Multi-Criteria Decision Making and Applications
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Lecture 41

Good very good morning, good afternoon, good evening to all the participants and the students and welcome to this course title multi criteria decision making. And as you know this is NPTEL MOOC course which is spread over 12 weeks with 60 lectures and each week we have 5 lectures and each lecture being for half an hour. And my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. So, the broader set of points which we are discussing and I keep mentioning them which is multi criteria decision making under that you have two main topics one is multi objective which we have discussed in details solved quadratic programming, linear programming, both integer programming for the bio objective case, for the multi objective case in the sense when the three objective functions dealt in the techniques showed the graphs, the results, the Pareto optimality frontier. Then we discussed multi attribute in details we did discuss about attribute theory or utility theory then multi attribute even though they were the only main points were mentioned. And based on the multi attribute we started the different type of qualitative multi criteria decision making and the main one which we had just started was the Topsis method.

And if you remember for this set of classes which are related to multi attribute theory all these non parametric decision making the details will be solved in the excel sheet and I will share the excel sheet accordingly. And whatever I will share only one excel sheet it will be kept I will keep updating it and inform you accordingly and obviously, the slides are all available with you. So, this is the 41st lecture out of the 60 lectures for this whole course. So, if the continuation of the Topsis method which is technique for order preference by similarity to ideal solutions.

Now if you remember we did mention that there was one matrix which was basically mentioned as x and in that x if I go to the excel sheet which was here if you see the first sheet of the excel sheet and this excel sheet main file has been saved as MCDM the first sheet which we were discussing was the Topsis method. Now when I mentioned about x and I also mentioned about the weights and also r. So, I will just recapitulate. So, you have 4 alternatives each alternative is based on 5 criteria and thus the column wise values which you see and why I am repeating it if it is a repetition please excuse me because this type of concept will be utilized for all the other methods also. So, the values which are highlighted in column C which is for the first column C_1 criteria the corresponding value corresponding to the first criteria accruing to A_1 to A_4 alternatives 1 to 4 are 10, 5, 15, 20.

Similarly corresponding to the second criteria the weights are given as highlighted 25, 15, 30 then we come to the third criteria values are 25, 35, 40, 30 then it is the fourth criteria which is C_4 30, 40, 45, 35 and finally, you have the fifth criteria again those values corresponding to A_1 to A_4 which are the alternatives are the values are 15, 10, 10, 5. Now

once this is done we need to normalize and I told you normalization can be done in different ways. So, the normalization which is there and which I will try to highlight using this ppt file it will be easier for me to highlight and obviously once you see the videos you will understand that. So, considering normalization first I will use different colors to highlight the concepts of normalization. So, we will normalize according to how the rows and the columns have been represented for our example we are taking the alternatives along the rows and the columns are the criteria.

So, based on that if I follow one normalization can be considering the values each cell is x_{ij}. So, when I consider x_{ij} which are the cells for the total matrix X, capital X. So, this will correspond to i = 1,….,M where capital M is the number of alternatives, j = 1,…,N where capital N is the number of criteria. So, row wise column wise. So, the normalization can be done based on $\frac{xij}{\sum xij}$. Now if you see I have not put any limits whether we are trying to calculate column wise or row wise for the x_{ij} that will again I am repeating based on the fact that how you have placed the rows and columns corresponding to alternatives and criteria. So, with this if I go back to the excel sheet. So, here it is the first value. So, the normalization which I just showed which is $\frac{xij}{\sum xij}$ is I am not going to use it now I will do it within few minutes which I have already done. So, this cell value is basically based on the idea that it is $(x_{ij})^2$/ the sum of the square values such that the sum would always when you normalize the sum would be 1. So, if I follow that the corresponding x_{ij}'s when converted into normalized values are for criteria C_1 are given as this which is 0.13 I will only treat till that second piece of decimal 0.13, 0.03, 0.30, 0.53 and the sum is 1. Similarly if I consider the same concepts or normalization corresponding to C_2 criteria, C_3 criteria, C_4 criteria, C_5 criteria the values are given as here for C_2 if you see again only reading till the second place of decimal 0.26, 0.09, 0.26, 0.37. Then again for C_3 values are given 0.14, 0.28, 0.36, 0.20 for C4 they are 0.15, 0.27, 0.35, 0.21 and last is C_5 is 0.50, 0.22, 0.22, 0.05 and check the values are 1 as highlighted in the yellow row hence it is mentioned as sum. Now in place of the normalization which we did which was like this so normalization which we did I will use a different color it is $(x_{ij})^2/(x_{ij})^2$ and summed up along the column. So this is the blue one is the normalization which we used. Similarly you can use the normalization shown as in the first part which is in darkish red other different normalization technique can be used you can use the diagrammatic normalizations.

So now the idea is the question would immediately come is that why are we using this normalization accordingly because if you remember we have considered different type of utility functions some were quadratic, linear, logarithmic, exponential. So based on the idea how the overall utility accrues to the decision maker based on different criteria for each alternatives we will use the normalization based on that concept of utility function. So once I come back to the excel sheet so based on X first I get the normalized matrix then the weights are there and the weights if you remember I mentioned they are the

importance you are going to place on the each criteria nothing to do with alternatives. So once you multiply so technically the weights are as shown. So I will just highlight it with say for example light blue so the weights which are given is 10%, 20%, 10%, 30%, 30%.

So if you add up is 3, 4, 5, 6, 7, 8, 9, 10 which is basically 1. So the weights add up to 1 so I am giving 10% to C_1, 20% to C_2, 10% to C_3, 30% to C_4 and 30% to C_5 importance. When I use weight which is given here W multiplied by R and obviously you should remember the row and column size of R and W based on that I have the values given as Y here. Now would come the main part. Now by the way here for the time being I am considering if you remember in the Topscis method what was mentioned about most ideal and the least ideal solution PIS and NIS.

So now based on the PIS and NIS concept we will proceed, but few important points I will keep mentioning as we proceed. Now if the decision maker decides on the set of the weights which I just mentioned the weights are given from by $w_1,\ldots., w_n$ and n if you remember is the number of criteria as I mentioned. Sum of the weights is 1 as mentioned in this first bullet point. So the weights are given here w and the $\sum w = 1$. Now consider the weights for this problem given as I will keep flipping from that excel sheet through here the weights are given as 0.1, 0.2, 0.1, 0.15, 0.25, 0.3 and based on that I find out y which is x shown here multiply r and w. Now as I said how do we rank? These values which I find out based on the multiplicative factors of r and w which we got is basically I will highlight this one is the weight I am marking in red and this is basically r normalized matrix corresponding to X based on that the values which I get is V matrix and the V matrix values are here I highlight that in green. So if I go back the excel sheet the R matrix normalized multiplied with the weights, weights is w I get V which is y given as the cell values and remember I have highlighted the cell values with red color. So they will keep changing depending on the normalized concept which I am thinking.

Now we will calculate the values of the most positive ideal solution and then corresponding to that the most negative ideal solution. Now if you consider what we will do is that for each value that means considering $i = 1,\ldots, M$, M is remember the number of alternatives which you have. We will find out that corresponding for each M every values which we have. So as i changes to 1 to M which is here in this case let us verify here in this case M is 4 and N is 5. So corresponding to finding the values I find out for each $i = 1,\ldots,M$ what is the max value.

If I consider i = 1 here and I will come to the negative solutions also for i is = 1,..,M each value. So I will do it. I find out y + similarly I will find out y- also what is my y- I

will come to that. So here so i = 1.

So this is the first one. I will write even though I am writing it below it is basically maximum. This is for the first one similarly so I will replace the fact i = 1 then y + i = 2i do it max of the second which is A_2 then I do it for the third. So I am just copying it. So I do it for 3 and finally I do it for 4 and this is the fourth value. So all the max values have been calculated.

Similarly so I will just highlight it as required with some color. So let it be light green here. So zoom in for better visibility. So obviously I will share this you will understand.

So now I find out for minimum. So this would be y- , but here rather than max it would min. So if I calculate this min of i = 1,…,M. Similarly I copy it and I will check and change it accordingly. What do I change? I change here i_2, i_3, i_4. So these are the max and min values and they are let me highlight this is basically used for PIS and this would be used for NIS.

So Y PIS, Y NIS let us go back to the excel sheet. Now here so we will need to find out V + which is Y the values which have calculated. This is as mentioned is the most positive ideal solution. So the values V_1 to V_N would basically depend on the values as calculated. So just one minute I should have so basically my maximization would be different here just one minute because there the rows and columns were different.

So technically it will be i = so if I have interchange the rows and columns they would be accordingly calculated. So I will do it here again for this case it will be max of if it was basically interchanged. So it would be maximum here then maximum found out for second values, maximum found out for the third value, maximum found for the fourth value and finally maximum found for the fifth value. So similarly if I have the NIS based on interchange the rows and columns I would find out the minimum for the first value as I am going. So I do it for the second, I do it for the third, I do it for the fourth and finally I do it for the fifth.

Now with this I proceed. So once I find out the max of the minimum so this is the maximum found out for each i is equal to 1 to m then I have calculate the negative that is the more negative solution which is the minimum. So maximum minimum we have found out. Now given the maximum minimum I want to calculate the corresponding distance from the best solution and the worst solution. So when I am talking about the best solution the worst solution is like this. The maximum of any set of values technically I would find it or consider is the best and the minimum I will basically technically find it as the worst.

So for each of this V matrix values, V matrix values if you remember is the multiplication of the normalized values with their corresponding weights. So I need to find out like this. This is consider the line, this is the best solution and this black spot is basically the V ij's and this the blue one is the best. Similarly I have the worst also because that is the max and min. So I need to find out the distance and use the blue color.

I need to find out the distance here and I need to find out the distance here. So this blue one I will mention as V +, this is V- and the distance function which we find out or utilize is basically simple Cartesian L_2 distance. Now the question would come depending on your requirement or whether you want to use the L_1 norm, L_2 norm or L_3 norm or L_{max} norm the distance formula will change and distance value which you find out would also change. So that would basically give you a weightage concepts. Weightage why? Because if you remember I had mentioned so consider the distance as even though it is written as S +, consider S + we are using.

So this would be S + from the positive, this is the S - from the negative for all these different criteria alternatives and based on that I want to find out what is the overall ratios of the normalized weights. Why? Consider theoretically if this V_{ij} is here which means it is furthest from the negative one. So obviously it is good that it exactly matches the positive value given such as the weights or the overall criteria importance I want to give or the collective criteria importance I want to give for the V_{ij}'s would be the maximum. In case if V_{ij} is on the red dot which is the most negative solution then obviously in that case it is furthest away from the blue one most idle one in that case the so called weightages or the importance I want to give to that particular alternative would be the least. So based on the maximum and the minimum or the highest value and the lowest value we will rank them and again the distance measures can change.

So, we calculated S plus based on the most positive idle solution given on L_2 which is squared norm then we calculate S i - based on the most negative solution again it is based on the L 2 squared norm and I find it for each individually for M, M = 1, 2, 3, 4 find out their maximum minimum value and then one is found out is here and this what I what I was talking about I can again draw the diagram. So if I have the real line sorry for the repetition plus please this is the best solution P_i s this is the worst solution N_i s V_{ij} is here I find out which is here already found out S i + I find out this which is S i - and based on that I try to find out and calculate C_{ij} s which will give me the ratios. The ratios are depending on what is the negative worth and the positive worth. Now the question would immediately arise cannot we use S i + in the numerator and do the calculation yes we can do the calculation. So that will give me a overall ranking based on the ratios of the best solution and ratios of the worst solution.

Now the question would be what would be the best and what would be the worst solution. Consider that if you remember one of the examples which you have considered was Mr. Murthy buying a house in the city of Vizag. So in that case price more the price is worse off or say for example, maintenance cost being high definitely not required safety being good that is a good point. Schools colleges being in a in a very nearby location is good.

So we will basically consider all those positive points based on which we will find out the best distance or the P_i s or the most ideal solution and based on the fact which are negative which do not give any positive value we will basically club them and find out the N_i s point and based on that we will do the calculation. Even though as shown here we are only highlighting one point but that can be expanded depending on how you have been able to divide the positive and the negative. Now if we come back to this excel sheet I did not mention anything about the criteria C_1 to C_5 were considered as they are but in case if you want to put importance on C_1 to C_5 consider C_1, C_2, C_3 are positive and C_4, C_5 was negative then obviously your P_i s would be based on C_1, C_2, C_3 and similarly the negative solution will be based on C_4, C_5 that means we will basically differentiate the criteria and the importance and their effect positive and negative in two sets and find out the P_i s based on the positive set and find out the N_i s based on the negative sets. So consider the problem is related to buying the house apartment among four choices where the decision to buy the house and apartment are based on eleven different parameters four choices means four locations four alternatives and the criteria s parameters are eleven in number. So, they are considered city, price, loan availability, location, number of rooms, safety, proximity to market, proximity to school, proximity to hospitals, facilities available, resale condition.

Now if the question is that how do we differentiate the positive and the negative one. So, let me highlight it consider here. If it is price higher the price is definitely not guaranteed it is not required or we do not want. So, we will highlight it as the one which will fall in the category of the subset of negative points. If I consider loan availability conditions of the loan are very stringent then obviously it becomes a negative set or a set for the negative points.

Now in this case in these two red one which are highlighted if higher the prices higher the conditions for the loan availability are it is negative that means more it is increasing it is having a negative effect. Lower the price lower the loan availability is good that means it is manageable. Now having said that if I look at the other side of the picture consider number of rooms, consider safety, consider facilities available and only highlight these three it can be done for the others. More number of rooms better, safety

features being more better is good, facilities available being higher is good that means if you consider the red and the blue one as subsets higher the values in the blue is positive, higher the values in the red is negative and vice versa when the values decrease. So, hence having said you can easily divide the positive set and the negative set into two different classes and do the calculation based on which you can find out the C i plus and the C i minus I will come to that in the next class. Have a nice day and thank you very much. .