# Multi-Criteria Decision Making and Applications

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**Week 07**
**Lecture 31**

Welcome back my dear students, dear participants, a very good morning, good afternoon and good evening to all the participants and students who are watching this lecture. The title of this course is multi criteria decision making and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur. So this is the thirty first lecture out of the total set of lectures which is sixty that is the whole course is spread over twelve weeks and each week we have five lectures each lecture being for half an hour. The broader set of concepts which you are still considering is multi criteria decision making, multi objective decision making, multi attribute decision making where we have been discussing time and again the difference between multi objective and multi attribute concept and then further consider multi attribute utility theory its implications how they can be utilized. The coverage for the thirty first lecture and if we spill over it will be in the thirty second lecture also which we will discuss would be further discussion about goal programming very simply. The concept of parity optimality in 2D space for linear programming and concept of parity optimality in 3D space which solved problems for a quadratic programming.

Again as I said that I assume and all of you have a good background in mathematics, statistics and operation research. So the ideas of how to program quadratic programming, linear programming, zero-one programming, then integer programming are you are all aware of that. So the concept of very simply trying to use those algorithms in the purview or in the concept of multi objective programming is being discussed and when I say multi objective I only am sticking to the bi-objective one because trying to convey the information how the parity optimality curves will look like is much easier in the two dimensional case and obviously it can be extended for the higher dimension also like three dimension, four dimension considering you have more than three or more than three decision variables and also the objective functions are also more than two. So as we were discussing this is the slide which we have discussed and based on which we ended the 30th lecture the idea was you have the maximization concept here and the minimization also is being discussed.

So that means there are two types of functions, functions can be different but there are two notions of what you want to do with these two different functions which are there. When I use the word functions it means that when you see Fi, i can be > 1, when you check Gj, j can be > 1. In one case I collect all the functions Fi's and I want to maximize them and another case similarly simultaneously I want to collect all the functions and G and try to minimize them. Now the question may immediately come up which I did touch upon in earlier class or earlier lecture but I will also highlight that these combinations which you see, I will write it accordingly, one case you can maximize when I am writing the word maximization is basically related to the first set of functions. Consider there are only two functions, $F_1$, i = 1, $G_1$, j = 1. So, let us make things simple to analyze that and then we can extend that. So, when I write max it means basically for the first function when i = 1. When I want to minimize I am using the green color and I am writing it in the next place which is in the second column which is related to j = 2. Then the other concept you can be I want to minimize the first function, so minimize first function means i = 1 and simultaneously maximize the second function. So, I will write it as let me follow the same procedure which we are using, so it is there is no confusion with what I display and what I talk and what is written on the slides.

So, when we are trying to maximize, so is maximization of $F_1$, I am not writing x, x can be in multi-dimensional case also but is a f(x) which is the decision variable and I want to minimize $G_1$. Similarly in the second case I want to minimize the function $F_1$, I want to maximize $G_1$. The other case can be I want to maximize $F_1$, I want to maximize $G_1$, the last combination is I want to minimize $F_1$, I want to minimize $G_1$, so these four combinations. And if you remember that in the space where there is objective function 1 and objective function 2 which you have drawn, so I will mark objective function $O_1$, $O_1$ can be considered is $F_1$, so mark it as $F_1$ and it is on the x-axis, it is basically $O_2$ which is $F_2$ on the y-axis and if you remember we had four different combinations, coloring scheme I will write down, so they were like this. The coloring scheme what I am writing down and I will highlight it accordingly which is written there in max $F_1$ min $G_1$ min $F_1$ max $G_1$ max $F_1$ max $G_1$ min $F_1$ min $G_1$.

So this was one, other one is I am using different colors, so in order to avoid confusion, so it should be drawn properly, so this is one and the final one I use the blue color, so the light blue color that was a little bit dark blue. So, I will mark it as A, so the curve as A is B + C, so when I am trying to consider these four combinations A, B, C, D, so let us see what is the overall idea one by one. I would not erase this graph which I had drawn A, B, C, D and I would not erase the equations also. So consider the first one A part, so in the A part when it is there technically the issue would be that we would try to and this is objective function 2 which is $F_2$, this is $F_1$ and technically for $F_2$ what we actually would be doing is that we would try to reach the corner point here, so corner point on the left top corner what does it mean, it means that I am trying to increase $F_2$ go up and decrease $F_1$ go left. So if I am considering $F_1$ and $F_2$, so $F_1$ is being minimized, so minimization color I have been using is basically green, so I will move in this direction and the maximization color I have been using is red which is in this direction. So maximization has been there for $F_2$, minimization there is for $F_1$, maximization for $F_2$ means basically G, so also let me remove that confusion also. I used $O_1$, $O_2$ because it was based on the diagrams which we had done, so I will write, so when I am using this so this is minimization of $F_1$ and maximization of $G_1$, minimization of $F_1$ and maximization of $G_1$ basically this diagram would correspond to this point, I am using the same color here as I am using here. So I am trying to minimize $F_1$ go to the left maximize $G_1$ go top. So this is the two dimension case, so obviously there is three dimension means first function F, second function G and third function H. Now do not confuse the functional form of the objective function H based on what is written on the constraints.

Constraints I have already discussed here it was basically written as two objectives with I and G can be more than one also. So H what I just mentioned is basically if there the objective function was denoted by H, so it will be in the three dimension case. When I use the $B_1$, B means B which is drawn here, so this is like this you are moving the bottom left corner going down which means $F_1$ is being minimized, $G_1$ is being minimized, if I consider both of them this is this part, both are being minimized. Now when I come to C again I have deleted it, but H can also be obtained or added as a third objective function and the diagrams would be considered accordingly. But only you have to remember that H can also be maximized and minimized.
So for two functions F and G you had four combinations max min min max max max min min then when you bring H obviously H having two combination means that now I will have basically six combinations. So one extreme mean all the maximization happening for F, G, H and the other extreme mean all minimization being have happening for F, G, H and so there are technically accordingly two for these two for these and two for these. So it will be $2 \times 2$, 4 to the 4 to the 8 sorry not 6, 8 combinations would be resulting from this. Now when I come to C which is this, so I am going to the bottom right condition. So here $G_1$ is being decreased and $F_1$ is being increased. So if I consider this $F_1$ being increased and G 1 being decreased, so it will be this combination and obviously H can be also considered as a third function. So if you remove this I will remove this color also in order to mark. So I just wrote max min for the color combination. When I finally come to the D curve which is here I have increased both of them which means increase of $F_1$, increase of $F_2$, so this would become the third part. So correspondingly I will mark which I have already done but just for A is for this, B part is for this, D part is for this and D part is for this, so using this we analyze.

Now considering that we have already discussed I will erase this and come to the constraints. Constraints I have already discussed but I will still highlight them. So the constraints are so there are one is > type as you see, the one is = and the third one is < sign. I am using different colors and the function is $H_1$, $H_2$, $H_3$ and the subscripts $K_1$, $K_2$, $K_3$ means the total number of such constraints are K in number where $K = k_1$ or $K_1$ which is for > type + means union here, $K_2$ mean for the equilibrate type again + means union and finally $K_3$ being for the < type and X is in the real line of n dimension. Now what I have explained is written here, so it will be easy for us to follow where we want to maximize $F_1$, i = 1, ……, I, while on the other hand we want to minimize $G_j$, j =1, …., J.

So here for simplicity I considered as I mentioned I and J as 1, subject to K number of constraint which I just mentioned few seconds back and they are again I am repeating greater than equal to less than and on the right hand side greater than equal to less than have been given by the constraints they are deterministic they are not stochastic that means constants. It is $B_{K1}$ for equality sign is $B_{K2}$ and the last one when it is less than sign it is $B_{K3}$ here, so here the suffix K1, K2, K3 which are written in the start bullet point which I am putting a tick mark, so small $k_1$ goes to 1, …., K1, $k_2$ goes from 1, ….., K2, $k_3$ goes from 1, ….., $K_3$ and here what I wrote the + is basically they add up to the total number of constraints. And obviously we consider the intersection of the greater than equal to less than is a null set because they are distinct and we can for simplicity we can differentiate them as constraints of the > type, < type, = type. Here X are the structural decision variables, $F_I$ and here $G_J$ are the set of objective functions and these constraints are basically the goals you cannot exceed them, cannot violate them. So $H_1$, $H_2$, $H_3$ where the number of such constraints for $H_1$ is $K_1$ in number, for $H_2$ is $K_2$ in number, for $H_3$ is $K_3$ in number.

$B_{K1}$, $B_{K2}$, $B_{K3}$ are the target values or aspiration levels associated with the respective goals. Collectively these constraints greater than plus equal to and plus the less than type are called the constraints, total constraints. So we will consider a problem and expand it accordingly. So this is a linear programming problem. Suppose one rich farmer in India wishes to grow the following four crops which are accordingly rice, wheat, bajra, maize, there are four crops. The selling price per kg of this rice, wheat, bajra and maize are respectively 28, 25, 30 and 24 rupees per kg. The cost price per acre for these four crops which are crops are again I am repeating rice, wheat, bajra and maize. The cost price like you have to use pump set to draw water, you have to pay the person who is tilling the land, use a tractor or a bullock whatever way you are trying to till the land, you have to use seeds, maybe you have to use organic ways of trying to do the farming or somebody use fertilizers whatever it is. The corresponding cost there would be many costs. The cost per acre for these four crops are respectively 33,200 for rice, 30,500 for wheat, 35,750 for bajra and maize is 27,000 respectively.

Corresponding the average output per acre, so there are different outputs, different cost also, different selling points also throughout India but I am taking a very average value. Output per acreage for these four crops rice, wheat, bajra, maize are 1250 kgs, 1150 kgs for the second one, 1200 kgs for the third one and 1100 kgs for the fourth one. The total irrigated land available with the farmer is about 1000 acres and for each crop at least 100 acres and at most 400 acres of the total irrigated land should have to be used which means for rice or wheat or bajra and maize you have to use 100 acres and cannot exceed 400 acres per the crops you have. Furthermore, due to water uses restriction total acreage of rice cannot exceed that because rice needs a huge amount of water, rice cannot exceed that of the combined acreage of wheat and bajra taken together and also as wheat consider the person is irrigating in the part of India where wheat is the staple diet. So, considering wheat is a staple diet hence its production should be more than 50% of the combined production of rice, bajra and maize because the person has to supply to the market.

Finally, for the movement of the tractor vehicles and other things you have say for example the person wants to lay down a fencing there are different type of animals like which come and destroy the irrigated land eat up these crops. So, the person has to put fence and has to use the pathway to take the tractors of the bullocks whatever consider that part of the land is not utilized for irrigation. So, here it mentions finally for movement of tractor and the vehicle etc to be used after crop harvest 2% of the total irrigated land cannot be utilized. Let us see the problem accordingly as evident the decision variables for these problems are related to the acreage to be given for rice, wheat, bajra and maize which we consider as decision variables $X_1$, $X_2$, $X_3$, $X_4$. Now there are revenue cost based on selling and there are cost also for irrigation for tilling the land and for production.

So, if I consider the corresponding revenues, so if I consider the two corresponding revenues it is the output per acreage for this four crops which I will mark are 1250, 1150, 1200, 1100 for rice, wheat, bajra, rice, wheat, bajra, maize and if $X_1$, $X_2$, $X_3$, $X_4$ are acreage for this four crops respectively and if I consider the selling price given as 25, 30, 28 sorry 28, 25, 30 and 24. So, obviously you have to multiply the corresponding factors to find out the total revenues which we are doing which is 28 for the first one which is here, 25 for the second I am again repeating it please do not mind 30 and 24. So, when I am multiplying is $28 \times 1250$ into $X_1$ which is the total acreage for the first crop rice, $25 \times 1550$ into $X_2$ for the second, $30 \times 1200$ for the third $\times$ X3 and $24 \times 1100 \times$ X4 for the fourth. So, this f(x) gives you the total revenues. Now when I come to the cost function, so if you note down the revenues are marked here in blue, cost are also blue.

So, I mark the revenue. So, now as per the concept given here the total cost price for the acreage for this four crops are given as 33, 200, 30, 500, 35, 750, 25, 7000 and per acreage. So, acreage for these crops are $X_1$, $X_2$, $X_3$, $X_4$. So, when I find out, so this is $35,000\ X_1 + 28,750 \times X_2 + 36,000 \times X_3 + 26,400 \times X_4$. So, this is the cost function and so this is if I consider G. So, now if I consider, so this was sorry sorry sorry sorry this was I just missed it, so my mistake.

So, this was basically the cost function given accordingly, my apologies. So, when I multiply them for the revenues they were coming as $35,000\ X_1 + 28,750 \times X_2 + 36,000 \times$

$X_3$ + 26,400 × $X_4$ and the corresponding equation for the cost are which I have not read I will just mention it 33,200 $X_1$ + 30,500 $X_2$ + 35,750 $X_3$ and 27,000 $X_4$. So, there would be analysis for them problem which we will do in the subsequent class. I will just show this and then continue the discussion for this lecture. Thank you very much and have a nice day. .