Welcome back to all my dear friends and participants for this course. Good afternoon, good morning, good evening to all of you and this course is titled multi criteria decision making which is under the NPTEL MOOC series. And as you know it is a course which is for 12 weeks with 60 lectures and each week you have 5 lectures with half an hour each for each lecture and after each 1 week classes you have 1 assignments. And as you know that we are in the 10th week for this lecture series and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. So, we were discussing about Electre process and I did discuss about the concordance discordance set and this concepts of so called nonparametric methods of multi criteria decision making which is a part and parcel of MCDM we are discussing after we have covered few simple topics of multi objective decision making. And we did discuss about utility theory multi attribute utility theory and all   those   things.

 This is the 48th lecture under the 60 lecture series course. For today we will be discussing the same method of Electre method, but it is known as the ϵ Electre. Why ϵ? I will discuss that and then we will go through the steps. Now before I start the class I should remind you overall the idea of calculation of the concordance index, concordance set, discordant index, discordance set then trying to find another condenses dominant matrix discordant dominant matrix then combine the concordance discordance matrix and finding out the final ways of analyzing which alternative should precede the other in the ranking system everything   is   same.

 There is only one subtle difference in the initial part where I will stress. So, the later part there would be lot of slides which will be repetition so I would only highlight it. So, this set of lectures which you will have for the 48th one would be more in number because I have gone through the concept of the Electre method in general details. So, I would not be repeating all the same thing, but where it is important I will definitely highlight. Now why we   mentioned   is ϵ Electre   I   will   discuss   as   we   consider.

 So, here the first portion will be going slowly to analyze the mean subtle difference between Electre and ϵ Electre. In ϵ Electre method we consider yes concordance set we consider positive one, on discordance set yes we consider the discordance set, but very interestingly the concordance set and the discordance sets are not mutually exhaustive because in concordance in discordance concept in the actual method of ϵ it was very start go no go black white you like it do not like it. It is there in the concordance set or it is there in discordance set, but in this ϵ Electre we are also will consider the concept of indifference set. That means where it is difficult to analyze whether the liking and the disliking can definitely be input into the concordance set in this is indices or the discordance set indices. They would be some gray area there is no stark black and white, they would be black white and   gray   in   the   gray   area   it   is   your   people   are   indifferent.

Now this concept of liking disliking indifference we have already analyzed in the concept of utility sense. So, in the utility concept or the risk concept we consider a decision maker can take a decision where he likes to take the risk, he does not want to take the risk and in another case he will be he or she would be indifferent to taking the risk. That means whether alternative one is chosen with respect to two or two is chosen with respect to one there is some reason or some region not reason, region based on the positive and a negative impact of the concordance and discordance no such differentiation can be made. Like to give an example that some person wants to choose a location of one apartment. So I always go to the apartment example buying a car and choosing the right college.

So you want to buy an apartment consider in the city of Bhopal consider and there are two locations one location is in the area which you really like but the cost is high and another one is in location two which you would not put in the same ranking as location one but the cost is low. So if I am trying to compare the cost with the location place obviously cost versus location you are indifferent that means you are trying to compromise or say for example, consider another thing person want job say for example, you are staying in your home city consider it is Madurai and consider you have got a job in Bangalore. So consider somebody wants to move from Madurai to Bangalore the cost implication would be huge there would be huge increase in transportation cost, huge increase in the daily expenses, huge increase in the apartment you are trying to basically buy as give a rent for or if some married person is going there would be cost of schooling for the kids and so on and so forth. So the cost implication on the cost front is quite high. So if it is not compensated with the huge amount of increase in the salary it would not make   any   sense   even   if   the   job   is   very   good.

So obviously a compromise has to be made so you are indifferent whether go to for Bangalore city for a job and get a good salary but the cost implication is high and in the other sense stay in your hometown Madurai which I gave an example where the   cost implication is low and the income is low but you can balance it. But obviously the person can immediately ask one of you who is watching this set of videos can ask what if the job opportunities the overall change in the profile of the job are not we going to take that yes we can we should take that into consideration but for the time being I am only considering the cost implication. Maybe on the job opportunity sense in Madurai the number of promotions which you can get staying in the same company would be less but the overall competition considering the number of people who are working there is much lower while in Bangalore the number such openings in the same company is higher but there is stiff competition also. So you have to again analyze with this right or wrong to make the decision. So these type of examples which I am giving may make sense when you are trying   to   basically   analyzing   this   problem   of   the   concept   of ϵ-electro.

So having said that let me continue so that if you remember I said I will give you the introduction and the later part of the calculation moves exactly as the electro method. In ϵ- electro method we consider the concordance and discordance sets are not mutually exhaustive. The C and D which we consider that means C being the concordance concept. The C and D is not equal to the overall set where C_{kl} is the set of those criteria for which

A_k is favored with respect to A_l while D_{kl} is the set of criteria for which A_{kl} is not favored with respect to A_l. So they do not make up all the sets of criteria some of the criteria are such that you may be indifferent between alternative 1 and alternative 2 which here in this case I am marking as A_k and A_l.

To account for this discrepancy one need to consider those set of criteria which we put in the set of I_{kl} which will denote as the indifference set of criteria between A_k and A_l such that when I consider the concordance liking, discordance disliking and indifference one would make up the whole set of criteria we are trying to analyze. There it was C D here it is C D I, C means concordance D means discordance and I mean indifferent. So now the process moves exactly the same. You have the set A, A is basically the cells or the values based on the overall value which you are trying to analyze between the alternatives based on the criteria. So given that the set A is given and it is 3 / 3.

Here as usual m is the number of alternatives, n is the number of criteria. You normalize them. I am using the log normalization scale here. So when I find out the log based on that we have the log values and find out the values and then normalize them. So normalization I am doing is basically based on the fact that I add up the log values and put in the denominator and then use that as a normalization case.

So you have seen that normalization being one case being $\frac{xij}{\sum xij}$. Another is $Ln[X_{ij}] / \sum Ln(X_{ij})$. So here using that the normalized matrix X comes out to be this. Then we have the weights, we know the weights are with respect to the criteria and such that the sum of the weights should be 1 which we already know. So based on that we have the weights and here we consider the weights as one third for all these three, so equal weightages. So multiplying the normalized X matrix with the weights gives me the Y values, Y matrix. So again remember the compatibility of the matrix should be such that you are able to get the multiplication of normalized X into weights would give you the Y matrix. Now comes the subtle difference which you have considered the indifference one that will be coming to the steps. Here you formulate the concordance index which is how A_k out ranks A_l and you put them in concordance. Discordance means index means where A_k never out ranks or dominance A_l and you put them in the discordance set.

And indifference means where A_k and A_l cannot be differentiated there you put in the indifference set. In the normal electro process that I or indifference set or indices were not there, they were stark black and white in the C set and the D set. Here there is C D and I. Now what does that mean? It means that if you have and again this same line which I am drawing if you remember in that case we have drawn the line as blue and red. Blue means yes concordance, red means no discordance.

Here I am trying to analyze the concordance and discordance along with the indifference. If you see the graph and obviously the slides will be shared with you. See the graph any portion on to the right along the real line is definitely in the C set. So, C I will basically mark with green in order to highlight the fact that concordance and discordance have the meaning. So, this would be the C set.

Any set which is on to the left not liked which does not outrank, does not dominate would be given in the D set. And the indifference part where you cannot say whether it dominates, you cannot say it whether it outranks, you cannot say it is negative, it is basically the I set. So, it is yellow even though yellow is not visible, but in the line scale you can find out C on those set on the right D and that means if I am looking and D set on the left and I is one portion in between. Now the question is, is the ϵ or the difference? Now if you remember in the concordance and discordance set of the ϵ method, I consider the difference. Now the difference this ϵ which we can consider can be made more or less depending on whether you are trying to consider asymmetricity also.

Now asymmetricity would arise from the fact that if you remember I did mention one very simple example even though it would not be an exact one that will come under the concept of reference theory and all these things. Very simple idea that if I lose 10 rupees and gain 10 rupees my level of satisfaction and dissatisfaction may be same, may be different. On the other hand to extend that consider I win 20, lose 20. So, I have lost 20 or I have gained 20 does not matter, consider it does not matter to person 1. But for person 2 winning or gaining 20 rupees, losing 20 rupees obviously has an effect positive or negative.

But the level of satisfaction and dissatisfaction are of different quantum. In order to bring that quantum of level of dissatisfaction into the picture, I would basically consider the ϵ can be tweaked in order to consider that if indifference region to be more on to the right or to the left. More on to the right means more towards the green side or more towards the red side depending on what is the amount of indifference concept the decision maker wants to bring into the discussion. So concordance would be given by the fact that for Y_{kl} if it for we will consider that for all j's will basically combine that if l is better than j will basically put them in the concordance set or else will put on the discordance set. Now how far it is more the difference is that will be given by the ϵj.

So, it would basically be dependent on case by case basis decision maker basis as well as based on trying to compare the alternatives based on different criteria. The question is that why did I mention the word criteria like example. I would consider for my case let me give an example that if I am choosing the college, so choosing the quality of the faculty members, quality of the education would be higher priority list than the cost. So criteria of the quality of education and cost are criteria when choosing the college. So if I am choosing a college one with respect to two then the quality a small delta amount change in the quality would be a great positive point for me.

When I consider the cost factor in trying to compensate or trying to basically win over that quality I am willing to basically incur a higher cost. So the cost factor I am not that fussy I am indifferent for a higher level of cost but for the quality of education I am fussy that means my level of liking disliking is totally different. Consider the example of buying a car. For me safety matter is more important than the cost. See the car's engine is very good, is a very sturdy car, safety features are very high then trying to basically expend some amount extra amount of money does not matter to me.

This is my viewpoint. Another decision maker can have a different viewpoints. So considering the criteria based for any alternative for the example which I gave for someone say for example rather than quality of education for him or her the job opportunities are more important. Cost of studying is not that important, job opportunities. So obviously he or she will try to analyze the alternatives which are the college based on the job concept and put a higher level of liking disliking or the indifference region would be much smaller. So these type of problems can be analyzed and this value of ϵ will depend on type of problems which you are trying to put into the picture.

So concordance would basically have all those where the outranks or dominates values are there. The green line would depict the region where it is more so you put on the positive side. The red and the yellow line would denote the discordance on the different region. Discordance means I definitely do not like and in the between region yellow one would be where I am indifferent. So any positive or negative in that band region does not matter to me and the examples which I gave.

So when I told about the color difference is exactly similar to the blue and red concept which I considered in the discordance and concordance concept. So the indifference one as I said can be symmetric, can be asymmetric. Symmetric one example gaining 20 rupees losing 20 rupees they would be of different concept if somebody is trying to analyze the problem from the point of view of the idea that what is the difference set or an indifference set based on which the decision maker is going to take the decision. So here the indifference set I am not going to the formulas they would be shared with you so you can go through but I am talking about the indifference set would be depending on the ϵ would be that indifferent region between which I would definitely put both the alternatives in the same group. So that would be analyzed by the yellow line and here it is.

If you see this yellow line it is an unequal length of ϵ j1 and ϵ j2. So, j being changing obviously the lengths would change that means in the positive direction I am more indifferent here on the region here and in the negative direction the length is smaller and it can be reversed also based on that which means a small amount of dissatisfaction I am definitely not going to what should I say. I am not going to tolerate but a large amount of dissatisfaction I am still happy. So that means if I lose 10 rupees that can be compared and given a hypothetical example my level of unhappiness of losing 10 rupees can only be compensated when if I win 200 rupees. So the happiness concept which I will get in trying to win or get 200 rupees can be compensated when I lose 10 rupees.

So in this way I am trying to analyze. It can be the other way around also level of dissatisfaction and satisfaction can be reversed. More level of dissatisfaction I am willing to withstand than the level of satisfaction. So the yellow line will depict the region where it will hold true that indifference region is given and the red one would be as usual with a part for the discordance. So in this way we need to basically give an ϵ and ϵ on the positive side and the negative side may not be of the same length based on that I try to analyze. Step 3, 1 you will basically find out the discordance set as you have found out the concordance you will find out discordance set which is the red line.

And the yellow line would be the indifference one. Once we have that based on y I am trying to analyze. Y means basically the normalized X multiplied by the weights. Weights were if you remember were overall one third, one third, one third. So when I am trying to compare C between the alternatives I would now have C, D and I.

I was not there in the initial case. When I compare along the rows of the columns I would basically have where $j = 1$, I have in the indifference region. Obviously comparing with itself with all the criteria would give me the indifference region. So I would have C concordance with null set, discordance with null set and indifference would be set 1, 2, 3 which is there is no difference between them. When I compare C_{12}, D_{12}, I_{12} based on the fact that I compare 1 2 2, 1 2 3, 1 2 4, 1 2 1, 1 2 2, 1 2 3 based on the next level of criteria. The sets are again please go through the slides to clear.

So I will have concordance 2, indifference 3 and discordance 1. Here indifference is coming to the picture. Similarly when I do C_{13}, D_{13}, I_{13}, I have C_{13} null, D_{13} null and I_{13} as 1, 2, 3. By the way the ϵs can be tweaked in order to basically get different sets of values. So if I continue I have I am only marking them C_{21}, I_{21}, D_{21}, then C_{22}, I_{22}, D_{22} and I D_{13} this is D_{22}, then I have C_{23}, I_{23}, D_{23}, C_{31}, I_{31}, D_{31}, then C_{32}, I_{32}, D_{32}, C_{33}, I_{33}, D_{33}.

One important thing when I am comparing C_{11}, D_{11}, I_{11}, C_{22}, D_{22}, I_{22}, C_{33}, D_{33}, I_{33}. In all the cases the concordance and the discordance set would always be null, indifference concept would basically consume have all the elements in the I set. So once I have that I basically have an asymmetric set and here is the asymmetric set. If I compare here the first cell which is 1 1, then second one principal diagonal 2 2, third is 3 3, just make a note here it is I_{11}, all the elements, here I_{22} all the elements, here I_{33} all the elements and based on that I have this concordance discordance matrices and it should be concordance discordance indifferent matrix important thing. Here when I compare this, when I compare this and when I compare this, here the idea comes out to be very start few important difference, I will mark them in red.

C_{21} and C_{12} when I compare it is 1 and 2 elements which are different obviously they are asymmetric. When I consider D_{21} and D_{12} it is 2 1, 2 1 means element being 2 1. Similarly, if I use this set C_{23}, C_{23} is 2 and null, D_{32}, D_{23} which is null and 2 that means trying to compare them based on the ideas of concordance, discordance and difference, indifference one would give you different results. Considering the concordance discordance we have the relative weights for the concordance concept I add up the weights. So, W C weights are given which I have the construct the concordance discordance matrix indifference and in discordance.

What I do is basically I add up the weights for which is positive or negative depending on concordance discordance and the indifferent concepts. Now, again important thing here very interesting is two things if you remember for the concordance set the weights were all adding up one for the mirror image of the elements according the principle diagram here it is not because we have divided into three sets C D I. So, they do not have to add up to one this do not add up does not add up to one and this also does not add up to one. What you need to remember is similarly for the indifference one they also would not add up to

one. For the three weight matrix these three weights the weights correspondingly for each element wise they should add up to one here it is   what I want to highlight.

 One third one third one third add up to one that means I am taking the element of one one comma two one third one third one third add up to one. If I take the element of three two three two three two it adds up to one third plus two third is one. If I take the element of two one two one two one it is one third one third one third is one. If I take the element of two three one third dash means does not have any point and two third is one. So, in this way you can verify the addition of the weights corresponding to indifference C D and I would   add   up   to   one.

 So, the ϵ steps in the electron would continue in this discussion and will basically try to wrap up the problem accordingly in the next class. Have a nice day and thank you very much. .