Multi-Criteria Decision Making and Applications
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Week 01
Lecture 05

Welcome back, a very good morning, good afternoon and good evening to all the participants and the students who are taking this MOOC course and this is the title of the course is Multi-Criteria Decision Making and Applications and my name is Raghunandan Sengupta from the IME department at IIT Kanpur in India. So if you remember in the fourth lecture we were discussing about the example number 6, which was basically to find out first the simple problem was to find out the minimization or the maximization of a quadratic function which we denoted by f_1 and I did mention there is a color coding there for us to understand and then there was an objective function 2, f_2 with the different color combination but the constraints for all are linear in nature. So the first part in the overall emphasis in fourth lecture was basically to find out the minimization of f_1 separately, f_2 separately and then try to find out the combined objective function its minimum value based on the fact that if the objective function is now $(f_1 + f_2)$ and using the diagram and also drawing and also with the graphs we discussed that how the minimum point for f_1, minimum point for f_2 and the minimum point for f_1 for f_2, minimum point means the decision variables x_1 and x_2, are all different and also the objective function values at the minimum points are all different. So we will try to extend that for the case when we are going to discuss about the maximization one. So this is the continuation as I mentioned about the introduction part and this is the fifth lecture under the title of the course which we have already discussed.

Coverage would be as I mentioned would be the continuation and we will try to wrap up example 6 and then consider another example 7. Now we try to maximize the functions. So highligh,t as I go continue the discussion I will highlight the points. So we will try to maximize the functions.

What are the functions? f_1 is this which we already know I am not going to repeat it much, f_2 which is this. Only thing, the center points for f_1 and f_2 are just interchanged. The constraints are similar exactly the same. The first one was $(x_1 + x_2) \geq 60$, which is same there in both the cases, $x_1, x_2 \leq 90$, $15 \leq x_1 \leq 90$, $15 \leq x_2 \leq 90$. Now when we try to find out the maximization point, very interestingly the first objective function which was f_1 is satisfied and minimized at the point on the line $(x_1 + x_2) = 90$, which is basically based on the constraint which I am trying to highlight here. The maximum point is (52.5,37.5) here and the maximum value comes out to be 1462.5. Now separately if we do trying to maximize the point for f_2 we do not think about f_1 now, the interesting part is the f_2 is maximized at coordinate system (37.5 and 52.5), which is just interchange with respect to f_1 here. I will just highlight it. The point 52.5 which you see here, when we are trying to optimize or maximize f_1 and the point (37.5,52.5), which you see here when we try to basically optimize f_2 are just interchanged; interchanged in the sense the x_1 and x_2 coordinates are interchanged. But interestingly when we put that and try to solve and try to find out separately f_1, separately f_2, the values come out to be same, which is 1462.5 in both the cases.

Similarly as we found out in minimization case. So let us consider the corner points. The corner points say, these are examples which I am taking and it will basically illustrate, why we are doing so, similarly as we did in minimization case. Let us consider the corner points (15, 75). Corner points means the corner points if you see, which was obvious I did not mention there. So if I consider x_2 here, x_1 here, the corner points I mean are these. These were the corner points for the minimization case which I am highlighting in black for the boundary. Obviously once it goes inside the feasible region the values of f_1 and f_2 will increase. So obviously corner points would be only that for the minimization case. The corner points which you are going to consider here now which are marked in red are these.

So (15, 75) would be this one, (75, 15) would be corner point this one. So we will try to find out the maximization at the corner points for f_1 and f_2 separately. So at the corner points (15, 75), which I am using a different color, say for example green, just to highlight. So consider this is B1 and this is B2. This 1 and 2 have nothing to do with the function f_1 and f_2 nothing just they are suffix.

So when I put the values of f_1, for the corner points separately the value comes out to be 2025. When I try to find out the objective function maximum value for f_2 for this corner point (15, 75), 15 for x_1, 75 for x_2 , the value comes out to be 3825. Interestingly, when I switch and go from B1 to B2, so these are the B1 corner points here. So B1 led to this. When I switch to B2 the corner points, as you remember is given by (17, 15) and the values which will get corresponding to f_1 and f_2 putting x_1 as 75, x_2 as 15, are very interestingly just interchanged.

So 3825 is the value of maximum for f_1, 2025 is the value for f_2 and these values which you see which are marked in green here for the corner point B1, (2025, 3825), they have been just interchanged and which highlights that the corner points would basically, if I find out the collective one which I will come to later on, they would give me separately the same values. Now the interesting fact would be that would that remain throughout the straight line $(x_1 + x_2) = 90$. So by the way the point (75,15) and (15, 75), all both of them lie along the straight line $(x_1 + x_2) = 90$. So $x_1 + x_2 \leq 90$ is one of the constraints to basically find out the feasible region. Now if you maximize the objective function f_1 and so let me use a different color, as it was highlighted in the first instance.

So if I find out f_1 here and this was f_2, which is here I am using the same color as it was given in the diagram simultaneously, under the same set of constraints. Now the common point which will give me the maximum value based on the fact we want to find out the combined objective function to be maximized, it comes out to be (45, 45). If you see this (45, 45) is a point which will satisfy both the all the constraints obviously and will satisfy the combined case such that we will get the combined objective function f_1 plus f_2 to be maximized. Now trying to optimize the value for f_1 and f_2 separately the coordinates are totally different. So if I consider that very interestingly the value which comes out and satisfies both f_1 and f_2 combined is at (45, 45) is 1125.

Now if I try to find out, and this 1125 is totally different from the single objective function case, as it was the case for the minimum problem. Now if I try to find out the values of f_1 and f_2 at different points and I consider a set of points like I will just highlight them. The first one which are given in the stars is (52.5, 37.5), (37.5, 52.5), (15,75), (75, 15). So these are the corner points for $(x_1 + x_2) = 90$, and one point being (45, 45), means what we have found out. If I do that the maximum value which you will find out for the case when they are single see for example combining them and trying to find out the values for (52.5, 37.5) comes out to be 2475. Similarly when I interchange the coordinates which is for the first and the second start point the value of $(f_1 + f_2)$ remains same but the coordinates gets interchanged and this is very much easily highlighted in the third and the fourth start point which are the coordinates are (15,75) (75,15), the values are 5850. But the last part which you see that the value which we found out in trying to find out the maximum case, which was (45, 45), which we just found out was coming out to be 2250 and this value if you see is separately the values of 1125 plus 1125 which was the maximum point separately for f_1 and f_2 and the points (45, 45).

So obviously looking into that the maximum value of 5850 is found out at the corner points which is 1575 and 7550. If I draw the graph again trying to maximize and we plot x_1 along the x-axis you could have done for x_2 also which I mentioned in the same example in the earlier class and if I try to basically plot $(f_1 + f_2)$ along the y-axis then very interestingly the maximum point comes out to be two corner points here the values are 5850, 5850 and the coordinates are **X** is for this point, this point is 1575, **X** is basically a bold because it is a vector not individual X values and here also X value again a bold comes out to be (75, 15), they are just interchanged. So the minimum case was the minimum we wanted to find out for the combined case. Now let us go into the seventh and the last example under introduction. So again this would not be quantitative it will be more qualitative but with also some additional factors quantitative in nature.

So the examples which I am discussing are basically to make one understand that the plethora of different type of problems we can solve under MCDM. Obviously we would not be able to cover all the techniques but we will try to basically go into many of them and appreciate the importance of them. Consider this case of Mr. Raman Murthy who is deciding to rent a 2BHK flat in the city of Visakhapatnam and in doing so Mr. Murthy considers the following points.

What are the factors or the points? The factors are 18 number and they are as follows primary cost, other cost, locality, community, safety, education, employment and entertainment. Under the factor the reasons are given primary cost can be related to rent, electricity, maintenance, other cost can be property tax, corporate tax, other expenses. Locality will be proximity to hospital, shopping mall, groceries, police station, schools and so on and so forth. Community would be how the conducive and how the locality is whether they are friends, whether they are the people of the same age, same societal backgrounds working in the same type of companies. Safety would be whether it is a gated community, whether the safety and that there is a low crime rate.

Education employment and entertainment would be based on how close the education institute is where Mr. Raman Murthy's son and daughter would go or his spouse would be working. So employment proximity would be how far his or her spouse employment place is, how long it takes, what is the cost, going in a car, go on a bus or a metro whatever it is and entertainment would be whether they want to visit the museum, the mall, go to the shopping plaza or go to the book store and all these things are the factors which are important for Mr. Murthy. And the scale of measurements for these 8 factors are the first two primary cost and other cost is in Indian rupees.

The last three which is education employment and entertainment again is in Indian rupees while locality, community, safety are given in a scale of 0 to 10. So if I consider few logical values which are applicable for the city of Visakhapatnam and the localities are given. So in Visakhapatnam I am considering the so called top 10 localities which are Seeta Madhara, MVP colony, Gajuwaka, Madhuravada and so on and so forth in Pendurati and if I consider the factors which are given there, 8 in number, they are again repeating price, other cost, locality, community, safety, education, employment and entertainment. I am clubbing and I will come to that reason. Factor 1 will consider price and other cost for the house. Factor 2 would be locality, community, safety which was given in the basis point of 0 to 10 and factor 3 would be education, employment and entertainment cost wise. So the values which I have taken, which I am just highlighting, the column which I have marked which is under price or under other costs or education, employment, entertainment are practical values which I found from the net. So they are more or less they would be obviously aberration but they are the values based on which we will proceed for the 10 localities based on these factors. Now what I do? I plot factor 2 and factor 1 which I have just highlighted in the 9th slide and all the localities are marked. So localities are marked by the points A, B, C, D, E, F, G, H, I.

So they would be 10, so A to I. Now if I consider factor 1 only which is price, other costs and I have multiplied with 10 to the power minus 3 in order to scale them down. There is no rocket science here, I just scale them down and factor 2 was locality, community, safety which was just combined weight of the factors each individually being between 0 to 10. Now if I consider factor 1, price, other costs obviously have to be decreased. So if I consider on the locality J which I follow accordingly if you see the slide, there are 10 localities and they have been marked as I am mentioning A to J. A for the first one B, C, D, E, F, G, H, I, J, J is the 10th one.

So this would be the first one according to the table and this is the 10th one. So I am only considering the 10th one. So obviously the analysis which I will do will be applicable for others. So factor 1 has to be decreased, so it will goes to the left hand side as low as possible. Factor 2 has to be increased, higher the points better for.

So if I consider all the sets of points A to I, A is so called best in the case that the level of factor 2 is the highest, but very interestingly cost fight is also the highest. So a compromise has to be made by Mr. Murthy. If I consider point number F, so obviously F is very low on the factor 1 which is the cost, but obviously if I consider factor 2 which is locality community safety, it can never compete against H which I am highlighting here. It can never compete against D, never against J or G or A.

The reason I am saying is that if I consider in a two dimensional case, factor 1 and factor 2 go in opposite direction. So trying to find out which is the best combinations between factor 1 and factor 2 for these 10 different localities in the city of Vishakhapatnam has to be compromised in the sense that one factor increases, other factor decreases would basically give Mr. Murthy a very good idea how he is going to balance them. Now the second part of this diagram which I want to highlight are the dotted lines. So there are three dotted lines parallel to each other, three bold lines which are parallel to each other. So if I consider and they have been drawn arbitrarily in order to explain something.

So if I am based on any one of these dotted lines, so consider I am taking the dotted lines, the first one which is when which I am marking and I am taking three different points here and I mark them as say for example Z1, Z2, Z3. So Z1, Z2, Z3 the overall value based on the fact that I am trying to find out the overall satisfaction level for factor 1 and factor 2 combined would be same for Z1, Z2, Z3. But individually the levels of factor 1 and factor 2 would be different for Z1 than with Z2 than to Z3. So at what level at Z1 or Z2 or Z3 one would like to remain would depend on number 1 obviously the combined effect and also on the individual factors based on which what is the level of satisfaction the person wants. Now why these parallel lines. If I go to the other parallel line, which I am now marking and I consider points Z4, Z5 and Z6.

So obviously Z4, Z5, Z6 are on the same straight line, so the level of satisfaction for the combined one for when I consider Z4, Z5, Z6 are the same. But interestingly overall level of satisfaction for the combined one for Z4, Z5 and Z6 are same but the level of satisfaction for Z1, Z2, Z3 which is also the same are totally different from Z4, Z5 and Z6 which means if I consider the level of satisfaction for Z4 would obviously be equal to Z5 will be obviously equal to Z6 and if I consider Z1 which will be equal to Z2 which will be equal to Z3. The combined value if I consider for this one and the first one would never be equal, they are on a different plane, different level of satisfaction like trying to have a different set of income based on that income I try to basically meet the needs. So obviously if I consider the bold parallel lines also this is the same concept which I have just highlighted but why did I draw two sets of parallel lines bold one and the dotted one. The reason being in which inclination the lines would be would depend on what type of satisfaction level one wants to achieve and one wants to have based on the fact there are two competing different type of decision variables to be made.

So we will consider I am just mentioning it very briefly we will consider this type of indifference curves obviously it is shown as a straight line in many of the cases the indifference curves which are the bold one and the dotted one need not be straight line. Now remember here we are trying to basically decrease f_1 and increase f_2. If I further on consider factor 1 and factor 3 where factor 1 is again price other cost again multiplied by 10 to the power minus 3 in order to scale them down and the values of factor 3 which is education employment entertainment which was also cost multiplied by 10 to the power minus 3 to scale down and I again plot A to J, which are the 10 different localities in the city of Visakhapatnam. Very interestingly now the scenario is a little bit different both factor 1 and factor 3 have to be decreased. So obviously in that sense point A which is locality 1 is highest both for factor 1 and factor 3 which is not wanted both of them are increasing.

If I consider say for example H on both the fronts f_1, f_2 and f_3 are the minimum so obviously we may be tend to go for that. Now why I mentioned this diagram which is shown in blue color all the points and the bold and the dotted lines with respect to the first diagram which we just finished is that, in the first one they were two competing factors. Here the factors are not competing against each other. Now also I have drawn bold parallel lines and dotted parallel lines again they would also give you the concept of the indifference curves which was much better explained in the figure 1 but we will consider such factors where both of them keep increasing. So obviously in the first case we consider one is increasing one is decreasing second one both should decrease in order to satisfy that level.

We will consider in more details as we proceed. Now why I mentioned these two diagrams is that concentrating only on factor 1 or factor 2 or factor 3 or a combination of any two may not give us the best result. We have to basically find out the best result based on the combination of all the three factors which will come to that within two minutes. If I consider the scatter plot again, the third set of diagram which is shown in green color in the slide 12 for this lecture, we are plotting factor 2 along the x axis and factor 3 along the y axis and factor 2 which is locality community safety based on the fact that higher it is better. So more points would be better locality and if I consider factor 3 which is education employment entertainment on the scale of 10 to the power minus 3 in rupees obviously that has to be decreased.

So again if I plot the localities from A to J the same analysis can be done say for example for F with respect to A if they are competing so obviously F would be lower on the factor 2 scale with respect to A and if I consider the factor which is A is better because A is giving much more locality community and safety factors. But if I consider on factor 3 which is cost F is lower with respect to A so both are pulling against each other A and F. So obviously this concepts of the bold and the dotted lines of the indifference curves would also come into the picture. We will consider that again I am saying in more details. Basically when I plot all the 3 factors which should be the case in a 3D plot the red points I have not marked these communities of localities A to J but it will give you a much better picture in trying to analyze that how you can basically find out the best solution based on the fact that Mr. Murthy has to find out locality to stay with his family such that his desire of trying to basically find out what is the best optimum point based on the 8 different separate factors. We combine them in order to simplify and come up with the concept in a much clearer fashion. So with this we will end the introduction part in this 5 lectures which we have done and slowly start with discussing all the nuances of MCDM both quantitative as well as qualitative.

Thank you very much for your attention. Have a nice day. .