Multi-Criteria Decision Making and Applications
Prof. Raghu Nandan Sengupta
Industrial Engineering and Management Department
Indian Institute of Technology, Kanpur
Week 10
Lecture 50

Welcome back my dear friends and students, very good morning, good evening and good afternoon to all of you at whatever time you are listening to this lectures, seeing this lectures. The course title as you know is multi criteria decision making which is an NPTEL course for 12 weeks spread over 12 weeks with 60 lectures each week we have 5 lectures each being for half an hour. Today we are in the last lecture for the 10th week, so we will have another 2 weeks of classes or recording which will be shared with you. Then obviously as you know that each week has one assignments and after the course ends you will have the ends end. So, the broader concepts we are discussing in multi criteria decision making we have already gone through multi objective decision making, utility theory both for the univariate case and the multivariate case and we are in the process of discussing the different concepts. We have already considered Topsis method, we are at the fag end of considering the two different electro method we are discussing the epsilon and today we   will   try   to   start   the   other   method   of   the   VIKOR.

 So, what is the coverage? Coverage is the last portions for epsilon electro and we will consider the VIKOR method. The concepts would remain the same but the way of handling the problems would be different. Concepts means in the Topsis method it was NIS-PIS, shortest distance to the positive solutions and shortest distance to negative solution. Then in the epsilon concept or the electro concept we considered the concordance-discordance, in the epsilon we are considering concordance-discordance and indifferent concept and Vicor we would have another different ways of trying to handle the problem.

  So, continuing with the last lecture which is the 49th one today being the 50th one. So, we will determine the cut off values star values based on concordance then based on indifference and then based on discordance. Concordance concept will give us the ranking we if we are able to consider the ranking based on only the positive values. Discordance matrix and then the cut off values which are the star values will give us the ranking based only on the negative concept and indifference would be that we are indifferent between the alternatives. Now why we are doing that because collectively you will take the concordance-discordance   and   indifference   and   then   rank   them.

 Because as I said and I am again repeating for taking any decisions on the alternatives it comes with a positive as well as with a negative baggage also. And negative baggages

have to be considered because if the negative worth of taking a decision is too high then obviously people or the decision maker would not be interested in taking that decision. So based on this I have for the case when we are considering the indifference one the values for when I am trying to compare the comparison is always remember based on the alternatives when I am comparing 1, 2 and 3, 1, 2, 3. So, I have the concordance-discordance matrix is given again the principle diagonal is 0 and the cut offs based on which I consider are given here. Because all the detailed comparison for the concordance had been done discussed in the last class and in the other classes also.

So, I will just give the values and summarize the results at the end. So we determine the aggregate dominance, indifference and non dominance matrices or consider the concordance, indifference and discordance matrices which will denote by ECC which is the concordance plus the concordance matrix ECI means the concordance plus the indifference matrix. So, I will highlight it the words are highlighted in red but I will particular highlight the color where concordance will be done with green, indifference will be done with blue and discordance will be done by red. So, let me do it concordance would be done with green, this is also concordance, the indifference would be done by blue and discordance will be done by red. So, we are comparing positive to positive, positive to indifference and positive to negative and for that we just multiply the corresponding cell values as calculated previously.

Now when we go into the indifference comparison considering the indifference again the subclass again you use the same coloring concept of green being concordance and indifference being blue and discordance being red. So, what we are doing we are comparing indifference with concordance, indifference with indifference and indifference with discordance and when I am marking it just note that the corresponding cell values would be multiplied accordingly. So if I go back to the last slide it is $f(C_{kl})$ that means for the concordance $f(C_{kl})$ for the concordance indifference. So, the first one concordance and concordance and similarly for concordance discordance the concepts are analyzed. Similarly it is the indifference $f(I_{kl})$ and what is being multiplied is here I will highlight in this slide and then go back to the other slide.

So it was indifference here and if I consider the concordance you can multiply with the concordance corresponding cells from the matrix then the indifference concept being multiplied for the second case. So, that is why it is blue into blue and last being blue into red and if I go back to the last slide even though I have marked in black that was just draw your attention. So, let me erase it and make it absolutely clear like the last slide which I did. So, it is concordance, so it is green in color concordance, concordance, concordance this is also concordance this is indifferent. So, it would be blue and red will be discordance.

So, we will multiply the corresponding again I am repeating the cell values for the corresponding matrices. Finally, when we have determined the dominant or indifference and non dominant matrices based on the aggregate of discordance the drawing scheme again we carrying the same discordance, discordance, discordance this is concordance this is indifferent and this is discordance. So, what do you do again multiply the corresponding cell values for those corresponding matrices the matrices what are they? They are based on the discordance here, discordance here, discordance here and discordance for the first case is being multiplied with the concordance for the second case is being multiplied with the indifference and the third case is being multiplied by again by the discordance. So, if I look carefully for the concordance positive one we have divided into liking concordance, indifferent, discordance again for the indifference again divided into concordance, indifferent, discordance and finally, for the discordance one we have again divided into concordance, indifference and discordance. Now, why this? It means that taking the positive value concordance we are trying to be much more logical much more decisive in the macro level in order to find out those alternatives where positive is always on the higher side which is concordance, concordance, when we are taking the positive and we are indifferent it is basically concordance and indifference and in the last case it is concordance with the differences or the discordance.

Similarly, we do it for the other level of classification of indifference and discordance. Based on that when we have we finally find out this when you are multiplying these are the values which you multiply. So, we get the concordance with concordance see here marked as green concordance with indifferent and concordance with discordance which we have already discussed when we multiply them. So, if you see it just so this symbol operator is just a simple case of multiplication any other operation can be used. So, if I check the values I will use now the black color.

So, black color to highlight the final value. So, the final value is 1. So, from where I will be getting 1 it is coming from here concordance being multiplied to give a value of 1. If I consider the value of indifference which is the second one concordance indifference this value of 1 and the value of indifference being 0 resents in the value of 0, 0 means here which I would highlight by black. So, that means even if it concordance the overall value comes out to be indifferent.

And finally, when I consider concordance with the value of discordance the value is like this concordance green discordance red the value is 1. Now, do not take this 1 as total discordance along the set where we have concordance there is obviously some value which are negative. So, I have taken that and then trying to find a compromise. So, based on that I am still positive that means I have taken the decision I have a value of concordance, but there is a values of discordance also, but still I am positive in taking that decision. Now,

when I do the aggregate dominance indifference and this discordance matrix based on indifference which I will highlight indifference been blue.

So, indifference, indifference, indifference second one is also indifference the first one is concordance hence green and the last one is discordance and red again the same values on the indifference set. So, based on that when I try to find out. So, indifference being 0 concordance being under that concordance being 1 the overall value when I multiply comes out to be 0 that means on the indifference set I would still be indifference because that dominance. Similarly, all the values can be followed finally, when I try to go for the indifference discordance, discordance, discordance the last one is also discordance with discordance this is concordance. This concordance means positive is green and this is blue.

So, if I consider the cell value this is discordant, this is concordant and the final value is 1. Now, do not look at 1 as positive in the other case. So, it was a sub class in the case primary class was concordance then discordance here the primary class is discordance then concordance. So, one means here which I have highlighted in black means that another class of discordance is still giving me in spite of the fact I am trying to pull in some benefits still the dis benefit is much more. In the other case when we are considering the concordance matrices the primary class was concordance.

So, under that I try to find out the negative benefits for the negative benefits were not overwhelming hence concordance still prevails. So now, the idea sorry I skip to the other slide now the idea comes that how do we would we be ranking. So, here is some discussion I would like to have and I will write down the matrix and discuss it accordingly one by one. So, I will create a blank slide write down the values and discuss. So, this is the blank slide I have created.

So, let me go back one by one this one. So, it is C C. So, I will take is basically 101001. So, this was matrix of E C C it was this one 101001. And I want to come and consider I have indifference of all being 0.

So, if I consider all being 0 E I C and finally, it will be D C discordance and concordance. So, this is 101000. So, if I have on the label of concordance and positive indifference positive and discordance positive positive what I am using for the concordance. So, if I consider A_1, A_2, A_3. So, let us consider on the level of the concept of positive values.

So, positive values if I take positive positive with I am considering A_1, A_2 only. So, this value is 1. So, on that frame it is positive. So, put a plus 1 here and I put a dash here comparing A_1, A_2 when I compare indifference and concordance it is dash dash. So, dash I am considering let us consider A_1 only I should not have marked A_2 I am comparing, but

it is dash. Then when I consider A_1, A_2 on the negative scale it is 1. So, in the discomfort level even though I have brought concordance it is still 1. So, this will be negative. Now, if I consider A_2 to A_1 A_2 to A_1 would be here. So, it is positive A_2 to A_1 on the indifference region it is dash is 0 and when I consider this on the level it is negative.

So, can any direct comparison be made between A_1, A_2 on the concordance set? No. If I consider the level of consider I am consider A_3 and A_2. So, on the level of A_3 and A_2 first let us consider A_2 with A_3. So, A_2 to A_3 it is does not accrue any benefit. So, it is 0 it is dash on the positive scale ECC then A_2 to A_3 again indifference is 0 and this benefit A_2 to A_3 is 0. Now, when I consider A_3 to A_2 this is 1 see this value. So, this is definitely benefit. If I consider on EIC scale it is 0 and when I consider EDC that means dominant discordance and concordance A_3 to A_2 it is again 0. Now see the difference in the other case comparing A_1 and A_2 on this concordance concordance in different concordance and discordance concordance we could not pass any judgment. When I consider A_2 to A_3 on the disbenefit scale whether you take A_2 or A_3 you cannot pass any comments convincingly.

So, they would be disbenefit to what level we do not know on the indifference scale also we do not know, but on the positive scale A_3 is always better than A_2. So, if I consider on the concordance concept the benefit which accrues for A_3 with respect to A_2 is always positive. Now, I have not finished the question would be immediately somehow one of the set of students or the participants who are taking this course would immediately ask the question that can we pass a judgment like this when we are considering indifference concordance, indifference indifference and indifferent discordance. Can such a judgment passed? Yes it can be passed. Also let me highlight such a judgment can also be passed when you are considering the discordance concordance, discordance indifference and discordance discordance matrix also.

So, conglomerated one overall idea which you will get by considering ECC, EIC, EDC then consider that concept. So, it should be EDC I think it is ECD sorry my mistake this is ECD whose concordance being the primary one. So, we have done the ranking based on first ECC with EC just one minute this was we have considered EIC and EDC. Then we will consider next level we will consider EC one ECI EII and EDI. Similarly in the third level we will consider EDC EDI and EDD C C I C DC CI I I D I we consider the concept of CD then ID and finally, DD.

So, we will have one ranking based on the first level, second ranking based on the second level and third level we will compare them and then find out the judgment. The one which you are seeing A_2 and A_3 was based on the first level of ranking only. Now if I so obviously in this cases no such judgment can be passed. So, let me do it for the case of this one E_{23}

based on of ECI. So, let me change it ECI EII and EDI which is the second case.

So, let us fill up these cells. So, let me erase them the principal diagonal U s dash dash dash dash. So, let us fill up ECI values are all 0s. Let us value fill the values of E_{II} 0 1 0 0 1 0. So, this is the value 0 1 0 0 1 0 0 0 0 1 0. So, let us see I have copied it correctly 0 1 0 0 1 0 finally, being EDI is all 0s.

So, EDI is this one is all 0s let us compare. So, this was the comparison which you have done based on the positive scale. So, let us do it for indifference one only concentrate on A_2 and A_3. So, when I consider A_2 and A_3 based on indifference being an important concept. So, it will be A_2 and A_3 is dash with respect to so on the indifference scale A_2 to A_3 is again a dash A_2 to A_3 again a dash.

If I consider A3 to A2 it is a dash dash means there are values. So, A3 A2 dash A3 A2 dash A3 A2 dash which means indifference concept and balanced. Positive one is still a positive factor for A3. Finally, I will take few minutes more even though the class may exceed 30 minutes plus please.

So, I have done this. So, now coming to the final part. So, it is ECD now. So, let me write down the values ECD.

So, this 1 0 1 0 0 0 1 0 1 0 0 0. So, this is CD. So, let us double check 1 0 1 0 0 0 done. Next will come EID. EID is all 0s and finally, EDD 1 0 1 1 0 0 1 0 1 1 0 0. So, let us concentrate on A23.

So, if I consider A23 A2 to A3 is 0. So, here A2 to A3 again 0 A2 to A3. Now, it is a if you see here A2 A3 0 A2 A3. So, this is a negative one. So, you can definitely put a negative side here because this is based on DD discordance discordance. If I consider A32, 32 is basically 0, 32 is 0, 32 is 0.

Now see the interesting part. If I compare, so this can be removed. If I compare this set and with this set interesting observations comes out. A3 wins both the ways positive side A3 is positive and negative side A2 would basically give a dis-benefit. So, it is always better to check A3 both on the difference concordance and the discordance set A3 is winning.

Indifference it was all balanced. So, similar analysis can be done between A1 A3 A1 A2 and so on and so forth. So, I only dealt with two alternatives in all the stages. This idea should be clear to you. Sorry I took a little bit long because I wanted to finish the epsilon electro accordingly. Thank you very much and have a nice day. Thank you. .