

Multi-Criteria Decision Making and Applications
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Lecture 49

Now, welcome back my dear friends and students a very good morning, good afternoon, good evening to all of you wherever you are there and at whatever time you are listening to this lectures. The title of the course as you know is multi criteria decision making it is another NPTEL MOOC series and the total duration of the course as you know is for spread over 12 weeks which consist of 60 lectures, each lecture being for half an hour and each week we have 5 lectures such that after each week we have the assignment. So, we are now in the 10th week and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. So, this is a lecture number 49 out of 60 and the main the title of the broad subject is multi criteria decision making, we have already discussed about multi objective decision making where it is more objective, there are equations to solve maximization or minimization based on different type of constraint, decision variables can be integer, can be continuous and we have considered different techniques of solving multi criteria problems. Even though the basic framework was try to discuss by objective because we can visualize how they are done and one problem was in the case where there was three objective functions maximization, minimization the overall procedure remains the same. Apart from utility theory we considered very briefly multi attribute theory and then we started on the nonparametric methodologies of MCDM, we have considered the concepts of top sets and then we are doing the electre method.

For the electre method first was the normal electre method then we went to the epsilon. The main difference between the first and the second was in the first case you can distinguish them into dominance, dominant concept and non-dominant discordant, concordance discordant. And in the epsilon concept of electre method we consider three different areas concepts dominant or concordance discordance and indifferent. Now where this the point of concept of indifference is coming if you remember in utility theory we did discuss a decision maker can love risk, can hate risk and can indifferent to risk.

So, third point of indifference is being considered here. So, based on the last discussion so you formulate the problem based on X which is the matrix based on alternatives and criteria's based on the values or the utilities which are now normalized after they are normalized they are multiplied with the weights, weights based on the criteria not on the alternatives. Once we have the multiplied matrix based on that we proceed then we find out the concordance indices, discordant indices then the indifferent indices and then we are ready to discuss the further set of procedures. So, the coverage is continuing of elimination and choice translation which is electre process which is the epsilon electre we

are considering. Now once you formulate and find out the concordance discordance and the indifference I need to find out the values of C_{kl} , C is basically for the concordance, I is basically for the indifference and D is basically for the discordance.

So, I will mark where what I mean. So, this C_{kl} is for concordance, this I_{kl} is for indifference and this D_{kl} for discordance. I am using different colors in order to highlight. Now in all these three cases we follow the same rule. We basically have for each J and J being the set which basically compresses the concordance. The next we formulate what is the indifference and finally formulate what is the discordance. So, based on that so what we mean by concordance, discordance and indifference is given here. So, here we have J is an element of C . So, basically this C is coming from here. So, concordance concept.

If I formulate the I , so again here J is an element of I , I is basically the indifference and finally when we have J is an element of D , D is the discordance. The important thing to remember is we are trying to find out the ratios. So, consider there are 10 different elements. So, some of them are concordance that they are positive, some of them are discordance negative. Negative I am using a word to differentiate and some are indifference. So, based on that I find out the ratios. So, ratios remember when I mean the ratios it is in the numerator. The denominator remains the same for all this calculation. So, this is the indifference and finally we have the discordance. So, based on that we find out and the values are as follows. So, once we have Y , what is Y again to repeat. So, initially we had X , X was the general the matrix based on the alternatives and the criteria's which were basically then normalized. Normalization I have considered based on few principle which can be considered how the normalization is done. Once it is normalized you have basically the R matrix. The R matrix is multiplied this star sign is multiplication with the weights and that gives you Y .

Only thing to remember I am repeating it please excuse me for that that mat when you are multiplying the matrix the rows and columns should be noted based on which the matrix multiplication is compatible between R and W . W is a matrix based on only criteria which are the weights and R would be based where you are where you are marking the alternatives along the rows and the criteria's along the columns. So, compatibility should be there once you have this we find out Y . Now once Y is done we will find out what I repeated the concordance indices, the indifference indices and the discordant indices based on which we find out the concordance, discordance and indifference matrices. So, C_{12} when we compare and only thing to remember again I am repeating in the denominator all the combinations are taking max of that max means you are trying to find out the maximum the ratios based on the maximum of the differences.

Differences means how further they are. So, I am using the word further because if you

remember in the first process of the concept of using the nonparametric method where we had considered the concept of top says it was based on the distance NIS and PIS. How close it is to the positive one, how close it is to the negative one and based on that we found out the ratios and then rank the alternatives based on the liking and disliking. The distance measure in the first methodology is been converted into the concordance and discordance concepts. So, consider positive one would be concordance, negative one distance would be discordance, disliking or the value is negative.

So, based on that we find out one by one C_{11} C_{12} which comes out to be I am just circling it C_{11} C_{12} comes out to be C_{13} comes out to be 0. Now C_{11} obviously you are comparing with itself so obviously that ratio would not be considered. So, when I find out C_{21} comparing C_{21} comes out to be 0.5567 and C_{22} C_{23} comes out to be 0. Now here I would like to pause if you consider C_{12} on the concordance concept that means I am taking 1 and not 2 then the value is 1 which means I am benefiting to a ratio if it the ratio is between 0 and 1 to the maximum ratio of 1. Now on the other hand if I take the other picture when I take 2 and not 1 the value is about 0.5567 consider is 0.6 which means my level of concordance level of positive satisfaction is much more when I take 1 and not 2 because in that case the overall level of satisfaction in the case when I take 1 is 100%, but when I take 2 and not 1 it is about 60% that means 0.5567 I am considered as 0.6.

Similarly you can compare, but this comparison only remember is on the positive front we have not yet come into the indifference front we have not come yet to the case of discordance front. Because when you choose a decision choose a alternative there are positive and negative benefits already there. So, if you remember in my the last lecture we have considered that you are trying to buy a car the power of the car the engine quality the quality of the car the safety features of the car the comfort of the car is very good, but obviously you have to shelf out much more money. So, obviously if money or amount of loan you have to take from the bank to buy that car if it is considered in the negative sense. So, obviously you have to basically pay a higher interest rate.

Similarly when you are considering buying a house similarly when you are considering trying to get an admission in a good college. So, those compromise have to be made like if you choose college 1 which is further from home. So, obviously travelling cost is very high when you come want to come back to home or if the quality of education is very high placement concept and all this benefits are very high, but the tuition cost is very high then obviously compromise has to be missed. So, in this respect we will consider concordance discordance and indifference. Similarly when I consider C_{31} and C_{32} is 1 and if I go back to the last slide C_{23} is 0 that means the benefit of taking 2 and not 3 is null is 0, but the benefit of taking 3 and not 2 is 1 which is 100%.

Now again I am repeating when you comparing two different alternatives the third,

fourth and the rest of the alternatives are not considered which means even though very simplistic in the practical sense it may not be true we are considering that the effect when you are considering two different alternatives other alternatives are out of the picture, but obviously you are considering all the criteria's at one go for two alternatives being considered. So based on that when I formulate the concordance matrix also remember it is asymmetric the principle diagonal obviously it would not make sense and asymmetrically you can consider here 1 with respect to 0, 0 with respect to 0 and 1 with respect to 0.6. Now here one just one note, so if I consider this 0 here and if I consider this 0 here in the diagrammatically opposite direction which means when I am considering, so let me compute it as 1, 2, 3, 1, 2, 3. So when I consider 1 and not 3 my total amount of benefit is 0 it does not give me much benefit when I am considering 3 with respect to 1 there also the total benefit is 0.

So the question would immediately come from your side if benefit is 0 in both the sizes when I take either on the left or the right that means taking 1 or taking 3 if it does not give me any benefit why should I compare. Now let us pause for 1 minute these you are only considering the benefit you are not yet considered the disbenefit. So, the disbenefit in one of them considering either you choose 1 or you choose 3 that means you choose 1 with respect to 3 or you choose 3 with respect to 1 if the disbenefit is higher in one case obviously that alternative should not be chosen. So, we will consider that in due course of time. When I come to the indifference, indifference means the level of satisfaction, dissatisfaction within certain range I am indifferent it does not matter.

So, if I find out there and if I find out the values of I_{12} comparing 1 with respect to 2 what is the indifference I_{13} . So, the values are already given is I_{12} is 0.1 I am not reading the last three decimals is 0.1 I_{13} is 1 then I_{21} is 0.1 and I_{23} is 0.28 and similarly I_{31} is 1 and I_{32} is 0.28. So, when I formulate the indifference matrix again it is symmetric I went a little bit fast with the slides because the level of type of calculation exactly the same for the normal electre method as well as for the epsilon electre method only in the epsilon electre method just for repetition we have considered the indifference set also. So, the principle diagonal is not required. Now interestingly this is what I want to highlight and this is an important fact.

Interestingly the matrix which you form for the indifference set is symmetric. Now this is very intuitive because if I am considering the indifference region between comprising of alternative I and J. So, obviously whether I take the i th side of alternative I or I take the j th side of alternative J if I am indifferent then the value accruing whether I take I or whether I take J are the same. So, which means that if I compare 1, 2, 3 here I am writing then one by one if I highlight. So, I am using a different colour.

So, if I take 3 and I do not take 2 the overall indifference region is about 0.3 because 0.2863 I am just making it to one decimal about 30 percent. Similarly when I consider 2 with respect to 3 the value is also 30 percent. That means in both the cases the indifferent region of disliking, lacking not being there not being there mark my words not being there would be of the same percentage when I compare 2 to 3 or 3 to 2.

When I am considering 2 to 1 so I should use a different colour. Let us use the green when I compare 2 to 1 and when I compare 1 to 2 the indifference concept or the inference score is also exactly the same which is now 0.1. So, I am not going to compare 0.

1 and 0.3 no because they are comparison of two different alternatives the blue one where it was 0.3 you are comparing 3 to 2, 2 to 3 and when in the green one which is 0.1 you are comparing 2 to 1 and 1 to 2. So, the indifference regions of two alternatives comparison remains the same. This was not the case for the concordance this would not be the same for the discordance and it was also noted down when you consider the simple electre process not the ϵ 1.

Similarly the observation was noted that the concordance and discordance matrices were asymmetric. So now we come to the discordant part. So, the discordant part if I considered D_{12} the calculations again are the same. So, I am not repeating it D_{13} is found out and just one point. So, this J is a \emptyset that means there is no such alternatives where the discordance level was there it can be possible.

When I similarly continue D_{21} , D_{23} and finally when I consider D_{31} , D_{32} utilizing that I find out the discordant matrix. So, initially we have found out the concordance matrix then the indifference matrix then the discordant matrix. The discordant matrix again let us pause the principle diagonal is just comprising with itself would not make sense. Again the results exactly agree to the first important observation which we made and which I am repeating time and again. Again you see if I put 1, 2, 3 with respect to the alternatives and I use a different color this is 0 this is 1 asymmetric the sorry my mistake.

If I consider this one 1 with respect to 0.567 again asymmetric and also one point the value is same, but asymmetry is the core characteristics of this matrix these values are 0. So, these values are exactly same that is coincidentally. Now, what does these values based on the greens values green values which are marked and the red mean. So, blue can also be compared, but I will only stick to the green and the red similar deduction can be done for the blue part also. Now, this is the discordant set which means that the negative impact the dis value or not the positive value.

Actually it means if I take 3 and not 2 the total amount of dis benefit which is accruing to me for taking 3 and not 2 is 0. That means all things for 3 are benefit which we have seen in the case when we are considering the C matrix concordance matrix. But if I consider the comparison of 2 to 3 which is 1 which means that if I choose 2 and not 3 then the level of discomfort dis benefit or the negative value is the highest is 1. But if I consider the value of 3 1 and 1 3 which is 0 0 in both the cases which means either way I win I am using the word win because if I choose 3 there is no dis benefit and if I choose 1 choose 3 means choose 3 with respect to 1 and again when I choose 1 with respect to 3 again the level of dis benefit is 3 is 0. So, we have compared the concordance positive we found the indifference where the symmetricity of the matrix was very important and it was noted down during the discussion and the third one was basically the discordant matrix.

Once you have that you would find out the concordance discordance and the indifference matrices and find out the cut off scores. Cut off scores the formulas are exactly the same which was if you remember summation of $(1/m) \times (n - 1)$ and the corresponding C values or D values or I values. Now one thing to be remembered is that m is the number of alternatives is nothing to do with the criteria because you have already considered the criteria when you are trying to compare the alternatives 1 by 1 with respect to each other. So, when I am comparing I to J alternative I am considering the conglomeration of the criteria at one go for the comparison. So, the cut off values based on the formula for concordance I will use the green colour it is about 0.2 I am leaving aside the other three decimal places. If I can consider the indifference which I mark in blue is about 0.8 and if I consider the value of discordance it is about 0.2.

So, having said that let us do a fast double check. What is the double check which we will do? It is we will try to add up the values. So, 8 + 5, 7 + 7, 14, 14 + 8 is 22, 2, 2, 6, 7, 8, 8 plus 4 is 14, 21, 1, 2, 8 then 15, 21, 1, 2, so 7, 8, 9, 10, 11. So, obviously it would not be 1 it not 1 when we considered only that concordance and discordance. Why I did that I will come to that within few minutes. Now based on the concordance based on indifference and based on the discordance I formulate the matrix F.

Now what it is let us see. So, for the values which are positive, positive in the sense from the concordance positive means higher utility, positive utility. Indifference means I am this way or that way based on the indifference concept which we have formulated and discordance means higher or lower whichever you see on the negative sense. So, it means disbenefit. So, if I consider it let us go one by one I will only compare 1 with respect to 2 not the number 1 which is written in the slide 1 means alternative 1 with respect to alternative 2.

So, let me write down A_1 and A_2 . So, if I consider first concordance I will use the color green. So, when I compare 1 with respect to 2 it is always positive for A_1 that means if I choose A_1 yes I am getting a benefit which is good. Now if I compare 2 to 1 also it is a benefit which means either I take 1 or take 2 it is always a benefit some benefits are coming which is fine with me, but I am not gone into the other two levels. If I consider indifference concept so in both the cases 1 with 2, 2 with 1 it is basically 0 that means both way the indifference regions are the same I took take the blue color I am putting a tick mark in order to analyze the situation. When I consider the negative impact of 1 with 2 and 2 with 2 with 1 it is also the same.

That means if I have taken A_1 the benefits would have been there which is the green tick mark along with that the baggages of negative benefits would also have come which I have marked in red. Similarly, if I have taken 2 similarly the benefit would have come with a negative baggage which is also marked in red. So, now if somebody ask can any comparison be done between A_1 and A_2 based on the fact concordance, discordance and indifference at one go it would be difficult to mention accordingly and rank them. That is the detail analysis I am giving between comparing A_1 and A_2 .

Now would such a ranking be possible for other alternatives. So, we will check in due course of time. So, let me erase it. So, if I use again let us use say for example, A_1 and A_3 because there are two comparisons. So, if I consider the concordance I am using the green color A_1 to A_3 and A_2 to A_1 nothing can be said because it is dash. So, there is no such positive on the positive value scale nothing can be said very categorically nothing can be said very convincingly.

If I consider A_1 and A_3 then on the level of indifference again these 1 1 which is indifference region being the same and if I consider the concept of negative benefit of A_1 and A_3 again discordance is dashed as that nothing can be said. Now if I compare this one with the previous one the ranking system in both the cases when you are comparing A_1 with A_2 and A_1 with A_3 cannot be done. So, we will see in shortly that how that can be resolved. Now determine the concordance, discordance and the indifference values which is I am only reading the two decimals 42, 46 and 42 % and based on that we will basically solve the problems accordingly.

So, let us solve it. So, now this is basically based on the concept of so first we had the concordance then we will have the indifference and then we have the discordance. So, all the concordance concept was considered. So, in here similarly when I consider the concept based on indifference, concordance, discordance and indifference I will have these values which I will discuss in the next class. Considering the time duration I would like to end it here and thank you very much for your attention. .