Multi-Criteria Decision Making and Applications
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Week 10
Lecture 46

Welcome back my dear students, dear participants who are watching this lecture and you have taking this course. This is the first lecture of the lecture. It is titled multi criteria decision making under NPTEL MOOC series. 12 weeks, 60 lectures each week we have 5 lectures and each lecture being for half an hour. And my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. So this is the 46th lecture and we are discussing about the different nonparametric methods.

We have started and finished TOPSIS method. We are in the process of discussing the electro method. And the coverage is the electro method which we have already started. We have discussed about what is the concept of concordance set, discordance set, how the contour concordance index, discordance index, the distance concepts, what are the steps we have already started.

We will continue with the problem as solved. And as usual the excel sheet will be utilized in order to make things much more clear. Now we were considering the different concept indices for the sets. Now the concordance set between two alternatives is only comprising the alternative not of the criteria. And another thing do not confuse the value of C_{kl} which is given here as criteria. No, they are the concordance index and similarly on the set. Similarly the discordance concept will be discussed by and given the symbol of capital T. Now the concordance set between two alternatives A_k and A_l, I am comparing the alternatives where the k and l are any values between $1,\ldots,m$ because if you remember m is the number of alternatives which you have, is defined as the set of all criteria for which A_k is preferred not A_l. So if it is good C would be populated with those criteria for each A_{kl} is better. Now the better would be in a very simple sense it is greater than some sort of thing and we will be utilizing that from the value which you have found from Y.

What was Y? Y matrix was multiplied multiplicative values which we formulated and found out by multiplying the weights and the normalized matrix. So we will put C_k as all these j, now j, k values are all the criteria. If you remember criteria have been mentioned as n, so it will be $j = 1,\ldots,n$ for all these cases. So I take the first value of j as 1 for criteria 1 compare A_k and A_l. If it is better I put in the concordance set.

Second one is not better so obviously I would not be criteria is not better I do not put in the concordance set and I continue accordingly. Now if you remember in the last class we were discussing about the concept of overestimation in under estimation I gave the

example of the dam and I said that under estimation is more problematic more concern than over estimation and I did mention that they can be examples where over estimation is more a problem than under estimation. First of all consider again from very simple electrical engineering there is a huge electrical circuit and there are very high ended trip switch vacuum circuit breakers where the average life is say for example 6 months. Before 6 months you have to definitely change them after 6 months change in them may be catastrophic because if accident happens. People are working and the average life is 6 months so if you try to replace that and consider the value has been in one case, case 1 estimated as 4 months rather than 6 and another case estimated as 8 months not 6.

In both the cases the difference is 2 + 2 and - 2 if I consider the square it is 4 + 4 in both the cases so if considering the concept of quadratic such loss over estimation it does not make any sense because we are equally giving weightages. But what happens practically under estimation means I stopped the work change it at the end of the 4 month so work stoppage happens man hours lost, but the probability of an accident happened is minimal. If I consider stop the work and I try to do it after 6 months 8 months then obviously the work continuity is there stoppage is less, but the probability of a catastrophic loss happening some accident happening is much more. So in this case under estimation is better than over estimation because over estimation is more penalized. So in the same case when we are considering the concordance set I will consider the real line onto the right is better   onto   the   left   is   not   better.

Now the difference moving from the mean point onto the right or the left the concordance on the right and the discordance which we will consider later on the left. If they are equally dispersed of equal length then we are considering that over estimation under estimation some sort of thing is happening where they are giving equal weightage. If I change the distance on the right or the left accordingly then obviously it will mean that I am not giving equal weightages. So first we will consider equal weightages. So let me read it the blue part of the line is the relevant portion of the concordance set between A_k and A_l for the jth criteria and in that case j as you know is 1, 2, 3, 4 till n, n is the number of criteria.

Now the discordance concept is basically between two alternatives A_k and A_l in the same case where I will only place those alternatives which in which case A_k is not preferred with respect to A_l. So I will put in that set D and if you remember I have mentioned D_{kl} would be the discordance and capital C_{kl} is the concordance set, C is not the criteria. So here the red part if you see on the arrow is on the left hand side is that part of the line which is relevant for the portion for the discordance set and to repeat I am using equal dispersion onto the right and the left. We will also consider in the same way j = 1 to n. Club them into concordance set club them into discordance set for each A_k and A_l where k and l are elements in the total set of alternatives from 1,…,m. Now let us consider that

we   are   doing   it.

Now let us do it here. So we have the concordance set I will put a C then I will basically have so I will consider for each j so C let me put it $j = 1$, j is basically number of criteria. If I put out so if I am considering the first with itself so A k and A l are same if I consider first with itself the value is true. So hence I will put because if it is true I put in the concordance set so I put A_1 in $j = 1$. When I consider the next one I am flipping between the slide and the excel sheet but   pay attention on the excel sheet.

So I will consider that for $C_j = 1$. A_1 which is the alternative one falls in the concordance set. When I go to the fact of $j = 2$ so I am going to consider correspondingly each of these values. So $j = 2$, I will put it and I also consider $j = 3$. If I consider the values so I compare this so 0.083 with respect to 0.0625 it is good so in that case the value of the alternative criteria 2 which I am considering would put a value such that so if I am comparing this one these two for $j = $ again 1. So 0.125 is better than point value wise is 0.083 so obviously in that case the discordance because I am losing something in trying to take the decision as A_1 so it will be coming in the discordance set. So if it comes to the discordance set so I   have   to   basically   make   discordance   values   also.

So I will say this is A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, A_{12}, A_{21}. So in that case A_2 goes here when I come consider 0.083 with 0.04 so obviously this is bad preferred so this would come in the set accordingly into the its preferred so obviously is better so with respect to A_1 is better. When I consider for the second one so I should move these values on the right so this is A_3 with respect to $j = 1$ when I consider these values correspondingly.

So I am considering for all the alternatives criteria 1 so obviously A_1 would be here so if I consider 0.25 is better so I am losing so A_2 is here if I consider 0.185 so obviously this is better so this goes. When I come to the last one so we have 0.1 with respect to 0.05 so 1 1 is same this is better so A_3 is in this set and 0.1 is equal so if it is equal we are indifferent. Now   this concept of indifference means if you remember when we were consider the utility functions of liking disliking or the concept of indifference set or this concept of risk loving risk hating so we had basically clubbed in with three regions liking disliking and indifference so indifference is basically I should use a set I and this set I would be made much more clear later on. So I would consider so here it is a null set I will put the null because there are no elements 1 to A_1, A_2, A_3 are all considered this is null set and in this case this is a non-null set because here both are same if I consider A_3 and this was basically A_2. Null set would be now here so I have formulated based on the first one when I comparing A_1 to A_1 itself A_2 itself A_3 itself I have this set so this is form $= 1$ comparison   when   I   go   here   so   these   values   have   to   be   filled   up.

So now I will compare 0.15 with respect to 0.08 so I will first basically first make it empty you have to fill up so if I make it so I am considering for 0.125 is better than 0.08 so in this case A_2 is better in this case when I comparing with itself so this is A_2 again and third case so A_1 would go here is not disliked so this I will put A_1 indifference is when I consider with respect to A_3 again this is better so A_3 would be going here and this is a null set. When I come here so I am coming for j = 2 criteria 2 so again 0.25 with respect to 0.0625 so A_2 is better so A_1 will come here with respect to itself 0.25 is better and with respect to third one this is better so A_3 will also come and this set so this is a null set. When I consider the last column for m = 0.05 < 0.1. So in this case A_1 is better. This is equal so this would basically go with itself when I am comparing so this would itself it is good and the last case again this is good so everything would be so in this case discordance because it is the worst 0.05 so it would be null here. When I consider m = 3, I will remove them so and I am considering the last one 0.04 with respect to 0.4 0.08 so obviously A_1 is better so A_3 is not liked because it is low. In the same case when I come to 0.125 A_2 is liked and in this case the null set would be here. When I consider the second column with respect to m = 3 this A_3 is better, because 0.1875 is better than 0.0625 then if I come to A_2 which is 0.25 it is not liked goes here and when I come to correspond to itself this would be A_3 and A_2 weight. Let me re check calculate it if I have missed anything in this cells I will just recalculate.

So, 0.187 with respect to 0.065 this is better so A_3 goes in concordance set then A_2. So this would be A_1 and this would be disliked so this would be put in the concordance set and this is same it will come to liking set A_3. So, this becomes null if I consider A_3 last one these are equal so obviously equality means indifferent 0.1 with 0.1. So, it becomes A_1 A_2 is not liked so A_2 and becomes A_3.

Now, few important points so if you consider this set I will mark blue colour here and make it bold mark blue colour here bold mark blue colour here bold and what is important is these. If I am considering any of the alternatives very interestingly the set based on which I find on the discordance and then concordance set are never symmetric why not symmetric because if I go here I will try to draw without the electronic pen the line so consider this is the line the line where we are considering on the right hand side left hand side and I have values like this I will put as see for example, blue and the levels I am going. So, levels I am going are on to the right so if I consider on to the right some equidistance and then I consider that on the left again a same equidistance let me measure it and put it considering this arrows on to the right and left are of same equidistance but the net values are same different. So, this would be red in colour because on to the left and this should be blue in colour because on to green in colour let me consider this because on the right. So, the liking-disliking set based on which I am trying to find out the preferences if they are symmetric which means liking amount disliking amount are the same thing but

if I am putting a value that the liking and the disliking are different it means equal unequal penalties are being given based on the fact that how the discordance and the concordance indices based on the liking set C disliking the set D and I have got I also here which will become much more understandable when I consider the indifference concept also.

So, based on that if I find out the C_i and D_i for this example the slides are there please go through that. So, I do all the step wise calculations for C_i D_i go each alternative and for each criteria the calculations have been done. So, let me come back to the final set. So, once this comparison been done alternative with respect to each criteria then I formulate the concordance and discordant matrices based on the indices. Remember that the relative values of the elements in the concordance matrix now it is capital C by the way this is capital C is not criteria is capital C based on the values of C_{kl} if you remember and k and l   are   the   alternatives.

They are based on the calculation of the concordance index. Now I want to find out that what is the concordance and discordance index values are the indices values based on which I can formulate the concordance matrix and discordant matrix. If you remember when we are discussing the hamming distance I said that if there is a match or more I put + 1 if there is no match is 0. So, this + 1 would be added up for all those concordance indexes where the value is more. That means if two criteria give positive value for alternative A_1 with respect to A_2 then obviously its benefit twice as benefit or I would not use the word twice because the level of benefits are not linear there is benefit and if the value of the concordance is not there discordance is there obviously it will be a disbenefit.

Now the quantum of concordance and quantum of discordance for the initial set of calculation where the length are dispersed are same we give same weightages if they are not we give different weightages. So what do we calculate? It is the sum of the weights associated with the criteria's contained in the concordance set. Now if you remember we have given the weights and the weights were given as 0.25, 25%, 50%, 25%. Now more the values of importance are the concordance are more would be addition of the weights.

So if all of this criteria comparison between A_1 and A_2 every point comes to A_1 which means the total level of importance we want to give for that alternative for all the criteria would be 1. It cannot exceed 1 because I am adding up the weights. So it would mean that for all the concordance matrix I would basically keep adding on the weights if they are positive, do not add the weights if they are definitely not positive, they are negative. As the concordance matrix indicates that the relative importance of the alternatives are given obviously the values of C_{kl} concordance concept would be between 0 and 1 and weights up to 1. Now as I mentioned the concordance matrix which we have formed is asymmetric along the principal diagonal and asymmetric concept would mean that the level of

importance   and   disliking   liking   disliking   are   of   unequal   scales.

So once we have that we have the concordance discordant indices based on which we find out the concordance and discordance matrix. Now once we add up the values are like this. After we construct the concordance and discordant matrix the principal diagonal obviously would be 0 because there the level of comparison is only happening with themselves. So there you cannot add up the weights. So the principal diagonal is a dash and the of the diagonal element it is asymmetric because if you see is 0.25, 0.75 here is 0.75, 0.25 here obviously it is same. Now what does that mean? It means two things. If I am comparing I will put this value comparison of these two. So when I am comparing it means based on the comparison of the alternatives and of the collective criteria has been already taken into case   then   I   would   be   willing   to   consider   by   the   way   these   are   alternatives.

So this one is A_1, A_2, A_3. This is A_1, A_2, A_3. So when I am considering A_1 with respect to A_3 the weight is 0.75 when I am considering sorry A_2 and when I am considering A_2 with respect to A_1 the weight is 0.25. Let me check. No, no it will be A_1 with respect to A_2 is 0.25 and A_2 with respect to A_1 is 0.75. So when I consider A_1 with respect to A_2 I have taken alternative A_1, I have not taken A 2. So my overall weightages based on the 100 or 1 unit of weightages which I should forgive for the criteria I am only getting a benefit say for   example   of   25%.

Rest 75% is lost. But if I come to the concept of trying to decide on A_2 with respect to A_1 for all the criteria is considered together I am getting the weights benefits for criteria as a 75%, 25 is lost which means a comparison of A_1 and A_2 it means that level of importance once would place on A_2 with respect to A_1 would be 0.75 divided by 0.25 which will be 3:1.   That means I would be willing to pay more attention to A_2 with respect to A_1 considering the overall set of criteria which I have. Now if I come to A_3 and A_1, A_1 both are 50-50 . That means either taking A_1 or taking A_3 I am giving equal weightages that means both ways I am equally balanced and finally if I consider the values of A_2 with respect to A_3. So A_2 gives me a weightages of 75 with respect to A_3 I get if I take A_3 it is 25. Now interestingly if I will come to the problems in the next class but if I consider this overall weightages in a very simple format A_2 with this A_1, A_2 is better. A_2 with A_3, A_2 is better. A_1 and A_3 both are equally balanced. So in overall A_2 would be better than A_1, A_2 would be better than A_3 but when you are trying to compare A_1 and A_3 we are equally balanced.

So definitely we can say in a very simple way that the idea of given weightages or more importance to A_1 alternative would give me better value with respect to A_1 as well as with respect to A_3. So with this I will end this class and continue the discussions about the electro method in more details later on. Thank you very much. .