Multi-Criteria Decision Making and Applications
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Week 06
Lecture 30

A very good morning, good afternoon, good evening to all the participants, students for this course title multi criteria decision making which is under the NPTEL MOOC series and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur in India. So as you know we are in the sixth week of the set of classes, total number of week is 12 spread over 60 lectures and each week we have 5 classes, each class being for or lecture being for half an hour. So this is lecture number 30 which is the last lecture or the last class to complete the sixth week. The broader guidelines, broader ideas, broader concepts which we have been covering for a long time for lot of classes and lectures spread over the weeks are as follows. Multi criteria decision making, multi objective decision making which is more objective, continuous, mathematical express, easy to communicate, answers are easy to understand and which we are covering now. Then there is multi attribute decision making, multi attribute utility theory which is more qualitative subjective where trying to analyze the problems may be more intuitive in nature.

The set of coverage or the set of topics covered for this 30th lecture would be Pareto optimality, Pareto curves and some ideas about goal programming obviously we will do problems accordingly. So in the 29th lecture which was the last one you are discussing a problem where there were three decision variables x_1, x_2, x_3 and for simplicity we took them as discrete and based on that we formulated functions f_1, f_2, f_3 and we were trying to analyze that how they would be considered the concept of Pareto optimality and the idea that non-dominating concepts were being discussed we will continue with that. So again consider the same problem for easy understanding I am going to repeat it. So you have three decision variables x_1, x_2, x_3 and they are discrete $x_1 = 0, 1, 2, 3$ similarly for x_2 and also for x_3.

The subject to condition which will mark in a different color green is that sum of x_1, x_2, $x_3 = 3$ and the functions there are three functions f_1, f_2, f_3 is as follows. The first one is $x_1^2 + x_2 \times x_3$, second one is $x_2^2 + x_1 \times x_3$, third one is $x_3^2 + x_1 \times x_2$. Based on that considering the sum is 7 we have this set of values for our analysis. So the first column, second column, third column are the corresponding values of x_1, x_2 , x_3 such that the idea condition that $x_1 + x_2$, x_3, x_3 is 3 is met in all the cases add up and they all add up to exactly 3. Now based on that set which are feasible we find out f_1, f_2, f_3 as given in the last slide which were these three f_1, f_2, f_3 and they have been marked in third column, fourth column, fifth column and in the sixth column last one I have just added up the sum of f_1, f_2, f_3 and if you remember we discussed what does this yellow cell mean, the blue cell, the dark yellow cell and the green cell and we discussed in details this was for f_1, f_2.

Similarly the same table, but here now we are considering f3 and f1 again same concept of coloring dark yellow, green, blue and yellow and the analysis was almost the same thing, but I repeated in order to make it much clear. Finally where we stopped the last class was now the comparison for the same table comparison of f1 and f3 considering the coloring scheme was exactly the same dark yellow, blue, yellow, green and here on all these three examples the Pareto concepts was repeatedly mentioned by different comparisons that is number one. Number two the concept of indifference curve was also highlighted with this simple example how they move parallely and remember in this problem it was a maximization concept considering f1, f2 or f2, f3 or f1, f3 which is in this slide. The other two slides were basically corresponding to f1, f2 and f2, f3. So, all these functions were maximized.

Now where we left the last day I will continue discussing there. Same problem three variables x1, x2, x3 all are discrete and the values they can take are 0, 1, 2, 3. Some of them has to exactly 3. So, the set of values which we will consider for our analysis is given in the first column for x1, second column for x2, third column for x3 and check the sum of all these x1 plus x2 plus x3 is exactly 3. f1, f2, f3 are the functions here marked in dark blue and finally, on the right hand column the sum of f1, f2, f3 is given.

Now consider the green cell set of cells the horizontal rows. There are two rows and considering the dark on the orange one the light orange one whatever you say. Now consider we want to minimize f3 there is no more idea of maximization now in all the three examples which we consider earlier which was considering two at a time function amongst f1, f2, f3. The idea was trying to basically maximize and based on that we discussed the Pareto and the indifference curve and the movement being in the parallel direction and Pareto points also. Now consider you want to decrease f3.

So, decrease f3 basically if I consider the point 2 the value 2 for this way f2 can decrease and as an example it becomes 1. Now if I am trying to basically find out the minimum for all of them remember for f1, f2, f3 separately then minimizing f1 f3 from 2 to 1 is fine. But let us see what are the consequences because the sets which are applicable such that x1, x2, x3 all add up to 3 has to be maintained. So, f1 stays at level 2 as I am marking with red I will use a different color consider use the violet. So, this is maintained 2 remains 2, but what happens to f2? f2 rather than decreasing from 2 it increases from 2 to 4 which I am now marking in blue.

So, it was initially 2 now it becomes 4. So, what is the overall effect? Overall effect is for the value that rather than getting a sum of 6 which was here now the sum becomes 7. So, obviously it is not desirable. Now if I consider on the other hand this set. So, say consider I will erase the colors here the same analysis thing happens f3 decreases on the another way is goes to 2 to 1 here f2 remains at the same level 2 and 2, but very interestingly it is not f2 which is being affected f1 will now increase from 2 to 4.

Hence the sum of them now increases from 6 to 7. So, if I consider the 2 mark points which are green which is 7 here 7 here total value is 7 same. So, the set of points 2, 4, 1, 4, 2, 1 they collectively give me the same points. So, if I consider on a 3 D space their sum 3 D space means I am considering the values of f1, f2, f3 and then trying to plot it, but actually we will consider the answers of the decision variables separately. The value 7 remains same and if I consider the set of points which is 2, 0, 1, 0, 2, 1 collectively they can be considered as pareto points because the sum of the functions $f_1 + f_2 + f_3$ remains as 7.

So, no one is better in the sense sum is 7 for $f_1 + f_2 + f_3$ and what combination you will try to analyze the problem would depend on what is the combinations of decision variable output which is possible for you. Now consider the idea of this example where the movements are minimization again. Now you are moving from 9 to 4 and the 9 moving from 9 to 4 are given in 2 different instances 2 different examples. So, I am not going to consider the example for yellow or dark yellow or light orange whatever you say for the second one which I am putting a mark. So, the same analysis will be repeated.

So, I am not going to repeat that and for this I am not going to say anything because whatever I will now mention wood can be replicated similarly so it is this blue points. So let us consider the analysis using say for example, the violet color. So, now 9 has to decrease minimization it decreases to 4. So, obviously the sum value comes down from 7 which is fine, but interestingly see that the total sum comes down to 6 in the other case it has increased because compromising trying to bring down f_3 was fine individually, but we saw that it resulted in the overall collective sum being increasing. Now here the f_3 value is decreasing from 9 to 4 in one case and another instance again 9 to 4.

So, this is instant one case one this is case two sum going from 9 to 7 total is good sum going from 9 to 7 total is good, but on the other side see very interestingly total sum is reduced, but individually f_1 and f_2 have increased how 0 has increased in one case to one and another case it has increased to two. Similarly f_1 0 has increased to 0 to 2 another case it has increased from 0 to 1. So, the combination which are giving this value 7 are like this I will mark it here one is the decision variables decision variables which are giving me this answer for 7 is x_{10} x_{21} x_{32} another is x_{11} X_{20} x_{32}. So collectively if I consider the sum is 3 based on the constraints perfect the functional value of f_1 is now 2 and f_2 is 1 another case is f_1 is 1 f_2 is 2 the value of f_3 has decreased from 9 to 4 in both the cases sum is 7 which is fine. So, if I consider this they are technically Pareto we have been able to reduce the total value from 9 to 7 is the cases, but the combination f_3 is fixed, but f_1 f_2 have been changed that means they are trying to compromise in order to maintain the balance at a value of 7 which is sum of f_1, f_2, f_3.

So, the analysis which I did or which I mentioned would all be repeated if I did these. So, here the value of say for example, 9 can decrease to 7 can decrease to 7 in both the cases total sum 9 decreasing to 7 which is perfect f_3 value is maintaining its position it is in one case is increasing its position that means for trying to bring down the total sum of f_1, f_2, f_3 to from 9 to 7 it is increased from 0 to 4. So, it was initially has decreased from 9 to 4 here it is increased from 0 to 4 and if I consider the values. So, the values are here f_2 is 0 it has it would either increase to 2 or increase to 1 and f_1 was initially 9 it will basically decrease to 1 or decrease to 2 that means in all these case 3 instances the sum is being decreased total sum $f_1 + f_2 + f_3$ from 9 to 7 in two different cases. So, I will consider this as case 1 this as case 2.

So, but individually if I see f_3 has increased from 0 to 4 in both the cases case 1 case 2 f_2 has increased from both the cases individually separately from 0 to 1 or 0 to 2, but f_1 which was initially 9 has decreased in both the cases substantially to 2 or 1 such that the sum of $f_1 + f_2 + f_3$ is being maintained at 7. So, in this case if I consider 7 7 where the sum of $f_1 + f_2 + f_3$ is 7 it is parity optimal the combination of points which is resulting this for x_1 x_2 x_3 are these 0 1 2 then 1 0 2 which is fine and the set of points which are optimal for the concept of f_1, f_2, f_3 are 2 1 4 and 1 2 4. So, based on that I can also find a functional form of the decision variables of f_1, f_2, f_3 which is resulting in this optimum solution. So, we will consider the concept of goal programming which is basically we did discuss, but I will highlighting it further on discussed in the sense that we mentioned and briefly and I said that I will be visiting that with the problems. Is a branch of multi criteria decision making or MCDA which was introduced in 1962 in the deployment of the antenna system for the Saturn Apollo moon landing system.

Goal programming as a technique is an useful and effectual tool for modeling, finding solutions and analysis of mathematical models that involve multiple and conflicting goals and objective type of models where as you have seen in the problems many of this compromises have to be understood in order to basically maintain the level of the collective goal. In many linear programming problem obviously we will consider later on for non-linear programming also objectives may not be single, but multiple objectives plus they can be soft as well as rigid constraints based on which we will try to analyze the problems. To take care of multi objective models one has to measure the goodness of the solution how good it is. So, whether it is able to meet all the criteria's whether it is a compromise solution whether it is optimal effective so that is the main answer. Minimizing the profit to the or maximize the loss if we consider they can be situations where rather than trying to basically when I mentioned the concept of minimizing the profit and maximize the loss would be the cases where say for example, I minimizing the profit in the sense I have to run the factory, but the overall sales of the product is very low.

So, I will try to basically maintain the minimum production such that my minimum profit is maintained because if I stop production so obviously all the salaries all the electricity bill the pension the gratuity whatever is to be paid the security the rent everything has to be paid as per the norms. So, if I maintain a minimum amount of production I may be able to reduce my loss such that in some way I am trying to maintain the minimum level of profit which is required. Any more production may either I stop my production which means this total loss or it may happen in many of the cases for any further production levels because if the goods are being not sold there may be instances that they may be piled up inventory which means for more number of production because I am paying labor cost on an extra hour basis like giving over time trying to basically buy raw materials trying to utilize extra electricity that may entail that the situation may be become much more worse such that rather than maintaining that minimum profit it now became maybe become a loss. To take care of multi objective models one has to measure the goodness of the solution that is to transfer the objectives into goals which are achievable by means of establishing an aspiration level or a target level. Consider objective function is of maximization or minimizing type then the converted model will result in three states of solutions which are less than optimal, more than optimal or exactly equal to the optimal value.

So, what you want to achieve is basically optimal solution but what is doable under this present circumstances under the practical situation have to be considered. It is evident then in the former two cases one has to minimize the undesired deviation more than optimal reduce it. Reduce in the sense that if your optimum value is I will come to that if your optimum value is 10 and if produced more which means that producing more obviously you would not be achieving the optimum value you exceeded you produced more which means the cost price is more even if the selling price is there but the cost would be increasing at a increasing rate. Such that selling more and more obviously you will be going into the negative region. So, you will try to bring it down to the optimum value while if you produce less it may be the case that any delta amount or extra amount of production may increase your profit the difference between the revenues of the cost and bring it to the optimum level.

So, in which way you want to operate you have to basically understand. So, let me continue reading it. It is evident that in the former two cases one has to minimize the undesired deviation which can be tackled following the three forms of geometric goal programming. One is the Archimedean concept also known as main sum or weighted concept where one wants to seek to minimize the weighted sums of all the unwanted absolute deviation from the goals. So, there are deviation the goals you want to minimize them and you give weights depending on what are the level of negative or positive importance you want to give to this deviations.

Chebyshev's concept also known as the min max one where one desires to minimize the worst or maximum of the unwanted goal deviation. So, if you remember the Chebyshev's inequality or the Chebyshev's bounds which were considered under the safety first norms when we are discussing utility theory. Non Archimedean concepts known as the preemptive priority or the lexicographic norm where one seeks the minimum more precisely the lexicographic minimum of an ordered vector of the unwanted goal such that you want to basically reduce a loss or try to increase your profit to the maximum level. When I am using the word profit and loss it is in very general sense positive something negative is a loss something positive is a profit. Linear programming as well as Archimedean and Chebyshev's concept may also be considered as special cases of non Archimedean goal programming ideas.

So, I will first consider the general concepts of the ideas of goal programming and then consider slowly different type of examples. The problem is trying to maximize a function f_1 or f_i which I will mark in blue and there I want to also maximize g_j where i and j are different. x is the set of decision variables which is of n dimension in the n space and the corresponding constraints are divided into three sets. One is greater than type which I will mark with greater than type which I will put one tick mark in red. The second one is equal to which I will put a two tick mark in red and third one is less than type which I will put three tick mark in red.

So, what is the constraint? Constraints are basically of less than type is k_1 = 2 is k_2 in number less than type is k_3 in number such that k_1 is < type + k_2 + k_3 = K, total number of constraints which are there. Now there can be instances where greater than time does not exist there is only equality and less than type. There can be instances when equality does not exist only greater than type less than type. And another case can be where when you are considering at least two combinations are there when less than type is not there only greater than any equality is there. The simplest would be where only greater than type is there equality to less than type is not there.

Only equality is there greater than type less than type is not there and finally less than type is there greater than inequality is not there. Now, what are the constraints? Constraints if you see, we have considered in a very simple way. The k_1 number of > r, h_1 f(h_1, $k_1(x)$) > bk_1, bk_1 is basically a value, deterministic one. Equilibrium type is h_2 again a function $h_2(k_2(x))$ = bk_2 again bk_2 in the set of constants. So, and finally, $h_3(k_3(x))$, x is the decision variable $\leq bk_3$ again bk_3 are constant.

So, k_1 can be 2 in number, k_2 can be 3 in number, k_3 can be 4 in number. So, it is 2, 3, 4 is basically you have 2 + 3 + 4 which is 9 number of decision variables. We will consider the concept of goal programming later on with examples as we proceed. So, with this I will end this 30th lecture and discuss further on about the ideas of goal programming.

Thank you very much and have a nice day. Thank you.