

Multi-Criteria Decision Making and Applications
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Lecture 43

Welcome back my dear friends and participants for this course on multi criteria decision making a very good morning, good afternoon and good evening to all of you. And as you know this multi criteria decision making is under the MOOC NPTEL MOOC course which is spread over 12 weeks which is 60 lectures and we are in the 9th week and as you know each week we have 5 lectures each being for half an hour. Now for the last one week or a little bit more we have been discussing about the multi attribute concepts, the nonparametric methods and we had started about the TOPSIS method even though I have been repeating many things time and again but once you understand the TOPSIS method all other methods which we will discuss one by one like the Elektra, AHP, the VIKOR method will all flow in a very logical manner. And my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. So this is the 43rd lecture what would be the coverage under the broader umbrella of a multi criteria decision making we have already discussed multi objective decision making we are now into multi attribute concepts and we were discussing about the TOPSIS method. By the way as in the last class or last to last class we have been discussing the methodology in the PPT and also giving the solutions in the excel sheet.

By the way when I was discussing about the TOPSIS method the concepts about PIS which is the most the best positive answer and based on that we find out the distance from the positive decision and NIS the negative one. Here in the slide it will be done only say for example for the NIS because the last ratio which we will be finding out which I will come to that within few minutes would be based on the NIS. But considering the problem which we were discussing about the housing like the whole set of criteria. So there were four alternatives and eleven criteria's the criteria's can be divided into two different sets one is say for example which accrue positive benefits another which give you negative benefits and will basically analyze the problem accordingly.

Now in the coverage part you see the TOPSIS method is there and I also added the concept of elimination and choice translation reality which is the Elektra method why because once we finish in this forty third lecture the TOPSIS method we will immediately start with the Elektra method and as usual discuss that with in the slide as well as in the PPT slides and the excel sheets accordingly. Now if you remember we were discussing about the weights and weights are specific to the criteria. And let me go back to the excel sheet so the weights are given based on that we had found the multiplying of the matrices which was

normalized of X which we named as R and the weights which we named as W. Based on that we found V and once V is found out which is shown in red numerical's in this set of excel sheet we find the maxima and the minima. The maxima and minima based on the fact that we want to find out the best solution and the worst solutions accordingly which will give us the PIS and the NIS.

By the way when I am talking about the PIS and NIS remember for the time being we have not differentiated into the subclass of the best set of criteria and not so best set of criteria. So, we find out in which we closed in the last class we find out the max and the minimum. Now let us go back to the slide so the weights are there they add up to one double check that one should double check that the weights which are there because there are 11 criteria's. So these are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 11 criteria's we they add up to 1. So weight would be 11 × 11.

Now given this I find out V and V which would you have found out, but remember the V which we are showing and I was experimenting to show to you that different normalization will give you different matrices of V, but the general concept of trying to find out remains the same. So, as it was not possible in the slide considering the whole set of V which you have is 4×11 remember Y_4 there are 4 alternatives Y_{11} criteria based on that I find out all the values and for your kind information the values based on the concepts of the requirement which you want to find out which is basically the matrix V multiplied by the weights gives you a value which is specifically pointed out here is 0.021 which is the second row and eleventh column. So, that is why it is mentioned as $V_{2,11}$. Similarly if I want to find out the V_{41} which would be V_{41} that value is given by 0.0558. So, they have been calculated. Now once we calculate I try to find out the max value and the minimum value. Now the max value and the minimum values are correspondingly found out, but now the issue is there. So, the max value and the min value which we have found out I tried to show it along the corresponding column wise, but that was based on the criteria, but your actual idea is to base to find it based on the alternatives because that is the final situation. So, if I want to find out the max I am doing it in the nth and oth column in the excel sheet max for A, A means alternatives.

So, I will mark it as A_i because there are $i = 1, 2, 3, 4$ and then I want to find out $\min A_i$. So, I do it. So, I find out the max for the first then I correspondingly find out the max for the second, max for the third, max for the fourth. Similarly, so I will mark them as yellow then I want to find out the min. So, they are copied for the first, second, third, fourth again I mark it as yellow to make it highlighted.

So, the max and minimum values are given based on the alternatives. So, given the values of V plus you want to find it if you remember if you see here, the values are based on

$i = 1, \dots, m$. So, I try to find out the maximum and I will come to the minimum soon. So, based on the maxima the first value correspondingly comes out to so obviously you have to find it out. So, corresponding to that for each $i = 1, 2, 3, 4$ we find out the values.

Now, the reason I am doing it along the row and along the column is based on the fact that how your representation of the rows and columns are. If the rows and columns are with corresponding to the alternative criteria or criteria alternatives you have to basically interchange them. So, these are the case when the values are found based on the alternatives being along the rows and these are the minimum values and the maximum values. Now, given that I find out the minimum. So, the minimum values are again for

$i = 1, \dots, m$ same thing which we did and the values are calculated accordingly and that I am showing it in the excel sheet.

Now comes you have to find out what is the solution based on the fact that what is the ideal solution. Now, here is what my repetition would be repetition is that based on the fact that how you have been able to divide the criteria is into good or bad we would divide into positive and negative. Here we are not doing that I will do that within few minutes for the other methods also. So, once you find out the distance now here important thing if you find out the distance is given by L_2 norm we are just using the distance measure which we know. Now the distance measures could have been L_1 which is Manhattan norm which is the MOD or else at the other extreme which could be L infinity norm which is the max value.

So, we will visit that for different sets of problems later on. So, once you have the V plus values which are the positive values. So, you find out the most positive ideal solution based on which I have X_{si} means based on the V matrix which I have I will find out for corresponding to either the rows or the columns based on how the representation has been done. So, if I have the values I find out s_i plus and corresponding in the next slide I will find out the s_i minus. So, s_i^+ is $s_{i1}^+ + s_{i2}^+ + s_{i3}^+ + s_{i4}^+$ why there are 4 alternatives.

So, the question would be would we be able to find out correspondingly s_i^- is $s_{i1}^- + s_{i2}^- + s_{i3}^- + s_{i4}^-$ the answer will be yes based on the fact we will find out the most negative ideal solution point 1. Point number 2 the question would immediately come up that as I find out the s_i^+ is $s_{i1}^+ + s_{i2}^+ + s_{i3}^+ + s_{i4}^+$ for this case the distance concept which is found out is the simple distance norm. Do we still stick to that when we come to the negative calculation of the most negative ideal solution the answer is yes. Once you decide on the distance norm you will stick to that and follow it both for the positive and the negative you would not change in between. What you can do is that for each combinations of the length concepts like l_1, \dots, l_∞ say for example, you do it for l_1, l_2 and l_∞ you will find it the both the s_i^+ s and s_i^- s corresponding to these 3 l_1, l_2 and l_∞ and then do the comparison, but do

the comparison in the sense that the distance measure does not change.

Similarly coming back we find out the most negative solution, ideal solution and again see here the distance measure is based on the fact the l_2 norm simple case. The s_{i1} -,

s_{i2} -, s_{i3} -, s_{i4} - which are the most negative solutions are given here as calculated. So, I have the most positive and the most negative. Now what do I do with that? Now based on that we need to find out the relative proximity index for each alternative which is what is was the problem to buy an apartment, buy an apartment based on that criteria. So, based on that I find out the relative proximity based on first the instance that I am only find out the relative proximity based on the negative solution.

Now what would the negative solution mean? Negative solution would mean that for which distances I would not like to take those criteria's in the importance for analyzing the alternatives. Even if this there obviously I will try to rank them as low as possible. Now the question can be is it always true that we have to consider only the negative set of criteria's in order to rank them answer is no. You can also do it from the positive side or else you can do it both from the positive and the negative and rank them accordingly. So, that means you are taking when you are doing it from the positive and the negative side you are trying to take a viewpoint where both the positive characteristics and the negative characteristics are analyzed in order to give you the best solution.

If you are only considering the positive one you will only consider the positive distance. If you are only considering the negative one you will only consider the negative distance. But a rational method would be where you consider both of them and then try to analyze it accordingly. Now here the my earlier statement would be true in the sense if you remember I did say that we can divide the criteria's we can divide the criteria's accordingly and take the decision and break them up into two distinct sets of criteria's and do this analysis. For the time being let us consider that we are only concentrating on the negative Euler's solution and doing the proximity analysis.

So, the proximity analysis would be based on the fact that I take this ratio. The ratio is I will use this red color proximate ratio in the negative concept would be S_i^- divided by $S_i^- + S_i^+$ which means that if it is the real line where we are considering I have drawn it, but I will few times I will again show it. I am my concentration is only in the negative one which I am marking as red. So, any point s_i which is there I need to find out and obviously there is a s_i^+ on the right hand side which I will mark in blue. So, I will try to find out the ratio of the red line distance divided by the sum divided in the denominator of the sum of the negative and the positive.

Now if the negative distance is 0 obviously the negative proximity would basically tend

to 0 because in a numerator you have a very small value which can be taken as 0. If it is the red the black point or the black decision alternative is on the most negative one. Now if it is positive then obviously the other set of calculations will come up which I will show within few minutes. So, based on that I find out the negative distances. So, these basically would be if I write it with the coloring scheme to make things much more legible and understandable I will use si^- si^- + si^+ .

So, this would give me the negative concept. Now if I want to do so what is this distance is here, distance is here, distance is here. If I want to do the positive cells so I will erase it and come to that write the formula. I erase everything so that is I can start a fresh and do the calculation. So, those calculations are not shown here they were only for the negative proximity.

The actual diagram would be like this. This is the line, this is the negative point, this is the positive point PIS and NIS. This is the point based on which we are trying to find out. So, that would be because they are the alternatives they would be four such points and you want to find it out. We would have basically the negative distance which I am marking in red, you have the positive distance which I am marking in blue and the formula would be I will write it down it will be the ratio of si^+ here also si^+ si^- .

So, the minus goes here, the plus concept comes here in the denominator and also in the numerator. So obviously the values would be calculated accordingly. So, if I want to find out the values based on the proximity in the positive sense. So obviously if si^+ the distance is exactly on the and is 0 and the point is basically on the blue point which means it is the best ideal solution. So obviously in that case it is furthest away from the red because in that case black being transferred on blue the distance between the negative solution and that alternative will be maximum.

So in that case si^+ blue ones would be 0. So now obviously in that case the ratio of proximity in the positive sense would be 0. Here in the other case it was negative sense which was 0. So you have to consider that in the negative sense is 0 that is worst positive sense is 0 basically is the best and you have to basically analyze the problem accordingly. So this is basically the way or the we are looking at the problem and trying to analyze the result.

Now here comes three ways of trying to analyze. So this the concepts which I mentioned about the proximity was for plus and minus. So if I erase it and only highlight the one fact and then logically the other ideas can also be extended. So if I erase this so I have in the negative concept ranking as this relative part. So if I consider the relative ranking then obviously it would mean in the ratio wise C_1 which is for alternative 1 is about I will only

read till the first decimal 2 decimal is 0.362, 0.23 for C_2 , 0.30 for C_3 and 0.68 for the last one. So if I go back corresponding to this thus the ranking when I am trying to find out would be that if I have this ratios given as this is S_4 . So if I find out the ranking so this is the line and the points are given so I will mark them one by one. So 1, 2, 3, 4 so the ratio for this case was about 68 or 0.68 which means if I am looking from the negative point of view the distance between the negative one and the ratio is the highest which means that it is more on to the right hand side.

More 0 means on to the left hand side which is negative point I am only considering from the negative side. So technically this would be C_4 because it is 68 then if I consider it will be C_1 then if I consider it will be C_3 and last is C_2 . So if I find out it will be C_4 is better than C_1 , C_4 is better than C_1 that is better than C_3 and according to the ranking C_2 is the so called the least preferred. So if I write down $C_4 > C_1, > C_3, > C_2$. So if I see this the answers are C_4, C_1, C_3, C_2 .

So C_4, C_1, C_3, C_2 . Now the question can immediately come up fine we have done it based on the negative part what about the positive part can we do that. So let us do that in the excel sheet and then rank them. So I will take these values of s_i+ and s_i- and do them. So I will write $s_{1+}, s_{2+}, s_{3+}, s_{4+}$ and similarly I write down $s_{1-}, s_{2-}, s_{3-}, s_{4-}$. So and here I will find out the ratios based on positive and negative.

Negative positive means the NIS and the PIS concept we are trying to use. So if I write down S_i plus the values are given if you notice them they are given in the denominator the right hand side. So I will write them accordingly 0.1045. So then is 1543, 0.1543 next value is 0.0870 and the last value comes out to be 0.0425. So these are all the positive part done. If I consider the negative one so they are given as 0.0589 negative does not mean negative value they are based on the distance and the square and then square root.

Then the value is 0.0477, then I have 0.0387 and last one is 0.0928. So all done now I want to find out the negative and positive. So let me put the negative as red in color and positive as blue in color. I would not put the cells because then it would not be able to see it and these negative being in red color again bold.

So let us do it. So positive one would be positive part divided by ratio of positive plus negative. So I copy it, this is the positive one and when I do it for the negative part is basically I am doing the negative divided by the ratio of the positive and the negative. And obviously if somebody is interested double check the sum would be 1 because the ratios is what the denominators are same and one time you are taking s_i+ another case you are taking s_i- . Now see if you have noticed based on the negative how far it is we had rank them as this.

So I will rank them rank positive. So again mark it as blue bold and if I rank them as just one minute and if I rank them as negative. So this would be now bold red. Now if I see it here, I will remove it once. So if I remove see it here so negative part was basically more negative it is further away. So obviously this becomes in the negative sense further one so it will be first then we will come to 0.36 which is the second then we will go into the third and the fourth. In the negative sense you are doing in the opposite direction. Now if I go into the positive direction positive means the value if it is 0 it is exactly on PIS. So I have to basically take the minimum value.

So this is 1 because that value is 0.31 then the value comes out to be 0.63 then the value comes out to be 0.69 and the value comes to be 4. Now see the discussion here and this is more of a discussion which we will analyze here. Now if I consider all the four alternatives in the benefit side obviously the ranking is always on the benefit side which is based on positive.

First I would have A_4 I am putting them in this order then I would have basically A_1 then I would basically have A_3 and last would be A_2 in a benefit side. If I consider the rank from the concept of the non benefit that means it is good that I have not taken that or what is the least negative effect it will have disbenefit. So, the fine of the disbenefit again the red one, but on the other direction. So obviously A_4 then A_1 then we come to A_3 and come to A_2 . So positive side the ranking I will just mark I will take few moments here.

So, this is the ranking would be I am using this will go here because and from the other side it will be just other way now. So, this is the ranking I will just mark and I will take few moments here. So, this is the ranking I am using and this is the ranking I am using and this is the ranking here. This will go here because and from the other side it will be the just other way now. So, I am using the blue color then if I consider the second one it comes here and I consider the third one it comes here and if I consider the final fourth one it comes here.

So, this comes here and similarly, if I do the disbenefit one let me remove the disbenefit. So, on the right hand side there is no crossing of the line. So, it becomes much easier for us put it side by side. So, symmetry will give us a much better view point. So, this was the answer when I go to the negative part I use the red color, but remember this ranking which I am giving the red and the blue are based on the positive negative, but in the reverse direction.

So, that should be important for us to remember. So, the first one goes here and I am using the red color in order to specify. The second one basically is this, the third one is basically

this and the fourth one is again the same. That means both in the benefit and negative scale and positive scale I get the same ranking. So, there is no dichotomy the question would be it looks very straight forward and obviously this dichotomy should not occur, but they would be cases where you can have the dichotomy occurring that when you are looking at the positive side and when you are looking at the negative side your rankings may change. Now, one thing again I am mentioning the ranking based on the positive or the negative are based on the fact that what are the distances.

So, in the positive sense if it is 0 best and negative sense it is 0 is the worst. So, you have to basically take the decisions accordingly. Again I am reiterating I am only reading the decimal places 0.31 is the best. So, which is A_4 then 0.63 is the second which is A_1 , third is basically 0.69 which is A_3 and last is A_2 which is 0.76. If I look from the negative side obviously the distances are 0.68 for the first which is A_4 then comes A_1 which is 0.36 then comes A_3 which is 0.30 and A_4 and last comes is A_2 which is 0.23. Now, in so if you have only considered positive or only considered negative the life would have been fine, but here I am trying to compare that how you can take both of them accordingly depending on the set of criteria's we can be divided into two sets. Now, the question may arise are we able to replicate this concept of negative distance, positive distance for the other methods in some cases yes and we will see that as in the discussion as we did in this lecture. Thank you very much and have a nice day. Thank you.