

Multi-Criteria Decision Making and Applications
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Lecture 32

Welcome back my dear students and participants for this course titled multi criteria decision making. Very good morning, good afternoon and good evening to all of you and my good name is Raghu Nandan Sengupta from the IME department at IIT Kanpur in India. So the broader concepts which we are still continuing to discuss is multi criteria decision making, multi objective decision making which will continue in this lecture. Then go into multi attribute decision making and multi attribute utility theory. So this is the 32nd lecture out of the 60 lectures whole course which is spread over 12 weeks and as we know that each week we have 5 classes with class being for half an hour. The concept which we will cover in this 32nd lecture would be I will try obviously if there is a spillover as I said, we will go into the subsequent lectures.

We will consider the concept of Pareto optimality in 2D space, concept of Pareto optimality in 3D space and try to highlight that with examples. So the problem was rich farmer in India growing 4 crops, rice, wheat, barge, dummies, selling price, cost price given, then output per acreage for these 4 crops given and all the constraints are given. So based on that we had the revenue cost function and revenue minus cost will give you the profit remember that. Now we come to the constraints.

The constraints are if you remember 2% of the land has to be left for movement of tractor and so on and so forth. The total land is 1000 acres, 2% of that goes so you have to basically have 98% of that 1000 which can be utilized. So this is the first constraint which I will put a dark yellow tick mark. The second one was related to that due to water restriction rice cannot exceed the combined acreage of bajra and wheat where bajra and wheat are the third and the fourth. So it is a third and the fourth or let me see. No it is the second and the third, maize is the fourth. So rice cannot exceed the sum of the bajra and wheat. So it is basically $X_1 \leq X_2 + X_3$ second constraint. Third one is that wheat is the stable diet hence its production should be more than 50% of the combined of rice bajra wheat. So rice is X_1 , bajra is X_3 , maize is X_4 , wheat was X_2 . So if I consider 50% more than 50% are combined. So combined is for first, third, fourth and more than 50% of that. So it is 1.5 and X_2 is basically greater than equal to that. And we have mentioned that each crop has to be in minimum 100 acres but cannot exceed 400. So this is the fourth. So these are the four constraints.

Now let us come to the problem. The problem is we want to maximize the revenue which is true and remember we are only going to maximize the revenue we have not considered anything of the cost till now. So you want to maximize the revenues, revenue function was

if you remember $35000 X_1 + 28750 X_2 + 36000 X_3$ plus $26400 X_4$ constraints which we had considered are all put here. This is a linear programming problem with linear objective functions, linear constraints when we solve it the value of X , the X_1, X_2, X_3, X_4 correspondingly becomes as X_1 is 0, X_2 is 400, X_3 is 266 and two-third and the value of the fourth X_4 is 0. So technically the amount which is obtained for the revenues is $35000 \times 0 + 28750 \times 400$, 400 for the second variable $X_2 + 36000 \times 266 \times \frac{2}{3}$, where 266 and $\frac{2}{3}$ is the third variable and 26400×0 because the fourth variable is 0. The total functional value for the revenues comes out to be 2110 lakhs. Now let us just double check the constraints are being met so $400 + 266$ and $\frac{2}{3} \leq 980$ perfect. Then if I have the second one so this is also being met because this first variable X_1 is $0 - 400 - 263$ and $\frac{2}{3} \leq 0$.

Then similarly for the third constraint it is $- 400 + \text{a value} < 400$. So $- 400 + \text{a value} < 400 - 400 + \text{a value} \leq 4000$ and all of them if you see X_1, X_2, X_3, X_4 are between values of so technically this value of 0 and 0 should have been met but we are taking we are trying to basically maximize the functional value. When I come to the cost, now cost factor has a different implication. The cost factor implication is that we are not considering any revenues we are only concentrating on the trying to minimize the cost. So technically if I consider everything as a cost so that means 33200 is the cost factor for producing one unit or one acreage of the first variable, 35750 for the third and 27000 for the fourth. So correspondingly if I consider everything as a cost, so that means 33200 is the cost factor for producing one unit or one acreage of the first variable.

So correspondingly the values which we will put, now technically if I see if all of them have to be produced at minimum 100 so obviously the cost would be increasing in the theoretical sense. Theoretical sense across are increasing like this. I have to produce 100 acres for rice so that would be 33200. So there is a cost already. So red color means negative even though I did use red color in the other combination but I did not mention whether it is a positive or negative it is here specifically negative.

Similarly for if I produce at least 100 for the second variable or the second product it was wheat, the cost will come out with this. For the third one again 100 minimum the cost will come out to be 100 into 35750 and the last one which is X_4 it will come out to be this. So this is the total cost component. So considering if I don't produce anything is the minimum amount of cost I can have that means don't do anything. The moment you start producing this is a cost factor because I have not considered revenue as such. Generally with the value of 0,0,0,0 for all X_1, X_2, X_3, X_4 the value comes out to be 0 cost factor. Now consider the net loss function so obviously you can consider from the net loss function and the net profit function. Why I am using the concept of net loss function and the net profit function is like this. If you see the two equations which is revenues and cost separately here this factor. So I will use red for negative value.

So there are no sign here so actually there are cost so these are all negative flow and if I consider positive one as blue or let us use the green one. So these are all positive. Very interestingly if I follow for the first variable which is rice if I take the value so X_1 is common if I take the differences this will be positive I am using green color for positive.

If I consider the next variable which is wheat this is negative because 30,500 is a cost which is more than 28,750 which is the revenue. If I consider the third which is bajra it is positive because $36,000 > 35,750$ and finally if I consider the maize which is the fourth variable here 27,000 which is corresponding the cost is more than 26,400 which is the revenue. So these are positive, negative, positive, negative. So if I consider the profit function and the net profit and the less loss function the signs would be reverse. So this part of positive which is green becomes red, red becomes green based on how you are try to analyze. So obviously you will try to maximize your net profit and when you are considering the net loss you will try to basically minimize your net loss function. So if I consider the net loss as the word is written here so it will be basically the loss function minus the revenues of the cost function minus the revenues.

So it was basically positive, negative, positive, negative as given. So now this becomes net loss means the first one rather than positive now becomes negative. The second one rather than positive negative being positive, the third one rather than being positive would be negative because there is a negative sign here and finally the fourth value rather than negative being positive. So if you consider that also in any combination the values would come out to be zero because the moment you produce it will basically be a loss and furthermore all the constraints are met. If I consider the net profit function so the values are just reverse, the negative becomes positive hence I am putting a green sign, the positive becomes negative a great sign again a green sign then a red sign and the corresponding values for group C the net profit function is also zero.

If we solve the maximization of only revenues with the cost as a constraint, now this is the point if you remember we had considered that for the goal programming or the concept of multi objective problem the idea was this. I will write it here and then solve the problem accordingly. So in case you had the maximization which I will use the green color, some max of F_1 and some min of G_1 and the constraints were given as say for example H_1 , I am not writing $X \geq B_1$ then $H_2 = B_2$ and $H_3 \leq B_3$. So here what we will do is like this, we will maximize F_1 and bring this G_1 function which was the minimization one as a constraint so the G_1 function has to be minimized so that means it cannot exceed a value of say for example B_4 . This B_1, B_2, B_3 which are shown in black colors are for the already existing constraint.

This B_4 which is shown in red colors is basically the so called value which cannot be exceeded considering is a minimization problem. So if I consider a value of the cost say for example 0, consider 0 it can be considered the it can be other values also it can be say for example loss has to be ≤ 200 or 500 rupees or 5000 rupees. So based on the fact that I am taking the constraint of the minimization as in the from the objective function brought into the minimization here the problem is the revenue function is already there I will put a green mark. So these are X_1, X_2, X_3 revenue functions. The constraints are already there I am putting a violet tick mark and the minimization has been brought back into the constraint I will put a red mark so it is $33200 X_1, 30500 X_2, 35750 X_3, 27000 X_4$ and this is < 0 . So that means I am considering this as B_4 . Similarly for the less than type so less than type is basically I am considering this, this so they are all less than type and this being a combination of less than greater than because actually if I consider any X's

so the actual equation would be $400 < X_i < 100$. So if I consider the concept of less than they would go here and if I consider this so this would basically become the greater than type because X_1 is more this would go into the greater than function. So this manipulation of greater than less than equal to can be done very easily. Now when I consider this I solve it considering the constraint is 0 I get a value of 0 for all the X's which is fine.

The main question which I have not solved here but we will discuss later on for other problems is what if the value which is < 0 can be considered as say for example ≤ 500 or \leq for example 500000 and so on and so forth would we be getting answers accordingly for that. That means a minimum loss has to be there because due to some reason you have to produce due to some policy decision by the government or you have to basically think that you should produce they can be different reasons. So based on that you will consider whether you want to maximize your revenue and what is the minimum level of loss you can sustain. Now the question is can this problem be considered where you want to minimize the loss and consider the constraint also includes the maximization objective function which was there. So if I write it, it is now minimization of G_1 because G_1 was the minimization as per the concept of the nomenclature using.

You already have the existing three sets of constraints which are $H_1 \geq B_1$, $H_2 = B_2$ and $H_3 \leq B_3$ and the objective function which was there on the maximization type that comes here it was F so it becomes $F_1 \geq$ to some B_4 because profit has to be more and losses were $< B_4$ there which was shown in the red color B_4 . So in this case the problems would be which minimization would basically bring the objective function as minimization and the constraints accordingly. Now the question is that we have not solved this for the case when you are bringing the objective function as maximization in one of the constraints. The question is that how would you choose this red B_4 and green B_4 depending on the problem formation. I am using the same symbol but I am mentioning the color scheme so if you have watched the video you would understand what I mean by red and green.

Red was basically corresponding to the cost and green was corresponding to the revenue. Now technically you can solve this problem and get an intuitive start accordingly like I will mention and we will consider that later on like solve the objective function first with what we have written, solve the minimization problem as a single objective function, solve the maximization problem as a single objective function, solve them get the answer. Now when we come and trying to basically put the so called multi objective case which we have written for the case when it is max of F_1 constraints $H_1 > B_1$, $H_2 = B_2$, $H_3 > B_3$ and there is a part of $G_1 \leq B_4$. This B_4 which also will be violet color is not the exact value it would be a relationship with the single objective function which we have solved. So, say for example in single objective function obviously in this problem it was 0 the final value when you are trying to solve the minimization problem consider it was 2 lakhs minimum value.

So we will consider some value of more than 2 lakhs here in place of red B_4 solve the problem and try to check that when you consider in the constraint of one of this objective function which was already there what is the change in the first objective function which

is shown in green here, green means maximization. In the similar way when you are trying to minimize the first minimization of the cost which is G_1 and this red in color and the objective function which is initially which is the maximization comes in the constraints. So this would be F_1 is now greater than equal to that was less than equal to. So here you have already solved F_1 initially separately, as I mentioned bring that objective function maximization means it would be a little bit less than what you maximum you can obtain and solve this problem. So in one case constraints are same in one case you have this is the minimization problem I am repeating in time and again please bear with me your maximization $F_1, H_1, B_1, H_2, B_3,$ and $G_1 \leq B_4$.

So the mean idea is if you consider so called problem 1 and problem 2 which I am writing case 1 and case 2 there is a subtle difference between them how they look like. One is a maximization another minimization considering that we are including in the constraints the other part of the objective functions accordingly. Now we solve the problem considering which I mentioned when we solve the problem considering the cost minimization with the revenue in the constraint. So this what I was talking about this I spend a lot of time in discussing in the last slide so which we will highlight here. So here is a minimization one so I am putting a red underlying for this the constraints are all same so these are the constraints the constraint existing one only the new one which is being brought is basically the value of the objective function which was maximization which is marked in green.

Only thing is this value which is 211 lakhs that can be changed perturbed and based on that I can basically find out what is the minimization of the cost which can be obtained. So rather than having 211 lakhs it could have been some value of $(211 \text{ lakhs} - \delta)$ value) so δ can be very large here just δ I am using as a symbol and based on that I solved. When I solve it the values here it is 0, 0 for X_1, X_4 that is a different question so but the values which we get it is basically 400 and 266 and 2/3 for the values of X_2 and X_3 . So if you remember when we are solving the problem for the maximization problem we got that based on then that when we put the values accordingly we get the corresponding output as 217 that means = basically 217 lakhs 33,333 and 1/3. Now the question is before we wrap up this class the question is what happens if we have three objective functions already max, max, min something like that.

So let me put I will only consider one already existing constraint which I will use the red color which I will use the black color sorry. So you have max of F_1 , max of F_2 , min of G_1 , constraint was $H_1 > B_1, H_2 < B_2$ so this is = B_2 , sorry, = $H_3 < B_3$. So this was the problem so what we can do is like this solve them separately well like we solve max F_1 as a single problem without considering max F_2 and min G_2 . Then we solve max F_2 without considering max F_1 and max min G_2 and we solve separately min G_1 without considering max F_1 and max F_2 constraints remain same for all this statement which I made just now. When we want to solve the so called multi objective case it will become like this.

The objective function first remains as it is and we bring $F_2 \geq B_4$, B_4 is a value which is the idea which can get once we had solved case 2 which we had just wrote down and

erased. And this case G_1 would be \leq , so this value \leq are the minimum values for G_1 and max value for B_4 this is B_5 and this we have got the idea after solving case 3 which I have just erased. And similarly we can do the combinations accordingly I will solve the problems accordingly and with this I will end this class which is the thirty second class have a nice day and thank you very much.