Multi-Criteria Decision Making and Applications
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Week 10
Lecture 47

Welcome back my dear friends, students. Very good morning, good afternoon and good evening to all of you who is taking this course. The title of this course is multi-criteria decision making and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. And this course multi-criteria decision making is under the MOOC NPTEL lecture series and as you are aware this is 60 lecture series spread over 12 weeks and each week we have 5 lectures, each lecture being for half an hour. For your kind information we are in the 10th week. If you remember till the 8th week we were discussing about different concepts of utility, different concept of multi-criteria decision making by objective and considering all the different nuances of that.

Then we started with the concept of top-seps and we are in this lecture, I will talk about the lecture number. The lecture number is 47th out of the 60 and we are discussing over the electro process. The broader context is under the multi-criteria decision making, we have already covered multi-objective decision making, few of the tools, some concept of multi-attribute decision making and we are now in the subjective ways of trying to analyze multi-criteria. So the coverage as we know that we started would be in the method of electro which is known as elimination in choice translation reality.

So if you remember in electre process the overall steps was to basically find out the discordant and concordant set like in the concept of top-seps and other method I did mention time and again we wanted to find out the most positive distance, more negative distance, find out the ratios and then rank them. This distance was basically based on the concept of L_2 norm in the Cartesian coordinate. Here the concept of the discordant and concordant set is not exactly similar to the NIS and PIS concept, most positive or more negative. It is basically how concordance means positive set of criteria which help in making the decision on the alternatives and another way the discordant is that negative way of trying to analyze. That means if you take it what are the negative impact.

So obviously making any decision would give you both positive and negative impact of the decision like buying the best house location wise, safety wise with more number of rooms and where the bus stop or the transportation nearby and all this in centrally located obviously will cost high or trying to basically buy a car which is really high ended in the engineering sense good safety features, good power of the engine, good space for passengers, good safety requirements would obviously would cost high. So in the similar way. So in the fourth step is basically after the indices concordance and discordant indices

are found out based on each comparison and we construct the concordance on the discordant matrices. Now if you remember concordance would basically add up to the value such that you give the corresponding weightages which you have which you had allotted for those criteria would have be added up and the matrix based on the concordance and discordance if you remember I mentioned it is not a symmetric matrix, but if you add up the sum of this opposite elements like IG and GI they should add up to 1. So the discordant matrix D as we had been denoting the discordant matrix D basically expresses the degree that a certain alternative A_k is worse than a competing alternative Ai.

So when I am comparing what I am doing is I am comparing A_i with respect to A_k alternatives. So obviously if I choose A_i and if it is concordance I will denote by blue. So obviously you give positive weightages or more weightages are the weightages are added up. So you will basically have the concordance set C_i am marking in blue color and if I have the discordance I would basically have the discordance set which I am marking red. So based on the indices I find out C and D.

So here if A_k is worse than it is competing alternative in choosing the criteria and the concordance and discordance elements would be coming from each and every criteria like consider power of a engine and consider you have see for example, a car 1 and a car 2. So if based on the power if car 1 is better so not choosing that is an disadvantage. So you will try to analyze that problem accordingly. So D expresses the degree that a certain alternative A_k is worse than it is competing alternative based on the fact for the jth criteria and if you remember we have analyzed and taken in consideration that there are m number of alternatives and n number of criteria. So when we take the elements of the discordant matrix we will basically try to analyze the ratio of the max.

So when you are taking the max of the difference we will only consider in the numerator we will only consider the discordant which we do not like and when we take in the denominator is basically set of all the alternate is based on the criteria which are there that means we are trying to basically find out what is the maximum the ratio of the maximum of the non liking divided by the total set of liking disliking combined together. So we want to find out the elements of the discordant matrix which is given by this. Now why we want to do that is basically we want to find out what is the maximum level of discomfort. Similarly the idea can be that can we find out the maximum level of comfort answer is yes but if you remember that when we were we had started discussing about the subjective way of trying to analyze the problems multi criteria's we have considered the TOPSIS method, we are considering the ELECTRE method, we will consider the VIKOR and the AHP method. There are other methods also MACBETH and all these things.

The focus is always on what is the loss because human beings want to basically minimize

the loss rather than the gain. Gain would be positive obviously but people are more concerned about the loss. So based on the fact the discordant matrix is being analyzed and we will try to basically take that discordant matrix in such a way that we are able to rank them based on the discordant values or discordant matrix values such that the level of dissatisfaction is the least. Disgordant satisfaction is basically the word I am using when you consider the discordant concept. So based on the concordance and the discordance concepts   we   have   the   values   of   the   discordant   matrices   values.

So here what I mean by, if I consider let me use a different color, let me use the green color. So when I am taking max of $J_{max} = 1, 2, 3$ or $J = 1, 2, 3$ in both the cases of the numerator you are considering all of this comparison together and we are taking the maximum of them and the numerator will only consider those which are dissatisfaction where you do not like, which you do not want to take the decision even if you have taken the decision what is our negative impact is going to have. So do not like discomfort or negative impact all these words I am using in a sense that I want to basically portray that what would be the overall discordance set based on which we will try to analyze. So when we find out when you are comparing 1 to 2, 1 to 3 the values come out to be I am so the red part would be the discordance is only 1 and 2 and here it is only 2 because where the discordant values are. So when I am trying to analyze it will be analyzing the discordance and   the   ratios.

So the discordance ratios based on D_{12}, D_{13} when you are comparing 1 to 2, 1 to 3 would be given by this value 1, 1. So what does 1, 1 mean that means the level of discomfort when I am trying to find out between 1 and 2 or 1 and 3 is 1 which would mean that on the level of discomfort I am facing the maximum amount of disadvantage based on that I will try to basically analyze. So here again I am saying we are not looking at the benefits we are looking at the discomfort disadvantage. So based on the disadvantage scale, so lower the disadvantage would be the highest and maximum the disadvantage would be the lowest. If I am considering from the point of concordance liking, maximum liking would be   the   highest,   least   liking   would   be   the   lowest,   it   is   just   the   reverse.

So based on that I find out D_{12}, D_{13} and if you remember if you go back to the slides are already there with you when you are doing this 47th lecture obviously would have gone through the set of slides for the previous one. So based on that I find out the values of D_{12}, D_{13} as 1. When I am trying to compare D_{21} and D_{23} the values comes out to be considering the comparison is for max of all those discomfort in the numerator and max of all of them in the denominator. So the values come out to be 0.31 and 0.64 which means that if I in a very layman terms if I consider D_{12} the value is 1, if I consider D_{21} the value is 0.31, consider as one third of that which means the level of discomfort I will have in choosing 2 with respect to 1 is one third of that value which would have arisen if I have taken 1 and

not 2. So 2 basically choosing 2 is basically is an advantage which my level of discomfort level of disadvantage is much lower. In the similar sense you can compare D_{13} with D_{31} also. So when I compare as I was saying when I compare D_{31} which is 0.37 I am just circling it with the value of D_{13} which is 1 which means that if I have chosen 1 my lower total amount of discomfort is the maximum level of 1. But if I had chosen 3 and not 1 and they are being compared 1 is to 1, 2 is not coming to the picture considering that all my concentration is there and I am trying to compare 1 with 3 or 3 with 1, 2 is out of the sea. There the value is 37, so about one third of the discomfort level is there that means it has reduced by two third. But very interestingly if I compare D_{32} which is 1 and I compare D_{23} which is 0.64 that means there the ratio of choosing 2 and not 3 the level of discomfort has reduced by one third because I am considering 0.67 or 0.64 as almost the same value. So here it is 2/3 and there it is 1. Another thing if I consider 1 3 as 1 and 3 1 as 0.37 which we have already discussed, so let me write down the values. So D_{12}, D_{13}, D_{21}, D_{23}, D_{31}, D_{32}, D_{32} is 1, D_{31} is 0.37 I am comparing these values then do D_2 0.64, D_{21} is 0.31. So, let do a rudimentary analysis I will use different colors of comparison. So D_{12} the green blue red whatever color I am going to use I am not placing any importance of the color, but only to link them up D_{13} has to be compared with 12 with D_{21}.

Then D_{13} has to be compared with this and finally, D_{23} with this. So, if I see the green color it is better to choose 2 with respect to 1. If I see the linkage of blue color it is better to choose 3 with respect to 1. If I check the light darkish yellow one is better to choose 2 with respect to 3. So, if I want to do documentary I will go into the details.

So, if I compare so obviously, I will mark them as 3 to 1. So obviously, the green one would give me 2 is better in a sense than 1. If I compare the blue one 3 is better than 1. If I compare 2 3 so 2 is better than 3. I am able to rank obviously, it would immediately give me the idea that 2 is better than 3 is better than 1.

But it could have been that here in place of 2 being greater than 3 you could have had this 3 is better than 2. I am not going to the values as such it could have come out that. In that case the ranking would be 2 is better than 1, 3 is better than 1 and if I say that 3 is better than 2 then obviously, the ranking would have been 3 to 1. So, you could have analyze them on the discomfort level. Now, which is basically looking at the other side of lower the discomfort more the profit or more the positive value.

Now construct the concordance and discordant matrices thus that discordant matrix would basically be the values which I have, which I have just written down. So, when I have the d values they are not the weights as I found out in the case of concordance. Then the principle diagonal where I am drawing this black line and the elements are not symmetric and also remember one thing in the concordance set as we are adding up the weights.

Hence the sum of the weights of the mirror image of the principle diagonal were 1. Here if we add up the values they are the levels of discomfort and they are nothing to do with the   weights.

So, the weights were used for the concordance set here 1 plus 0.31 obviously, it would not add up to 1, 1 plus 0.37 not add up to 1, 1 plus 0.64 not add up to 1. So, this is an asymmetric   along   the   principle   diagonal   as   already   mentioned.

Now we have done the concordance, you have done the discordance and only look in the concordance you would have been able to analyze, only looking at the discordance you would have been able to analyze. But what is needed is that trying to find out that concordance is positive, discordance is negative. So, in what sense which of the alternative basically scores a positive value in both the senses. Both the senses means, concordance also it is giving positive value, discordance it is also given positive value. Discordance means positive thing in the sense it is higher up and the discordance it would be on the lower side when basically lower side in the sense on the discordance or discomfort level.

So, once we find out the concordance of the discordance matrices, we need to find out a threshold of the concordance index and this threshold would be based on the fact that alternative A_K will dominate A_L if and only if the value of k when you are comparing the concordance values of k to l, if it is greater than some C^* value obviously the threshold index is positive in the sense it is better to choose the kth alternative with respect to the lth alternative on the positive front. So can we decide the threshold value? The answer is yes, the formula is given of C^* like this which is $(1/m – 1) m \times (m – 1)$ and you remember m and n are the corresponding numbers of alternatives and criteria. So, when you are comparing the alternatives it is based on the criteria's comparison. Alternatives can be compared based on the criteria's only the characteristics what you want to achieve and based on the m number of alternatives which you have you find out the C^* value and it is basically sum of all this concordance values for all l and k which is basically from taking together from 1 to m total number of concord alternatives. So once we have that putting the formula I get a value of C^* as 0.5. Now when the C^* is 0.5 I will find out if it is positive obviously it will assign a score that means yes threshold value is exceeded it is good I will take the decision. So 0.5 would mean that when if I go back to the set of comparison the concordance set based on that I have the concordance dominance matrix. So this is given by the factor F or matrix F and remember I am trying to basically compare with respect to the C^* which is 0.5 half. Now what it means is here I will try to draw some or on conclude some interesting points. Again principal diagonal the elements along the principal diagonal are not symmetric it is asymmetric and the values are 1 0 1 1 0 1 I just read the values and now let us compare. So here what we have along the rows we have the alternatives and we are comparing the alternatives with themselves. Hence the 1 1 2 2 3 3 principal diagonal

value elements are comparison with itself. So there is no use of trying to basically find a score or write up any score along the principal diagonal.

But when I compare A_1 which is the first row so with respect to A_2 it is 0 that means if I take alternative A_1 with respect to A_2 there is no such advantage on the concordance set. So it is basically does not cause the threshold so basically is the value 0. If I compare A_1 with A_3 the value is 1 look at the first list of elements on the first row it is 1 that means it definitely makes sense that you have taken A_1 with respect to A_3 I have chosen A_1 so some positive impact is coming. Now when I compare A_2 with A_1 the value is 1 which means that choosing A_1 did not give me any value positive thing, but when I chose A_2 with respect to A1 the value is 1 crossing the threshold it means it makes sense to choose A_2 with respect to A_1, but it does not make any sense in trying to choose A_1 with respect to A_2. Now if I compare the value of 2 to 3 so obviously A_2 is better than A_3 when I compare A_3 with respect to A_1 it is 1, A_3 with respect to A_2 it is 0 which means it does not make much sense in trying to choose A_3 with respect to A_2, but it makes sense in trying to choose A_2 with respect to A_3 that is why it is 1.

So based on that positive impact I basically compare positive things I am not coming to the discordant concept. Now in the similar way I try to find out the discordance remember again concordance based on the weights discordant based on the values which you have just seen. Again the concordance and discordant value the threshold is given by the same value D^*, D^* is calculated as 0.75 based on that threshold coverage is 1 threshold not being covered is 0 again I compare this is on the negative impact. Now on the negative impact if I see A_1 with A_2 it is 1 that means choosing A_1 gives me a lot of on the dis-benefiting scale it is definitely not warranted I definitely do not want to like.

When I compare A_1 to A_3 again trying to choose A_1 with respect to A_3 definitely on the negative impact disadvantage it is not at all advisable. Now there was an other side of the story also which we just covered. When I have chosen A_1 to A_3 A_1 gave me positive on the positive scale. When I am choosing A_1 with respect to A_3 I am getting a negative on the negative scale that means choosing A_1 comes with a baggage both positive sense as well as the negative sense it is basically being more tilted towards A_1. So obviously you would say that I am getting the benefit good but I am also getting along with that a list of dis-benefit also when I choose A_1 with respect to A_3.

So that is why it is 1-1 both on the positive and negative scale. Now if I compare the values of A_1, A_2 with A_1, A_2 is +1 in the concordance set. When I go on the dis-benefit scale is 0 that means on the benefit scale it is positive but on the negative scale or on the baggage scale is 0 that means if I compare that with A_2 with A_1 that means I am getting a positive thing only anything negative impact is not there which is totally different way

when I was trying to compare A_1 with A_3 and A_3 with A_1. So A_1 when it came it came with both the positive and the negative one. When I choose A_2 with respect to A_1 it came with the positive only not with the negative impact.

So based on that I find out the aggregate. So aggregate means I found out the concordance, I found out the discordance, I found out the benefits, I found out the dis-benefits and then I try to compare them. So if it is positive on the benefit scale, positive on the negative scale, positive means on the lower side it is giving me benefits as well as I am choosing it is also not giving me the dis-benefits obviously that is a win-win situation. So I will basically try to compare them. So I will basically combine the values to find out the aggregate dominance matrix which is E which is a multiplication of F and G in their corresponding cells. So when I do that, so what I have? I had elements of 0, 1, 1, 1, 1, 0.

So it was 0, 0, 1, 1, 1, 1, 0. So let me see 0, 1, 1, 1, 1, 0 and in this scale it is 1, 1, 0, 0, 0, 1. So benzene diol not being there would mean 1, 1, 0, 0, 1, 1, 0, 0, 0, 1. So if I compare the values in the sense that this is basically A_1, A_2, A_3, this is A_1, A_2, A_3 on the concordance, on the discordance of the concordance and this is A_1, A_2, A_3, A_1, A_2, A_3. So when I compare, multiply them, so 0×1 is 0, 1×1 is 1, 1×0 is 0. So all the values are calculated based on concordance discordance and the final aggregate dominance matrix based on concordance discordance is given.

Now from the aggregate one if I find out, so when we have been trying to do the discordance concept I was always mentioning the discordance values being high is not wanted, concordance value being high is wanted. When I am considering the discordance once I am putting the score for which the discordance concept is lower I put a higher score. So based on that when I find out the aggregate dominance matrix based on C and D I have this. From the aggregate dominance aggregate matrix we can derive a partial preference and what is the partial preference is that when I compare this values A_1 to A_3. So obviously I would always benefit on the positive scale, benefit negative scale is the least. So my comparison would be A_1 would always be preferred with A_3, but when I compare A_1 with A_2, A_2 and A_3 no such score is given they are 0 on both the scales positive scale negative scale they are 0 combined one, so no such ranking can be given.

So if kl is 1 then k is preferred then l, so eliminate any columns which have an element of equal to 1. So once we remove that we only have the case of 1 is preferred to alternative 3. So when I compare A_1 is definitely preferred with A_3, when I compare A_2, A_3 nothing can be said there is a question mark because both are negative what is scale no score. When I compare A_1 with respect to A_2 again nothing can be said. So 1 if I rank them obviously alternative A_1 is better than A_3, but nothing can be said when I compare A_2, A_3 and A_1, A_2.

So comparing concordance and discordance we have formulated a simple problem accordingly. So with this I will end this lecture and continue more discussion over the electroposits in the next class. Thank you very much.