So, it has been normalized and if that solution is black one in the numerator you have this distance marked in black and in the denominator I have this distance this marked in red color just for differentiation between the coloring and obviously you will give weights accordingly that weights will become later. Now, there is a part P what does it mean that means I am trying to basically find out the norm whether L_1, L_2, L_3 till the L infinity norm.

So, obviously if it is square L_P is 2 it is basically simply the idea of using the Cartesian coordinate L_2 norm. Here what we do want to do and want to make a note is this one C_jA+ is the maximum value and C_jA- is the minimum value based for each and every alternative for each criteria. So, each criteria I take I basically find out the maximum and the minimum and then find out the differences accordingly and I have to find out for each and every alternative. Remember here L_{pi} is used to formulate the ranking measure based on the fact which I just mentioned p = 1, 2, 3 accordingly is an integer and we want to find out the distance for each and every alternatives $i = 1,\ldots,M$, M as in mango. P is equal to 1 signifies the Manhattan norm which I am marking while $p = \infty$ denotes the infinity norm.

Now, what are the Manhattan norms which I will just I would have done it, but I will just go through it Manhattan norm is given by given two points and need to find out the summation of the mod of these differences. This is basically name relates to distance at the axis to drive in a rectangular street circuit and I have shown that and I will again refer to that. L_P norm is given between two vector points X and Y given by the norm which is here which I am circling in red and N infinity norm is the maximum value which I am circling in blue to give us this formula which you all know and when P = 1 P = 2 sorry you have the L_2 norm. So, given these two points x and y and x = 2, - 5, 20; y = -12, 15, 0 I have, I am calculating L_1 as 54 which I put a tick mark L_2 as 31.5 L infinity as 20 and I will put basically plot it, what I do? I do along the x axis I have P value P 1 2 3 4 and along the y axis I have the norm L P and if you see this curve is I am just trying to plot it this is a function like this where L 1 is the maximum value and L_∞ is the minimum.

Now, consider this one which I have already done if I consider the L_2 norm green line is for L_2 and the red blue and yellow are based on the L_1 norm because you can take different routes, but interestingly the length covered. So, if I consider the yellow one, so it is 1 2 3 4 5 6 7 8 9 10 11 12, so it is 12. I will mark it here. So, if I consider the dark blue one is 1 2 3 4 5 6 7 8 9 10 11 12 again 12 and if I find out the L_1 norm using the red movement it is 1 2 3 4 5 6 7 8 9 10 11 12 again 12. So, different ways to reach from point A which is in the left bottom corner to point B green line again I am mentioning is the L_2 norm one way distance can be found out L_1 norm is 12, but there are different ways to reach that point. So, this is what I discussed and I will just read it the green line L_2 norm is unique while red blue yellow which is L_1 norm is of same length 12 that is why L_2 norm has the unique solution while L_1 norm does not have any unique solution one can generate generalize this for the n dimensional case.

So, now let us basically go through the VIKOR algorithm assume M number of alternatives A_1 to A_{mj} number of attributes decisions C_1 to C_n and we state the pseudo code. The pseudo code is just stated here, but I will follow the steps accordingly. So, do not be too much bothered about the pseudo codes obviously this is slide will be shared with you they would be there, but rather than concentrating on the pseudo code we will just go step by step in solving the problem. Again the first initial sets are exactly the same. So, it is a repetition please bear with me assume the distance matrix decision matrix X is given and it is of size $m \times n$, m is the number of alternatives n is the number of criteria.

So, x_{ij} would basically be the corresponding decision matrix values for the jth criteria on the ith alternative. Why I am saying jth criteria to the ith alternative because I am writing the alternatives along the rows and I am writing the criteria along the columns. So, consider this the values are given I will just read the first and the last value along the column. So, for the first criteria the alternative values are 30 to 90 like 30 then 40 50 20 90 the corresponding C_2 C_3 C_4 for the alternative $A_1,\ldots, A_5$ are given I will just mark it with different colors. So, the second criteria I am marking with red starting from 100 to 150 the third criteria I am marking with green starting from 0.9 to 0.7 and the fourth criteria I am marking with violet starting from 9 to 5 and for each and every alternatives you can check. Once you have that I basically find out the normalized one and again normalizing concept I can use different ones. So, it can be normalization based on $\frac{xij}{\sum xij}$. It can be $\text{Ln}[X_{ij}] / \sum \text{Ln}(X_{ij})^2$ and they can be different ways of normalization. Once you normalize which is R I am basically normalizing by the sum only here even though it is given in the last slide as square I am just normalizing the sum. So, if you check they are being basically added based on the fact that the normalization is being done accordingly.

So, here only remember normalization can be done along the rows and the columns depending on how you have drawn the matrix of alternatives and criteria. So, based on the normalization I have these values these are the normalized values weights have not been yet considered because I will come to this value. So, once you have the I am not going to read the details. So, the values normalized for the R matrix which is based on X which is the initial decision matrix is this again it is still the size is 5×1. Now the decision maker wants to put weights $W_1,\ldots,W_N$ for each criteria and where the weights are between 0 and 1 where some of the weights are add up to 1 which I did mention again I am repeating and here for this four alternatives each weights are equal it is $1,\ldots,4$.

Now interesting thing the size even though it is very obvious to all of you have been following this course you have done a lot of studies. So, the criteria's are any number. So, obviously the weight matrix would be $n \times n$, m is not here. So, 4×4 which I am circling is the corresponding weights given for the matrix. So, once I have the weights I need to find out the multiplicative values of $R \times W$.

Remember which is very obvious the size of R and W should be in such a way that you can multiply the matrix which means here the size of R is given by $m \times n$ which is 5×4 and W is given by $n \times n$ which is 4×4. So, hence it can be multiplied. So, based on that I multiply once I multiply I have this matrix. So, the first one is R second one is W remember here is all the values are 1/4. So, once I have that I have the weighted normalized matrix first is the normalized matrix then is the weighted weight matrix multiplying them I have the weighted normalized matrix.

Now what I need to do is basically find out the maximum best from the criteria and the minimum best from the criteria. So, I will state that and definitely highlight few points which I will again repeat few of the important points in the next lecture. So, because I want to keep it within the time limit of 30 minutes. So, we need to find out the maximum which is the best and also we need to find out the minimum corresponding to the fact that how far or close it is. Now with this I will start from this slide which is the step 3 the VIKOR one and continue discussing the VIKOR method from here and plot all of the values accordingly based on the R the matrix which we had matrix what was that it was basically R into W which in many of the cases is given as Y in many of the cases is given as E.

So, we will be basically be working with the matrix Y. Thank you very much and have a nice day. .