Multi-Criteria Decision Making and Applications

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Week 07
Lecture 34

Good morning, good afternoon, good evening to all my dear students, participants. This is the course multi criteria decision making under NPTEL MOOC and my good name is Raghunan Sengupta from the IME department at IIT Kanpur in India. So, as you know that we this total course is for 60 lectures spread over 12 weeks and each week we have 5 sessions or lectures and each lecture is for half an hour. The broader ideas which we are still considering is multi criteria decision making and ideas we are still discussing for a long time in the weeks of the classes multi objective decision making. Then we will consider multi attribute decision making, multi attribute utility theory and then go into the concept of more qualitative decision making in discussion. This is the 34th lecture of the 60-lecture series course.

Now as we were discussing in the last class we were discussing linear programming with three objectives and they were marked as F_1, F_2, F_3 and to make things more clear I may have used different colouring scheme red for the first one, violet for the second one and blue light blue for the third one. The constraints are same. And also I did mention they were, all three was maximization. The question can be easily answered they can also be considered as minimization or combination of maximization and minimization, the idea remains same. In the first problem we considered objective function 1 as to be optimized which is maximized and constraints already existing were increased by 2 in the sense. The second objective came as a part and parcel of the constraint, the third objective also came as a part and parcel of the constraint and we will continue with the same logic for second problem solution, third problem solution. So, thus to make things move in the right direction these are the three objectives F_1, F_2, F_3 as shown.

So, this was F_1 in red, F_2 in violet, F_3 in blue. These are the constraints we maximize when separately, has the results F_1^* red for first objective, F_2^* violet for the second objective, F_3^* blue for the third objective. This was the solution for the first, this was the solution for the second, this means you have already seen it. So, I am not repeating it for the first objective the solution points decision points was (20, 0, 0), objective function maximum was 40 for the second objective which you are seeing now is the solution of decision variables are (0, 20, 0) maximization is 40. For the third single objective, the solution points are (0, 0, 20) and the objective value is 40. Now as I said I consider which we are solving the last day in the last means, the last lecture we consider objective function 1 which I will mark as OBJ number 1 which is red in color the constraints 1, 2, 3, 4. 1, 2, 3, 4 are there, fifth is here and the sixth and the seventh which is the additional one corresponding to objective function 2 and objective function 3 are shown in the corresponding color which is sixth in violet I am writing and another is the blue one which I am writing seventh.

They are all less than with their corresponding F_2^* and F_3^*. Why less than this is the objective function to be maximized. So, we will consider them accordingly we took different values of δ_2, δ_3 which you want to be subtracted from F_2^* and F_3^*. The combinations are given in the first two columns. Column 1 which is for δ_2 violet color column 2 δ_3 in blue color and the combinations of X's are given in the third column with F_1^{**} which is the corresponding changed values of objective function 1 are given in the first column. So, this is the so called solution of the first objective constraint second and third objectives are in the constraint. So, correspondingly we take other combinations of δ_2 and δ_3 and these are the 40 - δ_2, 40 – δ_3.

So, the δ_2 and δ_3 are given column 1, column 2 based on that I have the solution which is in column 3 and the finally, F_1^{**} in the column 4. They have been solved using simple linear programming problem. Having said that now we go into next stage or second case in the next stage I actually should be technically to be right next stage, where we are solving each objective function in the objective cases and the left out objectives are being dragged into the constraints. So, this is the formulation of objective function 2 violet in color the existing constraints I will use a green 1, 2, 3, 4 and the fifth one the sixth one is shown here and the seventh one is shown here I have used coloring scheme to make things clear and here also it is F_3^*- δ_3 and F_1^*- δ_1. So, this is δ_1 in order to avoid any confusion the coloring scheme. So, when I solve it interestingly the answers I get.

So, these δ_3 and δ_1 are the corresponding values of delta which are to be subtracted from the third objective function which is there in the constraint and the first objective function which is there in the constraint. So, that is why the coloring of blue and red. So, these are the values of 40 - here is 40 – 22, 40 – 23, 40 - 25 40 - 25, 40 – 22, 40 – 25, 40 – 27, here also 40 – 22, 40 – 23, 40 – 25, 40 – 22, 40 – 25, 40 – 27, 40 - 25. When I solve it the corresponding values X_1, X_2, X_3 are given here third column and the F_2^{**} value of which is for the second objective and double star wings because as they changing are given as here. So, if you see the values of 39, 38.5. I am reading the last column would almost be the same, but the combination of X_1, X_2, X_3 would change and I will come to that corresponding to more such combinations of δ_3 and δ_1. So, we have again these are all 40 -. So, this would be 40 – 30, 40 – 27, 40 – 30, 40 – 37, 40 - 39. On the right hand side we have related to the first objective being brought in the constraint again, 40 - 27. So, this would be again I am repeating on the right hand side. I will write it 40 – 32, 40 – 30, 40 – 37, 30 - 35, 40 - 39.

So, let us take one of these I should I would also do it for the first example on the case and the second stage. Second stage problem is maximizing single objective with other objective being brought into the constraint. So, if you consider this. So, actually this 40 – 30 is, I will write it with different with blue color is 10 and 40 - 32 = 8. So, what does 8 and 10 mean? So, 8 and 10 would basically mean this for objective function 2. This 8 and 10, this is $\leq F_3 – 30$, which is 10 and this would be $8 \leq 8$.

So, based on that we will solve it. Similarly, if I consider this, say for example, arbitrarily. Let me consider different value, 25 for example. This is the objective function 1, with objective function 2 and 3 in the constraint. This one. So, this would be violet color. 40 - 25 = 15 and for the blue one which is the third objective 40 - 25 = 15. So, if I go back to the problem this objective function 1, this ≤ 15. This is ≤ 15 and we will solve the problem for objective function 1. Now, we go into the last objective function which is the third problem under stage 2 or case 3 for stage 2.

This is the objective function which is F_3 which we all know. So, I will just write OBJ objective 3, constraints this is 1, we know already know this is 2, known to us. 3, 4. This is 5 and the red one and the blue one which are corresponding to objective function this is, again my mistake , this should be 1, which are corresponding to objective function 1 and 2 are as follows. So, this is the sixth one and this violet one is the seventh one. So, with this I solve the problem. Again I take λ_1, λ_2 as the part to be subtracted from F_1^* and F_2^*. So, these values would be I am again writing for (t-22) for (t-23) for (t-25) for (t-25) for (t-22) for (t-25) (t-27). If I come to the second column which is related to the second objective function being portion to the constraint. Again it is (40 – 22). So, this 40 value which I am writing is just as coincidence because I framed the problem very simply such that everything can be understood (40 – 23) for the second, (40 – 25) for the third, (40 – 22) for (t – 25) for (t – 27) for (t – 25). So, if I take any one set which we will solve for example, this one first one the values will I will write it, but the values if I for all of them are given here third column the objective values which is F_3^{**} in blue is in the last column and continuing different values being subtracted from F_1^* and F_2^* which are red and violet. The corresponding values to be subtracted from 40 are 30, 27 corresponding I am not writing it 30, 32, 35, 37, 39.

So, all have to be subtracted 40 and for the other objective function which is the second one, the values to be subtracted in 40 again are these 27, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 30, 37, 35, 39. The corresponding solution when they are solved these are x_1, x_2, x_3 values and the last common are values are the F_3^{**} which is the double star which is for the third objective. Now, I am coming back here. I mark this (40 – 22) and (40 – 22) which would be 18 in both the cases. So, if I put in this equation, so this would be ≤ 18. This is the one and this would also be ≤ 18 because they have been formulated for the key. Why 18? Because they are (40 – 22), (40 – 22).

Now, where are the important and this I will try to highlight using a diagram. I add a blank sheet and draw it. So, let me draw it. This is x_3, this is x_2, this is x_1. Now, the solution if you remember was 20, 0, 0, 0, 20, 0, 0, 0, 20. So, that is basically I will draw one by one. The points are coming out to be first objective we used red. So, consider this is here is the point. Second objective was violet, third objective was basically shown in blue. So, technically the points are these. Now if I solve the problem and as given let us go one by one. The value of the first objective consider, say for example, consider is this one. So, I will use the red colour because objective function one. So, I will use 37.5. So, it is 17.5, 2.5, 0. So, 17.5, 2.5 for x_2 it will go a little bit in this direction and for x_3 0. So, it will be on the flat plane x_1, x_2 no δ movement in x_3 direction. So, this is the point one and the value if you remember I will write it here 36.5. So, let us hunt for 36.5 for objective function two solved with objective function one and three in the constraints. So, this was objective function two with objective function one and three in the constraints. For 36.5 it is it was 3.5 sorry not 2.5. For 37.5 it was 2.5 and 17.5 and 36.5 is 16.5 and 3.5. So, I was so it is 16.5, 3.5, 0. So, this is the 16.5, 3.5, 0. So, I am not considering this one is my mistake. So, that was objective function one F_1 red.

Now for objective function two which is F_2 invariant this is this value 0, 16.5, 3.5. So, 0, 16.5 it is this and 3.5. So, it will basically be here. So, this should be 16, 0 for x_1, 16.5 for x_2 and 3.5 for x_3. These are the points. This is the point and lastly when I consider x_3 to be in the other point. So, I am taking the combinations and drawing it. I am not drawing x_1, x_2, x_3 as such. I am drawing only those combinations which gives me same values for F_1, F_2, F_3. When I solve the third objective function with the first and second in the constraint for 36.5 and do this one I am taking this 3.5, 0, 16.5. 3.5, 0 for this 16.5 would be on that plane. So, first was on the bottom plane if I consider where I am standing where the axis are x_1, x_2 and then the second point which is shown in violet is on the left wall. If I am standing in a room is the left wall and the third point which I just mentioned is away from me that means between you and the participants and me. So, this is the 90 degrees and the base is there floor wall 1, wall 2 and in the floor x_3 0 because we have measured x_1 along this direction from my left hand to the right hand x_2 is from the camera towards me or from you towards me and x_3 is from if I consider the vertical light from the base going up. So, if I plot it the point so the points were basically 3.5 on x_1, x_2 is 0 and x_3 is 16.5. So, this will be somewhere here. So, the first point which is red I will mark in red because it is related to the 1 then the violet one which I will use as 2 and the blue one which I will use as 3 are for the case when the objective function values are 36.5. So, they are actually not nothing to do with Pareto point, but I am just showing obviously there will be more analysis. So, technically I can I have not solved it I can plot such points where the value of these individual functions are 36.5. So, this I should also write and I can solve them to solve find out. Now, the issue is that can we find set of answers for x_1, x_2, x_3 such that the values remain the same and we get the points according to the case where the objective functions are fixed. So, we will consider that because considering even though there are 2 minutes left 2 to 21/2 minutes left, but considering that you have to go in depth into the problem and analyze that problem separately. Now, the diagram which I have drawn the question is that can we do the same for other points? Yes we can do the for the other points if you see for F_1 I have only considered 36.5, but there are other sets of points also which we gives me and similar combinations are available for F_2^{**}. So, using this and this is 3D one is easy for us to understand we can consider different problems. Thank you very much and have a nice day.