Multi-Criteria Decision Making and Applications
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Lecture 39

A very warm welcome to all the participants and the students, a very good morning, good afternoon, good evening to all of you and this is the course title multi criteria decision making and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. So if you know this course is of 12 weeks spread over 60 lectures and we are in the 39th lecture. The broader umbrella which I keep repeating is multi criteria decision making under that we have covered multi objective decision making and we have just started multi attribute decision making and the concepts of multi attribute utility theory. We have discussed, we will be utilizing it and for our example we will start one of the application side for multi attribute decision making where it is more subjective. The analysis answers would be more intuitive not straight forward based on the ideas of the concept of there are total objective functions, they are in numbers which can be discussed, constraints are there. Here many of the decisions would not be so straight forward.

Analysis would be easier in when you understand, mathematical complexity would be less, but trying to fathom understand the answers or give feedback has to be analyzed accordingly. So the coverage is obviously multi attribute utility theory between utilizing, but the main topic we are going to start is basically TOPSIS method. Now the TOPSIS method is basically the technique for order preference by similarity to ideal solution and what the word means. So consider there are many decisions and you want to take how close the decision is to the best possible one, ideal best possible one, how close it is, closer it is better, further it is not good and also as you are understanding the concept of how close it is to the ideal solution you are also comparing how far it is from the non ideal solution.

So basically you consider the ideal and the non ideal one, ideal is the best one, non ideal is the worst one and you want to basically consider the distance function of any alternative or decision from both the ideal solution and the non ideal solution. So technically the best solution would be what if I draw consider in the linear scale even though it can be extended for the multivariate case that means two dimension higher, but for simplicity let us consider. So here is the simple case of univariate around the real line and consider I am not giving any numbering system here I am just marking it. So consider the most non ideal solution is here and the best ideal solution consider it is here. So technically what will happen whenever I consider any alternative any decision closer it is the blue and further it is from the red is the best.

So technically the best solution would be what which is at point blue and it is furthest from red that is theoretical best, but it would not be possible in many of the cases to have the theoretical best you want to find out the practical best and in the similar way the worst would be when it is there on point the red and is further away from the blue one. So based on that we will try to consider. So the technique of order preference by similarity of the ideal solution was developed for the integrated human exploration mission simulation for the Johnson space center to assess the priority of set of human space flight mission simulators and study them and we will consider very simple examples for that not from the point of view of human space flight mission simulators, but from the simple examples like buying a house buying a car and all these things. Here one important thing the utility function is monotonic in the sense more or less you get more or less you want. So obviously, the overall feedback based on which you are trying to take the decision and utility   would   depend   that   more   I   give   you   more   you   want.

So based on that you will try to analyze. So coming back to the point which you are saying the basic premise assumption is based on the fact on which TOPSIS works is that selected alternatives should have the shortest distance from the positive solution which I showed the blue one and the furthest distance from the negative solution which I have showed in red. So best solution is the red blue point worst solution is the red point and we will basically have the points accordingly to assign the distance function. Now the question would come up immediately why do we need the distance function. So let me give you a example   or   conceptual   concept   about   that.

Say for example, there are two decisions and I marked them as A_1. So this A_1 and A_2 are nothing to do with the ideal solutions of the non-ideal solutions. So I am just using the color as it is. So   there are two decisions A_1 and A_2. Consider two scenarios.

Scenario 1: I take A_1 decision, Scenario 2: I take A_2 decision. Now due to some circumstances, so when I take the A_1 decision so I want to find out that how good or positive points I am getting by selecting A_1 and what are the negative points which I have been able to overcome with by not taking the A_2 decisions. Because if I understand what I said this is the bordering line. So if I take A_1 which means I have not taken A2, so taking A_1 has given me positive points and not taking A_2 has basically got myself rid from the negative points which means if I got myself rid from the negative points it is adding to my benefit.   So   in   that   sense   we   are   trying   to   understand.

Whether it is to A_1 better further away from A_2 is also better. On the other hand if I take the decision A_2, so obviously it is that I have taken decision A_2 and I am considering A_2 is not as good as A_2. I am not saying worst but is not as good as A_1. So taking A_2 is giving me the benefits pertaining to A_2 but on the other hand if I have not taken A_1 I   have

disowned some benefits which I should have got by selecting A_1. So here I am considering the benefits for getting A_2 and the disbenefits which I am getting for not accepting A_1.

So if the word which is used when we are not accepting A_2 is the positive disbenefits which I get by not selecting A_2 when I on the left hand side, left hand side means A_1. So this balancing of taking a decision, getting some benefit points, not taking a decision, getting both the benefit and the negative points is basically what is summarized in the concept of the solution. Now you may say that how is it possible. Consider this person X is working in a form and any form and X is basically not the variable, the person X, I am mentioning without naming him or her and there are two situations. The person X can continue staying in the home city which is a small metro city of tier 2 or else move to tier 1 may be it is Bangalore, may be it is Hyderabad, may be it is Calcutta, Delhi, Bombay, Madras, Mumbai means Mumbai, Madras means Chennai, Bangalore means Bengaluru.

Now these cities which I mentioned they are not his home town which means that he will get a higher package which is good but moving there obviously would entail other level of cost which would be a dis-benefit from him like hiring a house where he already has a had a house in his home town, more cost for travelling, higher school fees for his kids, his or her kids. So means the higher salary is good but this higher cost are adding in a negative sense. So what decision he or she will take basically entail the fact that what are the concepts of ideal solution and non-ideal solutions we are going to consider. I took a long time in trying to explain but when we come to the problem I am sure it would be easily understood. Now we will use the concept and very simple concept of PIS so which means the positive ideal solution and for when I am marking I will try to use red for the negative and blue for the positive.

So if I use the blue which means positive ideal solution is given by PIS and the negative ideal solution is given by NIS. So I want to choose the positive ideal solution of the original ranking process also choose the negative ideal solution for the original ranking process and find out the overall cumulative distance. So what we do is that as I mentioned in the diagram if I have the real line, the blue point which I just marked in a circle but is basically the PIS. Now you may be saying that why is PIS on the right, is there any strict concept by doing that? No, I am just drawing the real line such that it will be easy for us to communicate. So if you see the real line which I have drawn on to my right is positive higher numbers on to my left is basically negative numbers they goes to infinity.

So in that sense it can be understood and similarly this is NIS. So you find the distance if you read this third bullet point find the distance from each of these alternatives what is distance towards the PIS. So consider the alternatives I mark in black and I mention them as A_1 then A_2 and consider A_3. So for each A_1 I find my apologies I find the distance of A_1

from PIS. So I will do one by one not overwrite them.

So this is basically A_1 distance from PIS and similarly the distance from negative solution is this. So further it is away from the red NIS and closer it is PIS it is better and vice versa on the negative side it moves in the opposite direction. Similarly if I consider A_2. This is the distance from PIS and this is the distance from NIS for the second case. Similarly when I erase them the distance concepts this is for A_3 I should use a board one this is for A_3 distance from PIS and this is for NIS. So obviously the distance functions based on which we will try to find out the alternatives.

The question would immediately come up for people who have studied basic mathematics. The distance functions can be measured using different L console norm L norm concepts. So it can be L_1 which is the Manhattan norm, L_2 which is the quadratic which we all work with then L_3 norm, L_4 norm and obviously this is a L infinity distance also. Which means that the distance functions to be utilized in order to find out the distance of A_1, A_2, A_3 respectively from the PIS and from NIS would basically depend on what distance functions you are going to use. Now if you remember we did mention the concept of utility which will come within.

Now if I consider the Euclidean distance and I want to basically find it out and normalize them. So using the Euclidean distance is used and we ensure that our main motivation is to minimize the dispersion because dispersion means higher or lower the distance measure is from the NIS and the PIS would mean if the dispersion is very high that means for a change in the value of the decision your overall average answer is changing which is not expected because I want our answer or the methodologies to give us robust answers. So in this case one has to find out r_i by the way when I write r_i it is basically the symbol of the variable it does not mean the rate of return which we have considered in few of the cases when we are discussing about the utility. So in the case when I am using the distance concept and the metric the distance functions calculation based on which I find out or it is given is basically the ratio the difference of the from the alternative A_i to NIS negative 1 and I square them and then find out the square root which is basically using the L_2 and in the numerator I am normalizing them with the its corresponding distance from the NIS and the PIS. So if I again draw the real line the blue one being positive point and the red one being the negative one PIS and NIS.

So, when I take any A_i consider arbitrarily A_i is here. So I find out as given here and I will clarify that I find out the square distance square and then I will find out the square root of a i with respect to the negative point or negative PIS and NIS and divided by the normalized value which is this sum of these two. So in case if I do the ranking if the NIS value is almost so if the A_i value is almost or the decision is almost on to NIS. So obviously

the numerator will be 0 tending towards 0 and in the denominator whatever value you take the r value corresponding to that alternative will be 0. On the other hand if I consider the value of its corresponding value of a i is as close as to the possible to PIS which means in that case the distance concept which you are going to consider is that in the numerator the value of PIS because it is there on PIS only to be 0 the second term which is in the denominator will be 0 and the numerator will be ratio which will give you 1.

So it is basically being scaled between the values. The basic premise between Euclidean distance portrays the concept of utility function which we are considering as quadratic. Why quadratic? If you consider this distance measure being considered quadratic and if you consider the concept of normality to be true the idea of normality of returns with respect to the quadratic being the utility functions gives you the idea. So the question would be can we use any other distance measures can we use the mean of the Manhattan norm, Manhattan distance or we can use the max distance the ideas can be incorporated which would basically give you a different flavor of the problem it may not change the overall answer. So relative ranking would remain same but the absolute values of the ranking   may   change.

We have we already are aware the minimizing utility ensures minimizing the distance because you are trying to minimize the variance and this is what is given. Minimization of the variance is very important lower it is. If I consider so this is considered this is one distribution and this is the second one both are the same mean values for the dispersion or the variability for blue is higher for the green is less. So one would rank the ratios R i's which we get as I just discussed to get the best alternative and take the decisions accordingly. So let us consider the simple concept of the top-sis write is pseudo code or discuss the pseudo code and then proceed with the simple problem very simple problem.

The ideas would be I am taking a small sample size, sample size in the sense the value of capital N and capital N which is number of alternatives or number of criteria would be kept as minimal as possible in order to highlight how the overall methodology works. So we consider decisions on alternatives Ai 1,….,m and assume the attributes of the decision criteria are given from 1,….,n which is capital M for alternatives and capital N for the criteria. Capital means the can be capital M or capital N here it is small m small n. We state the pseudo code for the working on the top-sis method and rather than going to the algorithm I will just mention the important points which we will see in the problem. So you   will   basically   have   two   sets   of   matrix.

One is this X matrix. X is not any variable. X matrix basically gives you the comparison or the overall relative utility function values for each criteria based on the alternatives individually. So if the variable which you want to consider say for example mileage, so

mileage would be considered as the important factor such that you will basically rank or compare all the alternatives based on the mileage. Similarly you will take the second criteria which may be cost and then rank them for all the alternatives. Third criteria can we   see   for   example   the   safety   features.

Then again you will basically rank them for all the alternatives. Now as you are doing the ranking many of these criteria would be quality driven nature like safety I do not know what are exact points I need to assign, but mileage I know exact value is given. Maintenance I know exact value can be found out for the maintenance. Based on that I try to find out once given X, I normalize them and the nomenclature of trying to basically denote X is that it is a comparative concept of the criteria and their overall effect or the value for each and every alternatives. The second matrix which we will consider is the weight and this weight concept is basically again of certain dimension what is the dimension   I   will   come   to   that.

Weight gives you basically what is the overall importance the decision maker will place on each and every criteria. His or her main outcome is the alternatives. He or she is not looking at the criteria. He or she is using the criteria to rank the alternatives. Now when the criteria are important say for example for me price is not that important for buying a car,   but   safety   features   is   important.

So obviously the weights or the importance I will put on safety features will be quite high. See for example for some other person price is the style is more important. Safety features would come, but they are not as important as the style or the color of the car or the make of the car. So obviously that person the decision maker would consider the weights accordingly. The second important point for the weights is and this point which I mentioned for the weights would be repeated for the Topscis method and all other methods we are going   to   consider.

So it will consider the weights add up to 1 because obviously your overall priority which you are going to put for the criteria would be summing up to 1. So once you have the matrix X then you need to normalize, why you need to normalize and how you do that I will come to that later. Then the weights and once the normalized weights is multiplied by the normalized X factor is multiplied by the weights we will get single matrix based on the scores of the combinations which we will get for the alternatives of the criteria and you will rank them as given. So whatever I told verbally is basically discussed in this slides accordingly. So define X as I said which matrix consisting of priorities or scores assigned to decisions alternatives and you also have the weights, weights for attribute decision categories which I told such that the weights should add up to 1 and also you have a benefit matrix   and   cost   matrix.

So benefit matrix cost matrix would be benefit is what is coming as positive, cost matrix means what is coming as negative. So if I consider technically the benefit matrix it is basically adding up and giving you what is the p_i, p_i s point that is the point which I consider here the positive ideal solution and similarly we have the cost it will give me the negative ideal solutions. So based on that you find out different combinations of max and min, what is max and min I will come to that later. Then you multiply that value of X which you found out matrix considered a priority of the scores you multiply with W get a new matrix and then basically analyze the ranking accordingly. Now coming back to some concept of the distance I will show you very simply pictorially.

The Euclidean distance between two vector points is given by this. So in a two dimensional case it is difference of X_i s and X_j s square them up and find out the square root. The l 1 norm or the Manhattan distance between vectors and points is given by this sum of the mod. So Manhattan norm basically means if you look at city of New York from the top the roads are all parallel to each other parallel and orthogonal exactly 90 degrees. So I am taking the mod based on that I find out the Manhattan norm.

The name relates to the distance a taxi has to drive in a rectangular street grid which I will show in few minutes. There are other distance measures also one is the Mahalanobis distance where X is a covariance matrix for the corresponding random variables of X and Y you have. So obviously covariance would be present. So the first element would be if I consider the two random variables it will be with itself. The principle diagonals would be the covariances with itself which will give you the variance vector.

There is a hamming distance also between two vector points which is the number of positions at which the corresponding values are different. So you are basically comparing one and one comes they are not different. One and two comes one comes in the first strings two comes in the second stream. So that distance will be measured by the number of positions at which the corresponding values are different. So first position one one same second position one and three different.

So you basically keep noting it. While the L infinity norm is given by max which we know. So this is given by the max of the measures of X_i and Y_i. So you find out the difference between X_i's and Y_i's and take the maximum of them. And as we know the L p norm is given by this formula and here one important interesting part as p tends to infinity or p tends to 0 you will basically have the L 1 norm and the L infinity norm accordingly. With this I will close this class and continue discussing with the problem for the top-sis in the later classes. Thank you very much have a nice day. Thank you.