Multi-Criteria Decision Making and Applications
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Week 08
Lecture 40

A warm welcome to all the students, a very good morning, good afternoon and good evening to all the participants. This is the course title multi criteria decision making and as you know this is a 60 set of lectures spread over 12 weeks and each week we have 5 lectures each lecture being for half an hour and as usual I am repeating it please excuse me for that there are assignments for each week. So, 12 assignments and there is a end semester and my good name is Raghunathan Sengupta from IME department at IIT Kanpur. So even though the broader umbrella or the super set under which we are discussing is written there. So, that is basically throughout almost the set of lectures. So, because in the first part there was examples then those utility function which was quite given for whole set of slides and then we switched over to multi criteria decision making and there were two important points one was multi objective and one is multi attribute we are only discussing multi attribute now.

And the last class we started the TOPSIS method. So, the TOPSIS method idea even though I am going to start from the new slide, but the slides which you already remember I will discuss. So, there is X matrix which gives the overall scores and scores are based on the utility function the concept between the alternatives and the criteria's that is point one. Point number two is that you will give or the decision maker will give also weights for each and every criteria.

So, important criteria higher weights lower less important lower weights. We will also use the concept of normalization. Normalization would be depending on how you portrait the X matrix it can be along the rows or along the columns. And then we will multiply the normalized matrix with X find on the new matrix and then utilize the so called multiplicative factors of the weights and the normalized matrix and then try to rank them. Also remember both positive and negative concept was being used which is the positive distance and the negative distance or the concept of PIS and the NIS.

So, the idea which we are going which we are still continuing is the technique for order preference by similarity to ideal solution which is the TOPSIS. Now, as we give the and if you remember we have discussed the simple pseudo code and that was the idea which is being repeated. So, I will give the general formulation and then consider the problem. So, assume the decision matrix is given like this. So, if you see this matrix it is of size $m \times n$.

So, technically the matrix which I am talking about I will use the black color. And these are the alternatives. So, there are alternative 1. So, if there are m number of alternatives so along the rows they would be m numbers. So, $A_1,\ldots,A_m$, m small m or capital M.

And on the columns I will use the blue. So, this C_1,….,C_n, n as in Nagpur. So, these are the criteria. So, when I consider any value see for example, X_{11} I will write in black X_{11} would mean what is the overall benefit or of criteria 1 on alternative 1. Now, if I want to consider only the alternatives so only concentrate on the rows.

So, the first row if I consider this part I will use the light yellow color so it is easier. So, this one. So, the first one is X_{11} which is for criteria 1 weight or overall weightage or points for the first alternative. Similarly, when I go on the same row the second value. So, this will be X_{12} because X_{12} is the first one.

So, this is the first one. So, this is 2. So, this is the overall weightage for the second criteria on alternative 1. Similarly, I go to the last one which is X_{1n}. So, technically all the points which are there scoring which are there for X_{11},…., X_{1n} are corresponding to the first alternative.

If I consider the last column the corresponding values are X_{m1},…..,X_{m2} till the last one which is X_{mn}. So, which means that this set of benefits of the points which are being assigned for the by the criteria C_1,….,C_n are corresponding to the alternative m only. So, that is why it is $m \times n$. So, if the number of alternatives or number of criteria was same. So, it would have been a square matrix.

Now convert the entries in X into a normalized scaling. Now normalized scaling can be different ways. I will write them accordingly first let me erase this part. So, when I use the normalized coding it is one score can be R_{ij} as given here. So, basically the value X_{ij} divided by square root of the sum of the squares.

Another can be see for example, just it can be X_{ij} divided by just the sum it can be a normalization factor concept also. Another can be $R_{ij} = \ln(X_{ij})/\sum \ln$. So, this concept of trying to normalize would depend on the values of the cells which are there. So, when I talk about the values the utility one concept they would change depending on the criteria which you are considering or the weights you are giving or the importance you are giving. So, convert the entries into X into a scale normalized one and once you have that you have the matrix which is bold **R**.

So, if you remember I mentioned that you have to multiply the weights with the normalized matrix X that normalized matrix is R. So, once you normalize you will get the R matrix. Now, the question is the normalization has to be done row wise or column wise that would depend on how the overall the values of X have been given to you. It could have been like here if you remember the problem is there are m number of alternatives and n number of criteria. So, m was in the rows and n was in the column what it was just flipped transpose of that.

So, in that case n as in Nagpur would be measured along the rows and m would be along the columns. So, in that case your concept of normalization would change from row to column, column to row depending on how the nomenclature or the points have been considered. So, as I said that I will have values also to discuss. So, if I consider the

values so assume X is given by the score system as which is basically there are 4 rows and 5 columns. So, this is 4×5.

If I read it the first column was 10, 5, 15, 20, 25 and the second column is 25, 15, 25, 30. Now, the third column is 25, 35, 40, 30 and so on and so forth. Now, what does it mean? It means that if I only consider column 1 which is basically criteria 1 then the overall benefit which you will get based on the fact that somebody is analyzing the problem only from criteria 1 perspective for the alternatives would be as follows. I am reading the numbers 10 for the alternative 1, 5 for alternative 2, 15 for alternative 3, 20 for alternative 4. Similarly, which means that technically the overall value which you will be assigning based on criteria 1 goes highest to alternative 4 here.

Similarly, if you consider the second column, third column, fourth column, fifth column the corresponding values which you will get for the for giving the maximum scores would be for the second column which is C_2 again it is basically the last alternative. Similarly, if I consider the criteria 3 which is the third column the base score is given for the third alternative. Similarly, for the other two criteria the best score would be corresponding to the again third alternative because it is 45 and then in the last case it will be corresponding to the first alternative. So, scaling scale the values using the normalization factor and I do that when I do that obviously, this normalization factors which are to be considered as the case. So, let me do the normalization in excel sheet.

So, it will be easier and understood by all I will take a very simple problem. So, let simple problem in the sense we can consider this. So, I will put X here and I will zoom in the values which is for better visibility. So, here I will consider alternative 1 A_1 is alternative 1 A_2 is alternative 3 A_3 is alternative 3. Similarly, C_1 criteria C_2 criteria C_3 criteria C_4 criteria C_5 and that is why it is 3×5.

The scores are given as 10, 5, 15 that is I am considering what another one also it is 4. So, they would be A 4, 20. For criteria 2, 25, 15, 25, 15, 25 and 30. So, criteria 3 is 25, 35, 40, 40, 40, 40, 40, 20. For criteria 2, 25, 15, 25, 15, 25 and 30.

So, criteria 3 is 25, 35, 40, 35, 40, 40, 45, 45, 45, 45 and 30. So, these are the criteria 4. Wait let me check 1, 2, 3 this is 10, 5, 15, 20, 10, 5, 15, 20, 25, 15, 25, 30 then is 25, 35, 25, 35, 40, 30 this would be 30. Sorry my apologies then I go to the fourth column is 30, 40, 30, 40, 45, 35, 45, 45, 45, 45, 45, 45, 45, 45, 45, 45, 45, 45, and the last one is 15, 10, 10 and 5. See now the normalization which is done here, I am going to change it and also.

So, let me consider first R. So, I have R here, R here, R here, R here, R here, R here. So, I am going to consider normalization happening with based for the criteria. So, I am going to normalize the criteria and then consider for the alternatives. So, if I consider the simple normalization, simple normalization let me write it also. So, it is there is no confusion because when you see the videos it will be easy for you to understand.

So, let me consider normalization 1. So, normalization 1 would be basically be given as

this. So, I will use this normalization 1 concept here. This is equal to 10 which is cell first divided by sum of all the values which is there in column 1. So, I fix it because the sum is fixed.

So, I will have to do the same thing fixed. So, I fix it and then I copy it for the first column. Now, for just double checking the sum is 1 what you see. So, I will mark it with a light yellow color. So, this cell when is copied is not copied I have to change it.

So, this basically would become the values corresponding to D. So, this is also 1 then I copy it for the second column. For the third column again I change it. So, this would be E, this is also E, this is also normalized when I go to the.

So, this would be column F. So, this would be column F. So, this would be again normalized 1 and when I consider G again normalized 1. So, this is the first column. Now, what does technically normalization mean? If I consider this which I will just highlight with the color, but I will remove the color accordingly. So, consider I am highlighting with this color and should I zoom in more.

So, it means that if I consider criteria 5 only and on the normalized scale or so called important scale for each of the each alternative these are not the weights for criteria. So, for that criteria the overall so called normalized importance let me use the word importance which is being placed for A_1, A_2, A_3, A_4 which is alternately 1, 2, 3, 4 coming out from C_5 called criteria 5 are given as 37.5, 25, 25, 12.5 which means the highest amount of importance is being given to alternately 1 based on the fact C_5 is being considered as the criteria. If I consider this as I said I will remove the color.

So, there is no fill if I go to this. So, here for C_3 which is given as C_5, C_3 is given as C_3, C_3 is given as C_3. So, this is the criteria 3 the corresponding importance is highest for alternative 3 then it is for alternative 2 why 3 because it is 37.7, 30.7 the second one is 26.9 for alternative 2 the third is for alternative 4, 23.07 and the last one is alternative 1 19.23. So, it is for remove again this color. Now, with this also I have a weight and this weight there is some ideas which I want to give. So, this weights are basically the criteria weights importance you are going to. So, for me selecting the final answer is alternative not the criteria. So, my importance would be placed on the criteria based on which I am getting the alternatives.

So, obviously if there are 5 alternatives there would be 5 weights. So, let me consider the weight matrix. So, this you have C_1. Let me remove this one cell on to the right and this is the weights which I will put the capital W. Now, a question would definitely be asked from your side which I will answer, but not solve problem accordingly which will come the question will come later. First let me put it consider I am putting weights as 10 percent for all criteria weights.

So, this is the weight matrix. So, this is the weight matrix. So, this is the weight matrix. So, C_1 20 % for criteria 2, 10 % for criteria 3.

So, 2.1.2 which is 0.3.3 + 0.1 = 0.4. This is 0.3 for C_4. So, 3, 4, 5, 6, 7 and the last one will be also 0.3. Now, what does it mean? So, I have not put all the values. So, if I consider this principle diagonal which means I am putting 10 percent weightages for criteria 1, 20 % for criteria 2, 10 % for criteria 3, 30 % for criteria 4, 30 % for criteria 5.

So, corresponding to this the other values would be 0. Now, they would definitely be a question which I said which may crop up which I will try to explain. Now, the question can come up is it possible for a human being to change the criteria weights itself depending on the alternatives? It is possible. Like if I prefer a car made by say for example, Tata motors and I am trying to compare then I may be willing or may tend to give more weightages for Tata cars for the same criteria because see for example, the style concept is very good.

I am giving an hypothetical example. But this point when I am changing the weights based on the criteria and alternatives would be true when I am considering a whole set of gamuts of cars by different make. When I am considering the car for only one company and I am want to choose which is the rarity because in front of me there would be different cars from different makes. So, in those rarity cases where only one set of set of alternatives are there for buying a car where the manufacture is the same then only this both theoretical and practically sense would make true. But we will stick to that and continue. The reason is that matrix multiplication where rows and columns are important is maintained.

So, if I consider the multiplication. So, I consider this matrix as Y if I say matrix. So, matrix multiplication. So, matrix multiplication matrix multiplication. So, this is basically the matrix multiplication and here you will understand. So, if I multiply the concept and by the way this matrix multiplication this is basically of size 4×5 and that weights for the criteria is 5×5.

So, 4 cross 5 should be multiplied 3 and then 5 cross 5. So, the end result is 4 cross 5. So, let us check 4 cross 5 if it is yes yes it is there are 4 rows and columns 5 columns. So, not the other way round. So, this is not basically $(5 \times 5) \times (4 \times 5)$ because that incompatibility would be there. So, once you have this these are the multiplication of the normalized score with the weights which is important and we will consider and marking it as Y.

So, if I consider I will put the arrows in such a way. So, it will be easy for us to follow. So, first from the X we get R and then we will get Y. So, this is the normalized one. Once we get the normalized one we multiply R with the weights the arrows here means that we are multiplying and we get the multiplicative matrix which is Y. We multiply R with the weights the arrows here means that we are multiplying and we get the multiplicative matrix which is Y and we will check the second step.

So, this is the second step. So, I thought I will do the problems and then consider by the way normalization has been done in one way. So, just keep a look at the answer. So, I will just make it bold and also give a coloring say for example, red. So, you will check

the answers they change. Now, here the normalization was used the value divided by the sum.

That was a linear part. Maybe I can use the quadratic one. So, the quadratic one with this value squared up divided by the sum of the squares or logarithmic one log divided by the sum. Now, here if I use the square, so square sum would come here. I will continue using the normalization factors here in the successive classes, but let me show you the normalization once more other types. So, this would be square divided by not sum, so it would be this $5^2 + 15^2 + 20^2$.

So, this would be square divided by this. So, these values the column wise they are fixed as usual. So, this will be C_2 fixed, C_3 fixed, C_4 fixed, C_5 fixed and this. So, these values I will just copy it. Again check the sum which is yellow in color.

So, this comes out to be 1 as usual. If I do it for sum of the squares for this one and also keep checking because the video is there as you are watching keep checking the values as they are changing on the red cell which is y which I am multiplying. The weights I am not multiplied changed here, let it be as it is. If I check the values of here, so it will be D because I am considering the D column. D column is C_2, C_2 means criteria.

So, C again I am repeating A_1 to A_4 alternative C_1 to C_5 criteria. So, again copy it just here it is D_4, so D_2, D_3, D_4, D_5. So, C, the value again comes out to be 1 remember here which is in cell D_{13} where the cursor is. Similarly, I use the column E which is corresponding to C_3 criteria. I copy it just here, so C just check the value will come out to be 1 normalized.

Similarly, I take it for F column which is corresponding to C_4. Again I add up I like it is 1. Finally, I add up the value here. I do it for G column which is corresponding to C_5 G G G. So, if I copy it again the value is 1. And if you have kept checking this values which are there in the red cell which are multiplication of the normalized and the weights keep changing.

We will continue taking this from our end. I will save this file this excel file. So, we can use that in the classes of the next one and we will continue discussing about the top sets and under matters. Thank you very much and have a nice day. Thank you.