Multi-Criteria Decision Making and Applications
Prof. Raghu Nandan Sengupta
Industrial Engineering and Management Department
Indian Institute of Technology, Kanpur
Week 07
Lecture 35

Good morning, good afternoon, good evening to all my dear participants and students for this course and the course title is multi criteria decision making under the NPTEL MOOC series and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur in India. So, as you are aware that we are in the 35th lecture out of the total 60 lectures for this course which means it is spread over 12 weeks, each week we have 5 lectures and each lecture is for half an hour. So, the broader set of points we are or the ideas or the discussion we are going through and which we had for a long time are as follows. Multi criteria decision making we have discussed many relevant points we are still in the domain of multi objective decision making trying to formulate different problems to understand how the maximization minimization can be done for linear programming, integer programming or quadratic programming also. And then we did discuss the two very powerful tools in the course multi criteria where you give weights as λ_i depending on the i as the number of objective function or also we considered as we were considering in the last day, 34th lecture, we will continue doing so is how to operate and try to optimize one objective function with the existing constraint plus the other objective function being transferred into the constraints part. So, whether it is a maximization problem, minimization problem for any of the objective function the idea remains the same. And obviously the other points which we have discussed very briefly is the utility function will go into the multi attribute utility theory very briefly, but little bit more theoretical and then take the important points from multi attribute utility theory and try to understand the different concepts of other multi criteria decision making which we had mentioned long time back when we are discussing the syllabus those techniques being Elektra method, Topsis method, then the Vikor method and all the things.

The examples which we will consider in this 35th lecture is the continuation of the linear programming and the concept in MCDM and the problems which are there for the linear programming can easily be converted into integer programming. This should not be any problem provided the objective function and the constraints are linear. Obviously, if there are non-linear also we can consider that. The second coverage hopefully we can start or else it will be going into the next lecture is the idea of NLP where we will consider examples in the domain of multi criteria decision making.

So, the problem which we are considering and I did mention quite a few times that this is a three dimensional problem x_1, x_2, x_3, y we are considering a three dimension because it would be easy for us to visualize even two dimension is also possible and all of them are maximization f_1, f_2, f_3 and I told time and again that f_1 would be considered in the red color, f_2 in violet color, f_3 in blue color and the constraints are all the same for the case when we are trying to solve them individually. So, considering that we want to solve them individually the constraints are given here. So, there are 1, 2, 3, 4, 5 constraints.

The first objective function was denoted by f_1 with the red color, f 2 with the violet color and f_3 as the blue color and we also consider that for single objective f 1 star would be the max value for f_1, f_2^* would be the max value for f_2 and f_3^* would be the max value for f_3 and for all these single objective cases when we get f_1^*, f_2^*, $f_3^*$the constraints are the same as shown here. These are the constraints.

They will be applicable for case 1, case 2, case 3. Without going too much details the first case which is objective function 1, f_1 maximization the points are 20, 0, 0, f_1^* is 40. We have discussed that. For the second case f_2^* being found out the decision variables are 0, 20, 0, f_2^* is 40. Third case the decision variables or x values vectors are 0, 0, 20 and $f_3^*$is 40.

So, it is 20, 0, 0, 0, 20, 0, 0, 0, 20 and f_1^*, f_2^*, f_3^* all the values are 40 each. Now, we bring as we are discussing I am repeating the problem, but please bear with me. We want to maximize f_1. This is f_1. The constraints I will just put tick mark.

These sets these four here and the fifth one is here and the two objective functions have been brought into the constraints. One is this the one which I am marking in violet which is related to the second objective function. Another is where I am putting the tick mark blue which is the third objective function. They are less than because we are maximizing. So, obviously it cannot be more than a maximum value.

It has to be less and we will tackle the problem accordingly. The problem solution depending which I told δ_2, δ_3 are the values which are to be subtracted from f_2^* and, f_3^* in order to have a feasible solution and δ_2, δ_3 are given in column 1 where I put a violet tick mark and column 2 where I put a blue tick mark. The corresponding x vector which is x_1, x_2, x_3 are given in the third column and the corresponding values which we get for f_1^{**}. ** is basically the other f_1^* was the max with a single objective. So, now with the two other object is being in the constraint how does f_1 vary and we are denoting by f_1^{**}.

That is given in the last column where I put two tick marks and the values are there. One interesting thing is that I have just noted down the values as they change. So, a darkish yellow then the value of 39 there are two sets of 39 which I get depending on δ_2, δ_3. So, δ_2, δ_3 rather than that being important what is important are what are the values of decision variables x_1, x_2, x_3. They are same for 39 if you see they are same 19, 1, 0, 19, 1, 0. Similarly the green one is related to the objective function f_1^* being 37.5 and the values are I am not marking it here but you can see is 17.5, 2.5 0 then we consider 36.5 which is there the values are 16.5, 3.5 0 and for a value of 35 when I have to talk about the value 35 it is f_1^{**} the values of x_1, x_2, x_3 are 15, 5, 0.

Now I again I go to the second objective function which is f_2 maximize and all the constraints remain same this is fourth and this fifth one. So, 1, 2, 3, 4 and 5 and the two objective function pushed into the constraints are corresponding to the blue one which is objective function 2 on the red one is objective function 1. Again less than type like $f_1^* - \delta_1$ and $f_3^* - \delta_3$. So, I again take different combinations of δ_3, δ_1 which are given in column 1, column 2 and the corresponding values of x_1, x_2, x_3 are given in column 3 and the corresponding values of f_2^{**} is given in the last column where I put a double tick mark black.

Again the combinations of color like dark yellow, green, blue are done here in the same way and only change which you see is the decision variables. So, for 39 it was if you remember when the 39 value was for f^{**} . f^{**} is f_1^{**} here. So, the values were 19, 1, 0 here also you get 39 for f_2^{**}. The values are 0, 19, 1. So, the other combinations are just flipped and the values come out to be exactly the same and again this δ_3. δ_1 which is there in the first column, second column does not make much sense only our concentration should be on the decision variables x_1, x_2, x_3. These are again replication of the same way for f_2^{**} 35, the decision variables are 0, 15, 5.

Now we come to maximizing the third objective which is f_3 subject to the same set of initial conditions which are marked here as 1, 2, 3, 4 and 5 and the first objective there is there in the constraint which I put a red tick mark and second objective in the constraint which I put a violet tick mark again they are of the less than type. So, I have to be subtract a value of δ_1 δ_3 in order to obtain feasible solutions. The table which is shown here we do the exactly the same analysis first column is related to δ_1 values which are to be subtracted from f_1^*. Then the second column are the values of δ_2 which are to be subtracted from $f_2^*$and the third column and fourth column where I put a single tick mark and a double tick mark are the corresponding values of x_1, x_2, x_3 the decision variables which is in the third column and the fourth column are the corresponding values of f_3^{**}. And if you see f_1^{**}, f_2^{**}, f_3^{**} if the values come out to be same the x vector which is x_1, x_2, x_3 obviously changes.

So, in the first case when it is 39 for f_1^{**} it was 19, 1, 0, I am going back 19, 1, 0 here I will highlight with say for example, green color 19, 1, 0 for 39. Then for 39 when I am interested for f_2^{**}, it is 0, 19, 1 where I am circling the decision variables and 39 is also circled. So, this is the repetition for the other dark yellow which I am not repeating and for f_3^{**} again it is 1, 0, 19 this 39 is f_3^{**} again a repetition. So, the green blue and the yellow which you are seeing in the slide had been already discussed and in the same way we can obtain that we have already obtained when we are trying to maximize objective function 1 with second and third objective in the constraint. Then maximizing objective function 2 with first and third objective in the constraint and this one which you are seeing is maximizing objective function 3 with objective function 1 and 2 in the constraint.

So, these are the repetition. Now I take all the values from this combinations and make a table and I have marked it with different colors with a specific reason and I am going to highlight that first concentrate on row 1 only. If you see row 1 it has x_1, x_2, x_3 given in red color and the corresponding f values. f_1^{**} , f_2 , f_3 are given in the fourth column where I put a double tick mark in red. So, the first column would go hand in hand in trying to analyze with the fourth column. If I consider and obviously that becomes very obvious if you see the heading of the first column which is x_1, x_2, x_3 is in red.

Similarly, the heading which is in the fourth color f_1^{**} is in red f_2 , f_3 are in their usual black font size or font color. If you see the second column and the fifth one this one the second column is corresponding to when I am only optimizing the second objective function with the first and the third are in the constraints that is why it shown in violet x_1, x_2, x_3 in violet and the corresponding heading of the fifth column is given as f_1 is black font color f_2^{**} which is violet and f_3 is black font color. Finally so I am just mentioning what are the heading then it will be easy for us to go through. Finally, you have the blue one which is the third and the last column and it basically corresponds to optimizing the third objective function with the first and the second in the constraints. So, that is why it is shown in blue.

So, x_1, x_2, x_3 third column blue and in the last column is f_1 black font color f_2 black font color and $f_3^*$is blue as it should be. Now let us see why did we make different coloring schemes for the values which are there in the table. Let us consider the part corresponding to the yellow one. So, if I consider the yellow one the corresponding values of x_1, x_2, x_3 corresponding to f_1 being optimized in the objective function are 19, 1, 0 and the values for f_1 which is first objective being optimized are 39, 18, 18. 18, 18 are the values for the constraints corresponding to the second and the third which was basically 40 because it was δ_2 δ_3.

So, depending on the values of δ_2 δ_3 we obtained $f_2 - \delta_2$, $f_3 - \delta_3$, those values are 18, 18. Now when we consider the values yellow in the second cell it is 0, 19, 1 which is for second objective being optimized first and third in the constraint and the corresponding values of f_1 f_2^{**} and f_3 are 18, 39, 18. So, here it was done by $f_1 - \delta_1$ and $f_3 - \delta_3$. So, corresponding to δ_1 and δ_3 as 22, it was 40 – 22, 40 – 22 gave us the value of 18, 18 as you see. So, I am not writing δs here, I am writing 40. My 40 was basically the max value which we obtained for optimizing f_1 , f_2, f_3 as single objective.

So, we are writing that difference of $f_1 - \delta$, $f_2 - \delta$, $f_3 - \delta$, f_1 f_2 f_3 are basically the max values f_1^* f_2^* f_3^*as you want to say. Similarly for the third cell which is the heading is shown in blue for x_1, x_2, x_3 the values of decision variables are 1, 0, 19 and the corresponding values of f_1^* f_2^* f_3^* are 18, 18. f_3^{**} are 18, 18, 39. 39 is the value which you are getting which you have already seen in the slides if you go back to the slides you will understand and this 18, 18 are values of $f_1^* - \delta_1$, $f_2^* - \delta_2$. So, before moving on to the other color like the dark yellow green pink blue and subsequently. So, the values which you have seen in the yellow one and what I was talking about this 18 being obtained from 40 – 22 it would become clear here.

If you consider the first one it was basically for the. So, consider here. So, this is basic 22 which is δ_1 and if I subtract 40 – 22, this is 18, here also 40 – 22 = 18 and the objective function f_1^{**} was 39. So, it was 39, 18, 18 which we saw and the values of x_1, x_2, x_3 which were red in color are 19, 0, 1, 0 here it is 39, 18, 18, 19, 1, 0.

So, this one right. Now when I go to f_2^{**}. So, again it is δ_3 δ_1 is 18, 18. So, this is 40 – 22 = 18. Here also is 40 – 22 = 18, and this is 39. So, this was basically I should basically use the sorry my I should use the violet color because we are following a coloring scheme which is much easier for us to discuss.

So, if I. So, this was 40 – 22 = 18 and 40 – 22 = 18. So, the values of f_1, $f_2^{**}$and f_3 would be 18, 39, 18 and the values of x_1, x_2, x_3 is 0, 19, 1. So, here you see 0, 19, 1 and 18, 39, 18, 0, 19, 1, 0, 19, 1 which you see and 18, 39, 18. Finally, when we see this case or the values here. So, for f_3^{**} let us go back to f_3^{**}.

So, I will remove this and go to. So, this is equal to 40 – 22 = 18 and then 40 – 22 = 18. So, this value is 39 and the values of x_1, x_2, x_3 are 1, 0, 19. So, actually the value for the decision variables are 1, 0, 19 and f_1 f_2 and f_3^{**} are 18, 18, 39. So, this is here 1, 0, 19 and 18, 18, 39.

Now, with the same logic I repeat. Now, before I repeat an interesting point to be noted. If I consider the sum of $f_1 + f_2 + f_3$ as the overall objective function then very interestingly you see the following. So, I will remove the colours not to make it too cluttered here it is. So, if I consider the objective function f_1^{**} f_2 f_3 if I consider the total sum as a case of the multi objective not singularly as a case for multi objective. So, the value is 8 + 18 18 36 36 is 5 75. Here also 75, here also 75 and the all the objective functions are same and these points which I am marking as in red as 1 violet as 1 white colouring scheme the same thing to differentiate blue as 1.

Then the red violet blue are the Pareto points with different points for the decision variable the objective function f. If I consider as a sum of the objective functions of f_1, f_2, f_3 with ** being for the first case second case third case and the sum being 75 is the case or a nice way of trying to explain that for different Pareto points we get the same objective function. Now, if you repeat the same concepts considering the sum of the objectives f_1, f_2, f_3 as the multi objective case then let us see. For the case when you have the value of f_1^{**} or f_2^{**} or f_3^{**} as 38.5 then this dark yellow set of values would corresponding correspond to the case where you would have the Pareto points this is number the one of them second for them and third for them and what is the objective function add them up.

So, you will have it. So, here the objective function if I denote it would be 0.5 and this is 33, 33 71.5 71.5 Now, if I go to the two ones which are not in light and the the C green or light green color the last one others can be repeated for the pink one and the green one we can repeat the same thing, but I am going to the last one. If you see the set of points very interestingly 15, 5, 0 I will mark it as red one red one because we are trying to optimize f 1 then the violet one and the blue one.

So, I should mark it as sorry I should mark it as and the corresponding values which I get is very interesting even though that would not give the exact idea Pareto optimal points because the indifference because concept do not cross each other. So, if I consider the sum for the second last column. So, this would be this 8 58 58 for all for the objective function which is sum of f_1 f_2 f_3 which is 58 the Pareto points are given the last two columns as 1, 1, 1 which is 15, 0, 15, 5, 0, 0 15, 5 and 5, 0, 15, but for the problems as we have considered very interestingly then the last values is given as 8 + 3, 3 1 4 5 53 here 53 here. So, value for 53 58 we get same sets of points. Theoretically it may not be possible because technically then in that case for the same set of points if you considering the utility you are getting different values we will take this problem later on.

So, with this I will end this lecture and continue discussing more about it in the subsequent lectures. Thank you very much and have a nice day. Thank you. .