Multi-Criteria Decision Making and Applications
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Week 09
Lecture 44

Welcome back my dear friends and students. This is the course on multi criteria decision making and which is a course for 12 weeks spread over 60 lectures and each week we have 5 lectures each lecture being for half an hour. And my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. So, if you remember in the 43rd lecture we were discussing about the TOPSIS method and time and again I mentioned about the concept of PIS, NIS, how you can find out the distance, how you can divide the criteria into two sets and based on that you have can have the negative distance, positive distance and based on the negative distance and positive distance how you can rank them. The discussion also came up that and it was very obvious that when you are trying to rank them you need to find out that how far it is from the positive, how far it is negative, the distance measured from the negative one is 0 that means it is not at all preferred and the distance measured from the positive is 0 it is definitely preferred. So, based on that the positive one, negative one we found out the ranking for A_1, A_2, A_3, A_4 which was the alternatives and why it was done because it was a double check that both from the side of getting the benefits and trying to reduce the disbenefits I am using the word reducing the disbenefits.

So, benefits should increase, disbenefits should decrease. So, both from this point of views the alternatives were perfectly balanced, balanced in the sense the ranking there was no dichotomy, there was no confusion. But we can have a situation or situations where these dichotomies can arise where when you are taking the decision or the benefits and the disbenefits the decisions can vary. We will see that.

Now having discussed TOPSIS we will consider the next method which would be our discussion for this 44th lecture. As usual the main umbrella under which we are discussing is multi-criteria decision making. We have already gone through multi-objective concepts using different linear programming, non-linear programming, integer programming considered both the concepts how you try to find out the optimized values for one two functions and more than two functions and three functions as usual. And in one case we did discuss about how they would look in the three dimension one and two cases we discussed how they would look in the two dimension case. It was easy for me to explain.

Now, we have started for the last few lectures about the multi attribute, the non-parametric methods having finished TOPSIS we will consider the electro method. The electro method even though it was mentioned in the last class the discussion went on so that means I could not start it in the 43rd lecture. So, the coverage would be starting on elimination and choice translation concept which is known as the electro. Now the concept of elimination and choice translation reality is basically a French word and it was basically found out by Roy in 1990 and it is one of the MCDM tools based on the

concept of out ranking. Now if you do remember in the TOPSIS method is basically how they are close to the ideal best and the worst.

The concept of the out ranking would be also be used in this manner as we have considered in top-sis method, but here we will be considering the concept of ideal solutions and the non-ideal solutions in a different way, but the ideas remain the same. Now in the course of time generally it is different variants of Electre method came which were known as Electre I, II, III, IV, IS and TRI, but we will only stick to the simple method in trying to understand the Electre method. Now the Electre method which we will discuss here would be a little bit lengthy. Lend in the sense that the ideas of some loss functions would also be covered in electro method. Now the question may immediately turn up from your side or from the student side, could not we have done the concept of the loss functions for the top-sis method or cannot we use the concept of the loss function in the other method.

The answer is yes we can do it, but rather than trying to repeat it for each method I will only concentrate in the electro method and it can be replicated in the other concepts of nonparametric methods or out ranking methods accordingly. So there I am different text and the text have been given in the slide in the first time when we are discussing and as and these are very recent concepts which are used in big projects trying to analyze where nonparametric methods are there, where decisions are not very objective they are subjective. So for our simple understanding as I said few minutes back we will consider the electro one method and a variant of the same. The first question which will come up when you are doing the electro method is what do you consider as the concept of out ranking. So the out ranking would basically consider and I have already mentioned in a different way when we are doing the TOPSIS method.

Our ranking would be the concept that on the positive front what are the good points or what are the good qualities the alternatives has and on the negative points what are the dis-benefit the alternatives has. Like for buying the house price more is definitely not required we want to lower the price. Maintainment cost for the house definitely required not required we want to lower it. But if the number of rooms are high and the same price is good, safety is high good, locality is good it is good, it is nearer to schools, nearer to shopping centers, nearer to hospitals, nearer to the transportation all are good. So we will basically consider out ranking in this concept try to divide them into the positive and the negative benefits.

So stated simply the method of out ranking is the idea in which one compares one decision or alternatives against the other based on some set principles. Now the immediate question which would come up is that what are the set principles do we know it. Yes we have already considered that that if I am trying to analyze buying the house there were eleven different concepts based on which the decision was taken. So those would be the criteria based on which the out ranking would be analyzed for each alternative. Now remember one point was mentioned from my side time and again at least twice in the discussion when we are doing for all these nonparametric methods the ideas of trying to compare the alternatives based on the criteria can be drawn only on a

one to one basis.

That means I cannot collectively consider all the criteria and do the ranking for two alternatives. So I will consider one at a time and then combine them based on the weights. Now coming back to the concept of the weights would the ideas of the weights will also be replicated? Yes they will be replicated. Would the concept of utility function would also be replicated? Yes they will be replicated and we will see that. Only the last stage of how you do the out ranking is to be done.

Now continue reading this in doing so one assures that the end of this pair wise ranking for the alternatives we have a unique ranking system amongst all the decisions or alternatives based on which we can take the decision in a much more objective manner. Obviously subjectivity is there but our analysis will try to make it as objective as possible. Now here also the background story is the same. We will have basically A as the vector which consist of A_1 to A_M number of alternatives, C vector is basically C_1 to C_N as the number of criteria and we will try to analyze them accordingly. When we try to basically accomplish we will basically do the same concept of the ranking ideas which have been doing is as follows.

So if there are two alternatives consider this as A_i should use the black one A_1 and A_2 any arbitrary in the set A. So you can basically have A_1 is better I am using colors red does not mean bad or worse I am just using different colors to maintain them A_1 is better which is I get the benefit and then the disbenefit side is lower for A_1. If I consider equality even though I use equality sign just generally it will be utilized like this. So that means both balancing the benefit and the disbenefit, benefit positive disbenefit should be on the negative side they balance each other. And the third one is basically $A_2 > A_1$.

Now this red one and the green one may try to give you the same ideas. So what do we mean by the red and the blue red and the green which is greater than sign and less than sign. So it means that consider A_1 and A_2 is there in case one I take A_1 and in case two I take A_2. Now when I am taking the decision A_1 both the benefits of A_1 are coming as well as the disbenefits or A_1 are also coming. When I take A_2 in case two I am getting the benefits of A_2 as well as getting the disbenefits A_2 also.

To expand this concept so when I take case one is remember this I am getting the benefits of A_1, I am getting the disbenefits of A_1, I am not getting the benefits of A_2 and I am also not getting the disbenefits of A_2 that means if I have taken A_1. In the case when I am taking A_2 and not A_1 what are the ideas, ideas are I am getting the benefit of A_2, getting the disbenefit of A_2, not getting the benefit of A_1 and also not getting the disbenefit of A_1. So if I consider this four combinations for case one and case two there are basically two positive and two negative. What are the positives in case one, case one positive is benefit of A_1, what is the other benefit or positive one is basically not getting the disbenefits of A_2 and what are the negative, negative are getting the disbenefit of A_1 and not getting the benefit of A_2. So if I go to case two the answer would be exactly the same but trying to basically analyze the problem from A_2 side.

So remember that what I said when you are taking A_1 or A_2 you are getting the disbenefit and benefit in a set, so the two sets would be benefits and two sets would be non-benefits or the disbenefits. So obviously when you try to analyze that the ranking concepts would basically be a collective of all these things. So this idea which I just mentioned benefit, disbenefit was also there in some way when we consider the PIS distance and the NIS distance accordingly. So one advantage of the electro method is the fact that the final result when presented to the decision maker, so the decision maker makes the decision presents in front of the whole set of people who will have an analysis of this tool and take the decision. It is easier for him or her to take the final viewpoint rather than what he or she would have taken with only A_1 or only A_2.

So A_1, A_2 both being given is that case where disbenefit and benefit all would be analyzed. Because in that case it may be only if you consider A_1 then you are only going to judge all the alternatives based alternatives being m in number based on either the benefit or the disbenefit. But if we give collectively both the sets of disbenefit and benefit would be considered when you basically do the out ranking for all the alternatives. Now expanding on that in case if the ith alternative by the way when I am considering the alternatives these are what the decision you want to take like buying the house, buying the type of car or taking admission in one college and what would be the criteria would be I am sorry I am repeating this examples time and again please bear with me. The criteria would be for buying the house would be the price, mental immunity, maintenance cost, locality, safety, nearer to schools, colleges, hospitals, shopping center, family members being there.

So all this can be the criteria. For the car price, maintenance, what is the mileage, what is the safety features, boot space and so on and so forth. For the college it can be what is the rigor of the course, what is the type of internship the student get, what is the final placement, what is cost of that, what is the ranking of the college, what are different type of programs which are there. So these would be the criteria. So in case of the ith alternative let me continue reading it.

A_i outranks jth alternative by the way i and j I am considering so obviously actually j is basically for the criteria but for our discussion we are considering I and j are the index based on which I measure the alternatives. ith alternative A_i outranks jth alternative A_j then we denote this relationship A_i outranks A_j. So if it is the other way round so A_i and A_j would get flipped which means that and this is important that the risk of the loss whatever one would like to say or the benefit of the disbenefit. So risk and loss I basically mean the disbenefit for A_i is not as much as A_j or A_i is as good as A_j and not as worse than A_j. So I am trying to look at the picture as I said few minutes back both from the benefit and the disbenefit side.

So ideally would be what I get all the benefits and I also then do not get any disbenefit that means disbenefits set of the overall point is 0 which cannot be. Now how one decides the so called relative ranking between the alternatives A_i and A_j is a matter and which is of prime importance for us. It is also noteworthy to mention that the collective cumulative effects of the N number of criteria's or the M number of criteria's would

come into play. Electro ranking is non-transitive it means that if here it is important if $A_i > A_j$ and A_j is better than A_k it would not mean A_i is better than A_k. But if you consider this transitivity property from the real line is very simple to understand that would be true in the sense if I have the real line and if the points are I will mark in green this is 1 this is I am taking real line means going from my side to the right is positive higher and this is 3.

So if I say 3 > 2 and 2 > 1 so now obviously 3 > 1. But when I am say that say for example analyzing quality attribute then this dichotomy may occur with transitivity property may not hold which means that if I have or if I find out the ranking and I put this ranking as see for example ranking means this 1, 2, 3 are not to be confused with A_1, A_2, A_3 which I am not writing. So see for example I write A_1, A_2, A_3, not on the real line just like this which may be $A_1 > A_2$, $A_2 > A_3$ but it may be possible that rather than $A_1 > A_3$ it can be $A_3 > A_1$ and here is what the discussion is all about out ranking. In electre method if you remember in the top system method what we did PIS and NIS in electro method we will use the concept of concordance and discordance. PIS was positive distance good NIS was negative distance bad here we are trying to basically use a different concept of concordance agreement discordance disagreement.

So in electro 1 uses the concept of discordance and unconcordance and discordance which is a sort of level of liking disliking which I just mentioned respectively which one has when comparing two criteria's based on the decision makers utility of the net worth which he or she is going to place. So if we say that if we like Ai with respect to Aj when one chooses Ai in place of Aj then one assigns some scores or points or quality points which are positive and quantifies this level of liking in the concordance level in the positive level. On the same basis if we due to some reason one is forced to choose Ai with respect to Aj then the level of disliking is objectively summarized in the discordance set. Now the question we immediately come up that when I am taking A_1 with respect to A_2 is not it possible that I have some of the criteria's which are positive and some of the criteria's which are negative yes it is possible like to give an example I am talking verbally I will come to that when you consider the house price being high maintenance cost being very high which is basically discordance it will technically go into the discordance set which was in the NIS concept. Now when I am considering the positive like room number of rooms is more safety features is there family members nearby bus stop nearby schools nearby hospitals nearby so they would be in the concordance set.

To deal with this out ranking relationship by using pair wise comparison under each of this criteria we do it separately. So you take criteria one do the ranking for A_1, A_2 do take the second criteria do the ranking with A_1, A_2 and do it accordingly then you take criteria one do it for A_1, A_3 then take criteria two do it for A_1, A_3 and continue in this way. The out ranking between two alternatives A_i and A_j which we have already seen is denoted by A_i is better than A_j which generally means that even if the ith alternative is not dominating the jth one yet the decision maker can choose ith one because the bundle of positive things concordance is more than the discordance concept. Alternatives are dominated if there is other alternatives which excel them in one or more of the criteria's

and equals in the remaining one. So they can be different alternatives and if there are ten alternatives nine of them are equal benefits collectively but one of them for the first alternative for criteria one is much much better than the same first criteria for the second alternative.

So obviously the ranking would definitely be more tilted towards A_1. One important thing when I am considering the out ranking concept and when we write the function in the some matching index like this and here it is important to note now even though it is written as gi Aj actually the ideas would be done accordingly like this. So I need to find out of specific threshold based on that I need to find out the concordance and discordance. And the concordance and discordance concept should be remembered is based on this. So let me draw the line which will be coming up.

So this is the real line. Now when I consider the concordance and discordance concept so consider my liking if I take A_1 with respect to A_2 is say for example +2 whatever the unit is +2 that means I like. But it does not mean that when I take A_2 with respect to A_1 the value is - 2. Because this concordance concept and discordance concept and the function based on which we are trying to basically analyze A_1 and A_2 is not a symmetric function. So if I am trying to analyze or trying to understand the overall concordance concept and the discordance concept or the liking disliking is a matrix then it is not a symmetric matrix. Because in the sense that if I take A_1 and not A_2 or take A_2 not A_1 the points obviously they can be different and even if they are same it does not mean they are basically positive negative.

We will consider that in the concept of the real line accordingly. One should specify the threshold which depends on the decision maker choice concept of utility function is important. The decision maker assigns weights or importance same way we have assigned weights. Important factors to the criteria and this is done in order to express one preference to show the relative importance. This also comes from the fact that the utility function of the net value concept of any decision maker process is an important driving force for the choices and how the rankings are made.

The method is convenient when there are few criteria and a large number of alternatives because you have to consider this electro method and as I said I will consider two different concepts of electro method. The line which I have drawn about the ranking of better than worse than will be coming up later on, but first let me just specify very simply here. So what we are trying to find out is based on the fact that what is the distance measure on to the right hand side and the left hand side of taking the decision and not taking the decision. Now if you see this line which I have drawn is equidistance. So if I basically mark point see for example, x here x has no meaning I am just putting x x_1 here and if I put y_1 here and y_2 here the distance measure from x_1 to y_1 on to the right hand side from my side from your side it will be left.

So it is on the right hand side the distance between x_1 and y_1 and the distance between x_1 and y_2 are equidistance which means they are equally dispersed, but the answer would be in many of the cases it may not be true. Now few of the concepts of the distance

measures which we I did mention in the topis method the simple distance measure, but I will just elaborate them accordingly. The Euclidean distance and they are all based on the same formula of the LP norm or LP distance. The Euclidean distance between two vector points x where X is a vector from x_1 to x_n and y = y_1 to y_n is given by this formula which we all know. That means finding out the difference between x_1 and y_1 squaring up then adding it the difference between x_2 and y_2 then squaring up and doing it for all the n number of points which are there and adding up and then finding out the square root.

So in the case when it is the l1 norm I need to find out the mod and when it is basically sum of the mod whenever I want to find out the l infinity with the max. The l_1 norm as we know is the Manhattan distance between vector points and it is given by the mod difference and the sum. Why Manhattan norm? It looks like if you look at New York from top the roads are all running parallel and perpendicular to each other that is why we say it is a mod based on the Manhattan distance. There are other distance also consider between two random vectors given x and y are in the n dimensional and s is the covariance matrix we also can find out the Mahalanobis distance which we would not be mentioning, but I just thought I will mention different type of measures which are distance measures. Another distance measure which is used is the Hamming distance it is used in case of many biological studies is the distance between vectors of points and where the vectors are from x_1 to x_n for X vector and y_1 to y_n for y vector is the number of positions at which the corresponding values are different.

So, you count down the numbers and then basically find out the Hamming distance. The $L\infty$ norm is given two vectors X and Y in n dimension it is given by the max and the general concept of the L_p norm is given by this concept which is mod of the sum of the mod of the difference between x_i's and y_i's to the power p and then finding out after the sum you find out the 1^p. So, now, if you put p as 2 is basically the L2 norm which you have already obtained. So, I have considered few examples of the distance and I will tell you why they would be much more relevant as we come up. So, I will take few minutes to discuss the distance concept if there are two points x and y and in the three dimensional case and they are the points are x_1, x_2, x_3 are 2 - 5 20 and y_1, y_2, y_3 are 12, 15, 0 then based on the L_1 norm where it is basically sum of the mod it comes out to 54 this is important why I am mentioning the values is important.

If I find out the L_2 norm put in them the formula the value comes out to be 31.55 and if I find out the L infinity norm the value comes out to be 20 very interestingly the values are decreasing from L_1 to $L\infty$. So, can we have some visual flavor of that. So, let us do it. So, I have just plotted the values of p, p is 1, 2, 3, 4 till infinity.

So, and depending on when p is 2 you have the L_2. So, when I measure them along the x axis is 99 it will go till infinity and when I measure the distance for this three points for x and y which are in the three dimensional state the norms of the distance very interestingly looks like this. So, obviously there is a difference in values as p increases, but just for information I have drawn the values. So, the last value which you have for infinity was basically 20 the value which you will have basically for L_1 is was 54 yes it was 54 and it was 20 and the last and the first value second value which was for L_2 was

31.5. So, it is somewhere here. So, I have not marked the points, but this is the general characteristics with this I will end this lecture and discuss the problem solution for the electro and also the other method of the electro method accordingly. Thank you very much and have a nice day. Thank you.