

Multi-Criteria Decision Making and Applications
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Lecture 45

Welcome back my dear friends, participants and students for this multi criteria decision making course which is under NPTEL MOOC series and as you know this is spread over 12 weeks with 60 lectures and each week we have 5 lectures each lecture being for half an hour. And my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. So, if you remember for the last few classes about 2 weeks we are discussing about the multi criteria decision making where the decisions alternatives, criteria's are more subjective not objective as like in MODM. And we were discussing concepts of TOPSIS method and we are in the process of discussing the electro method. And in the electro method obviously, you will see in the slides we are discussing about the different about distance measures like the Manhattan distance, the L^∞ , then L_2 which is the general distance measure between two points the coordinate system. We did mention about Mahalanobis distance, Hamming distance, but all our focus would be in different LP norm or the LP distances considering the Manhattan at one end and the N^∞ distance at the other end.

And this is the 45th lecture which is the last lecture in the 9th week. Broader Amorella as I keep repeating is basically MCDM we have already covered MODM few of the techniques we are into multi attribute decision making and the different techniques like as I mentioned few minutes back TOPSIS, Electre, AHP, VIKOR and all these things. So, the coverage is where we started and left in the last class was Electre method which is elimination and choice translation reality. Now, if you remember we had drawn and considered the two different points x and y .

If you follow the last lecture you understand X and Y and they are then three dimension like x is a vector which has x_1, x_2, x_3 , Y is a vector which is has y_1, y_2, y_3 are the coordinates. And based on that we found out what was the distance based on Manhattan norm then L_2, L_3 till L^∞ and we plotted that if you remember that. And I showed you that for Manhattan norm it was the max and L infinity it was the minimum. Now further analysis of that I have drawn this graph where the same ideas has been converted into the concept of different coloring and if you notice there is a red color, blue color, green color and yellow color. Now if I consider first the green one, so let us consider two different points I will mark them as A here, B here and if I have the coordinates this is the two dimension one.

So, it is easy for us to visualize. So, if the coordinate system for A are x_1, y_1 for B are $x_2,$

y_2 obviously I can find out the distance between A and B which is based on square root of $(x_2 - x_1)^2 + (y_2 - y_1)^2$ and we take the square root. So, that would be given by the green line which I will put up mark in the this one I will just again redraw it. So, this is the I will hash it. So, there is no confusion this is the L_2 .

Now if you see the red, blue and yellow they are as follows. The green line which I just hashed is the unique shortest path while the red, blue and yellow are based on the L_1 norm and out of the same length, but you can take different paths. Why of the same length? So, if you consider the total distance covered same length means the L_1 distance of the same length. So, if I consider the yellow one, so it goes I am not comparing with the L_2 I am only comparing the L_1 . So, if I take the yellow one, so I go 1, 2, 3, 4, 5 units right then go one vertically up which is 5 + 1 then I go right + 1 and then again go up 1, 2.

So, it will be I will be going 1, 2, 3, 4. If I see the red one, so red one basically I would go 1, 2, 3, 4, 5, 6 and on to the right I go 1, 2, 3, 4, 5, 6. So, let me count if I have taken it correctly 1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6. So, 6 + 6 and in the yellow one that we count if I miss something 1, 2, 3, 4, 5 plus it goes 2 up sorry my mistake it goes 2 up then goes right 1 then goes up vertical again 1, 2, 3, 4. So, the total amount of movement if I follow the L_1 basic concept if I follow the yellow path it is 5 + 2, 7, 7 + 1, 8, 8 + 4 is 12.

So, yellow is difficult for you to see, but it is 12 number of units and if I consider the red one also it is 12 units movement. So, the L_1 norm different it can have different paths, but the distance covered is same. So, that is why the L_2 norm has unique solution while L_1 norm does not have any unique solution lengths are same, but you can take different paths. One can generalize this in the n dimension one we did it in a two dimension we can have in a three dimension like corner points or some points in a cube and I can find out the distance based on L_2 and the L_1 . Now, why did we discuss the distance it will be clear later on.

Now, the ideas if you remember that we were discussing the steps of the electro method and in the concepts of what we did in the TOPSIS method. So, this these excel sheets which is named as MCDM and there are different sheets inside the excel sheet. The first one I have already sent the for the 8th and the 9th class I will send along with the slides the excel sheets would also be shared with the participants and the students. Now, why did I go back to the excel sheet if you remember we were discussing about the distance based on PIS and NIS and I categorically mentioned there the PIS and the NIS distance were based on L_2 . So, the idea was that was can we convert that into any other distance concept yes it can we will see that in the electro method.

So, electro method sheet is still empty I will fill it up with the discussions as we proceed. Now same step set of steps. So, the steps which we had followed in the TOPSIS method was first have the X matrix, X matrix is based on the number of alternatives and alternative of criteria the alternatives were marked along the rows or criteria along the columns. Then they were normalized, normalization is based on the different type of Euclid functions or normalization concept we used. And I did show in the excel sheet and it is also there in the video which we are which I was delivering and that the normalization was based on simple normalization like X_{ij} .

If I consider each value in the matrix X capital X which is matrix as $\frac{x_{ij}}{\sum x_{ij}}$ is depending on whether you are trying to normalize along the rows of the columns or you can take the logarithmic value or you can take the square values. So, in the square values it will be $X_{ij} / \sum (X_{ij})^2$ along the rows of the columns. In the logarithmic scale or when you are normalizing it will be $\text{Ln}[X_{ij}] / \sum \text{Ln}(X_{ij})^2$ along the rows of the columns. We will follow the same concept here also. So, here we convert the entries in the decision matrix and normalize them.

Now in the slides and in the excel sheet or in the discussion generally the concept of trying to convert X into the normalized vector multiply it is the weights, weights are based on the criteria only then proceed accordingly. The concepts and the methodology remain the same may be the nomenclature of what you want to mention as X or R normalized vector that symbol would be different, but they should not be any confusion. So, you convert the entries in the decision matrix X or A into a scale normalized value. So, this would be square. So, I have A_{ij} X_{ij} would be $(A_{ij}) / \sum (A_{ij})^2$

So, this should not be a square root it should be just be the $\sum (A_{kj})^2$ and when I mention about the squares. So, based on that I find out the normalized values along the rows of the columns. Now what are these? These are on a scale factor of 1 or 100 if I multiply the scale factor by 100 it becomes on a 100th scale. I am trying to find out the level of importance because the utility function based on which the person is analyzing the points of the values for each and every criteria for each and every alternatives need to be scaled and found at the same level such that comparison becomes easier. So, once we get the normalized vector we should remember again the point which I am mentioning repeating please bear with me.

Here m would be the number of alternatives n will be the number of criteria. So, here if I read the second last bullet point here m is the number of alternatives shown along the rows and n is the number of criteria shown along the columns. The normalization which you do implicitly depends on the utility functions or the word which the decision maker thinks what is the value which accrues to each and every alternatives based on the criteria what he or she is analyzing. So, say for example, the car example. So, the car are

there are m number of alternatives in front of you the criteria can be I am just repeating can be the mileage, can be the cost, can be the maintenance cost, can be the safety features, can be the color, can be the luggage space, can be the number of passengers who can be carried, it can be the resale value.

So, based on that the points are given. If I consider the example of college choosing a college it can be what is the tuition fees, how far it is from home, it can be what is the quality of the education, what are the job facilities, internship, what are the final job facilities, how are the teachers, what is the environment and all these things can be considered. Similarly, we can consider different ideas when somebody is buying an apartment I keep my discussion limited to these three because they can be easily communicated with the participants and the students. Now here let us consider so the normalization has to change because it should be not the square root a square here. So, let me do the problem accordingly.

So, I will consider of M as 3, N as 3. So, let me mark it as A_1, A_2, A_3 this is C_1, C_2, C_3 I should zoom it more so it is visible to all and C_3 and this is C for example, the matrix X and I would have also have the matrix of weights. So, they would be correspond I will fill it up just C_3 and there were also C_1, C_2, C_3 and the values are given based on 2, 1, 2 then I have 3, 3, 4, 1 I am taking the same values 1, 3, 2. Now, I write the normalized vector as say for example, R. So, here also I have A_1, A_2, A_3 .

So, the corresponding criteria is come. So, now I normalize so let me use first. So, if I go back the normalizations are based on the columns. So, if I consider first example I am normalizing based on just $\frac{x_{ij}}{\sum x_{ij}}$. So, it will be sum of along the columns.

So, I need to fix the sum value because sum value along the column does not change. So, I find it out and it is the sum I want to verify. So, it is equal to 1. So, I highlight it with the color. So, it is easy for us to follow.

So, now I take as D so D columns again normalized sum is equal to 1, I go to the third one means C_3 . So, it will be sum of E. So, I sum them up and the value comes up to 1 perfect. So, this is I should mark this with some orange color here. So, there it is because visually much more noticeable and this sum is yellow and this was the weights.

So, I should shifted one box. So, it is easier. So, this is the weights I also mark it with light orange. So, we need to find out the weights. So, based on this I find out the weights. So, same concept once the weights are found out I need to multiply the normalized vector along the weights.

Weights remember are based on the criteria only what is the level of importance of the criteria which are in some way the constraints because alternatives are the objectives which you want to choose, car you want to choose, buy the house, check with the college which college you want to get admission. And once the weights are given based on the decision maker we will multiply the normalized matrix with the weights matrix. Only thing to be remembered which I did mention twice be aware that what is the dimension of W what is the dimension of X. So, based on that you multiply. So, step two would be weighting the normalized decision matrix which will do.

So, consider the weights are $1/4, 2/4, 1/4$. So, it is one-fourth, two-fourth, two-fourth is fifty percent and one-fourth. So, double check the values of $0.25 + 0.5 + 0.25$ should be one. So, the other values are all 0. So, I have done this. So, let me add few rows. So, what I did was I will insert.

So, I found out from X to R. So, I will put arrow here this is the calculation which we are doing. So, I will put the arrows in black color. So, this is the calculation. Then I need to multiply the weights and the normalized vector R.

So, I will put the arrows accordingly. So, but first let me put the matrix it will be $M M U$ $L T$ of the first array into the second array I have all the values. So, these are the corresponding values. So, once I do that, so these I will basically mention it as Y and then mark them with red values and bold them. So, based on that we will do the calculation. So, what we do is multiply the weight with the normalized vector this we do and we get Y vector.

So, I put the arrows accordingly in the same way if you remember in the top axis also I put the arrows. So, that was to avoid confusion. So, once you have Y, so I would not put any color here. So, this is Y and these are the cell values for the weights I will put as blue in the same way as I did for the top axis. So, remember the weights is one if you think or perceive that for criteria 2 you will assign two times more weightages than criteria 1.

Now this is a point in qualitative context I am mentioning it will become clear when we consider more of these other different techniques. So, twice more importance, thrice more importance, 10 % less importance based on the criteria. So, here it is mentioned criteria 2 is twice as important while for criteria 3 the level of importance of the same as criteria 1. So, if criteria 1 criteria 3 are same obviously criteria 2 would be twice with respect to 1 twice with respect to 3. So, based on that the weights are there I immediately put on the weights, but the qualitative statement would be like this what is the percentage,

what is the importance, what is the level of importance, how many times.

So, once the weights are given I multiplied and find out Y, so the values are given. So, the values are what we have calculated in the cell. So, this set of point I am only reading till three places of decimal 0.083, 0.062, 0.100, 0.125, 0.250, 0.050, 0.041, 0.0187, 0.100. So, these are Y which in this even the different because the Y calculations are different they are based on this. Now, we will take a pause if you remember in the TOPSIS method we considered the PIS and the NIS the distance from the most positive and most negative. Here the idea was if you remember the concordance set and discordance set, liking set, disliking set.

Now, here also I mentioned one point in details that liking set and disliking set does not mean that we are only taking one alternative. So, considering there are two alternatives A_1 and A_2 for A_1 I find out the liking set, disliking set for A_2 also if I find out the liking set, disliking set. So, what does it mean to repeat consider I have taken alternative A_1 . So, obviously there are some positive points, but obviously it can be possible that they can be negative points on two accounts negative points based on the account that there are some disbenefits accruing for alternative A_1 . Consider buy the car very good car, but the maintenance cost is very high.

So, that is the disbenefit or consider the college, college is very good, but cost of studying is very high. So, that is the disbenefit. Similarly if I take the other alternative they can be disbenefit also. Benefit say for example, for buying a very less costly car that is the benefit I am paying less, but the disbenefit may be longevity of the life or later on the maintenance cost exponentially increases because for wear and tear or the road conditions are not good the car has to maintain much more often. So, we will consider this levels of benefit and disbenefit.

So, we need to determine the concordance and the discordant indices sets based on the indices set will formulate the concordance matrix and the discordance matrix. Now what is the concept of concordance and discordance what I mentioned I will repeat it. Concordance index is defined as the amount of evidence to support the conclusion that alternative A_i outranks dominates or is better than A_j . Now that is the index so obviously I have to compare that based on each and every criteria. Similarly what does the discordant index means is the counter part of that and it in very simple mathematical terms one can say there is the discordant index is the complement of the concordance index based on the disbenefit which I am getting by choosing A_i over A_j .

Now the distance concept and here the distance concept I will slightly go into a different analysis of trying to analyze the distance. Now when we are discussing the liking

disliking consider this example and I will give you a mathematical background of that also. So, if I consider first the mathematical background I will draw it here it has nothing to do with the problem solving is basically to do with the analysis. So, consider I have axis x and y and I am trying to find out or estimate in simple statistics. So, if I estimate I need to find out the variance if you remember the variance is square quadratic in form as per the formula.

So, if I put the concept of quadratic equation for trying out to find out the variance, the variance function looks quadratic like this. Now why I drew it if I plot the value of δ what is δ ? δ is the difference between the actual value and the estimated value parameter and its estimate. So, this point which is the origin where both the estimate and the parameter exactly match. So, $\delta = 0$ on to the right it is positive on to the left is a negative.

So, if I consider a parameter value of say for example, 4. So, in one case we estimate 4 by a value of +2 in another case we estimate 4 by a value of say for example, -2. So, in this case or consider for I will give you example +2 - 2. Let me take the value of I will take a much better example wait. Consider the value as theta which I want to find out and theta hat I am using a symbol as the estimate. So, difference between theta hat minus theta is delta which I am measuring.

I will come to the values of the parameter and the estimate later on. First let me come to the concept of delta only. Consider in one case $\hat{\theta} - \theta = +2$ and another case the difference between $\hat{\theta} - \theta = -2$. So, $(+2)^2 = (4 - 2)^2 = 4$.

So, if I basically try to find out the quadratic value. So, in both the cases for negative and positive was of same equal distance on to the right and the left. But the answer is that when I come to the analysis here is it true answer is no. Why not let me give you two examples before I proceed. Consider an example where and in concept of a real consider an analysis I win or I find out 100 rupees and in another case I lose 100 rupees. This is a very simple and rudimentary example, but try to understand that.

So, my net worth or the value of the satisfaction of winning 100 rupees with respect to the value of minus 100 rupees being a loss my level of dissatisfaction that I have lost 100 rupees would be much more than winning 100 rupees. So, my loss I am quantifying much out of much higher level even though 100 rupees losing and gaining worthwhile is the same in units. Consider another example and here I am trying to basically say for example, if I consider from a civil engineering perspective I want to build a dam. The dam's height is basically actually should be 120 feet, but consider there are two instances in one case the dam's height is 122 and it has been built and another case it has been built

to 118. So, what is the difference between 120 and 122 is in one case if I consider it is + 2 another case 120 118 is - 2.

If I square them up considering the quadratic of the concept of variance both case $(+2)^2 = (4 - 2)^2 = 4$. So, is equally penalized, but the question is that in the practical sense is it right the answer is no wrong it is not right why? In the case if I build it to 122 feet there would be some extra cost manpower extra materials extra time, but why 122 has been decided because on an average the flood comes and the level of the water max can reach 120. So, in the case if it is built on 122 then the probability of the water breaching the dam is much much less. Now, the answer is much much less with respect to what? Consider if it is built to 118 and the flood comes and the average height of the water is 120 in that case the probability of it breaching the dam is much higher. So, initially when you are built it to 118 the amount of manpower amount of material amount of time taken was much much less, but the catastrophe it loss it will be happen due to the environment impact of the flood overrunning and breaching the dam would be much more.

So, in that case here for this example over estimation of 120 by 122 with respect to 120 and 118 over estimation would not be penalized under estimation will be penalized. And they can be other examples of over estimation being penalized under estimation not being penalized and so on and so forth. So, with this example and with this thing in the mind we will discuss the problems accordingly. Thank you very much and have a nice day. .