Multi-Criteria Decision Making and Applications
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Lecture 03

Welcome back my dear students, dear friends and participants. This is the course title multi criteria decision making and application and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur in India. So, this is the third slide or the third lecture we are going to do for this course and if you remember this is a 12 week course each week we will have 5 lectures each being for half an hour. So, as you see this is the slide number 3 out of the 60 such slides or lectures we will have. So, what is the coverage? The coverage if you remember in the second lecture which was slide number 2, we just finished example 4, but there is one important point which I thought I should discuss for example 4 and based after that we will cover 5, 6 and 7 which is also under the part of introduction for this multi criteria decision making and application MOOC course. So, if you remember in the example of 4 which we were discussing I did mention very in depth that all the constraints which we are considering on the revenue cost function all were being met.

Having said that when we went into the second objective function which was the cost function again if you see the slides it was mentioned there in the slide, but I did not mention it verbally that also all the constraints will be met. Similarly, they have to be met. Similarly, when I come to the net profit function which is the difference between the revenues and the cost all the constraints will be met and here I would like to draw the attention of all the participants and the students and here it is. So, if you see and if you put in the constraints accordingly in the problem that few very interesting observations are noticed for the net profit function.

If you see there would be unutilized part of and if you remember there were 3 raw materials and 2 products. So, if I see that the propylene which is one of the raw materials would be 180 tons unutilized that if you have procured for the month you would not be utilizing that. For the industrial chemical procured again unutilized will be 1040 total production unutilized will be 140 tons and if I see the output which is industrial rubber and industrial plastic which we have denoted by the variables x_1 and x_2 unutilized will be 15 tons and if you remember maximum capacity for each of this product 1 product 2 which is X 1 and X 2 is 175. So, 15 tons would be unutilized for product 1 and 175 would be the capacity of unutilized for product 2 and why product 2 is totally unutilized if you remember the value for decision variable X 2 was 0 why because in the net profit function the parameter based on which we are multiplying the output for X 2 find out the net profit was negative. So, any value of X 2 which was positive would bring down the net profit hence the total unutilization concept would be there for the second variable which is X 2.

So, all the concentration would go for product 1 which is X 1 which is industrial rubber. Now, if I consider the first three points of unutilization of propylene for industrial chemical and the total capacity only if I consider in the first one which I am denoting say for example, by numeric #01 and numeric #02 as I mentioned, if you procured at the beginning of the month they would be unutilized and can be utilized in the next month which is fair, but for the time being if I only concentrate on the first month as such unutilization of production capacity may not be a direct issue which is fine in the sense that obviously, electricity cost is saved then maintenance cost is saved so on and so, forth. Obviously, there is a cost you have to pay to the labourers to the workmen because they are on a monthly basis, but what is more important is to note that if you have procured the raw materials then 180 tons of propylene which I will try to highlight using a different colour, let me use the blue one. So, this 180 tons of propylene and 1040 tons of industrial products they will be as locked up inventory and may add up to the cost. So, obviously these ideas have to be taken into consideration when you solve different type of multi criteria decision making as such in very simple single objective problems also, but they would become much more relevant in the MCDM methodologies.

Let us consider a fifth example so this is in a total different area of portfolio optimization. So in the area or the idea of portfolio optimization is basically we need to optimize two things. So till now in the fourth problem we have only considered single objective: revenues to be increased cost to be decreased and net profit to be increased, all separately. Now, let us consider the problem where we have two objectives: one is maximize the portfolios return because more it is better for the investment and the second point is minimize the portfolio risk. So, return and risk generally would be considered in parlance of portfolio optimization investment analysis are considered as the first moment which is expected value second moment which is variance risk is basically the variance and they would be different type of practical constraints also and we will see that soon and try to analyze that.

Consider we are taking the scripts from NSE 50 for the time frame of first of September 2023 to 30th September 2023 and the scripts I am only taking few of them not all they are Asian Paints, Nestle India, Larsen Toubro (L&T), State Bank of India, Infosys, ONGC, as considered from NSE and they can be considered from BSE also from other stocks from other countries also but I have taken this simple example and if I consider the returns not the prices based on the closing prices. So, if you open up the and check the prices there is an opening price for the day closing price of the day maximum minimum so on and so forth. So, I am only considering the opening, closing and also there is a closing and a closing adjusted I am just simply taking the closing price based on that I use the formula for the returns which is returns can basically can be total returns and which is basically can be return concept can be denoted by capital R and small r, rate of return small r, total return capital R. They have implications we will come to that later on. If I consider the rate of return, which is basically for, in general, will be [price 2 for day 2 - price 1 for day 1] / [price 1 for day 1]. Day 1, day 2 are the prices we are considering. It can be seconds also, minutes also, hours also, but I am considering the day unit. So, based on the concept of the prices and how the returns are utilized considering the logarithmic concept, the for the returns of the stocks are considered as ln[(price of day 2) / (price of day 1)].

So, basically R rate of return will be Napierian log, ln[P2/P1], which you have considered here in the formula and based on that for all the 6 stocks which we have, if you calculate the returns I have the data was for 1 month. So, I have not noted down I have only given the actual result. So, based on that if I utilize the problem and what are the two optimization problems. The first optimization problem by the way, the returns which are given I will denote as R1 to R6, which is not given in the slides but I am just noting down. So, R1 is for stock 1 R6 is the stock 6, the six names which are given.

So, if based on the 30 days we have the returns for 29, because it is ln[P2/P1]. So, based on that if I find out r_i, i is 1 to 6, if I find out the average. it will be given by total number of days of readings I have, average of that. So, this is the average return for each stock and based on that I can find out the variance also. So, this is the risk which I was mentioning. So, utilizing the concept of return and risk. So, return is here. So, risk is here return was here. So, I am just. So, this is the first one which I am marking as 1 and the second one which is risk I utilize the data which is given. Based on that I will try to find out the two objective functions and the constraints.

I will try to keep the constraint as simple as possible. What are the objective functions? The first one which I mentioned is basically related to minimizing the risk of the portfolio. Portfolio being formed by the 6 different scripts. So, I want to minimize the variance and if I just formulate that in a simple concept of portfolio optimization, the variance for the portfolio is given by double summation for i to j because variance is basically if you see it will be in the matrix concept of variance covariance matrix.

So, w_i and w_j what are w_i and w_j, I will come to that. σ_{ij} is the covariance. Covariance basically considers three concepts ideas of standard deviation of i, standard deviation of j and the correlation values. So, based on the data which I have I can find out the standard deviation of all the 6 different scripts the correlation between them and obviously the returns. Now, these ways which I mentioned few seconds back are the decision variables which you want to find out which in all other problems we are denoted by x_1, x_2 and accordingly.

So, those ways are to be found out because these ways will give us for the first optimization problem which I am again highlighting such that it will minimize the overall risk of the variance of the portfolio. Now, in this problem I made it very simple in the sense if you see the weights are basically the total amount of investment proportion wise I will make in the 6 different scripts. So, if I have, 100 rupees say for example, and I invest 20 rupees in the first one. So, the weights for the first one will be given by 20/100. Similarly, if I have a third and I invest 5 rupees in the second one which is 5/100.

Similarly, if I have the third one if I invest 10 rupees out of 100 it is 10 /100. So, the first one which is 20 /100 is w_1, second one which is 5/100 is w_2, third one which is given here as 10/100 is w_3 and obviously you will have w_4, w_5, w_6 for this problem and very logically all the sum of the weight should add up to 1. So, this is what I am considering as the constraint in this first objective function minimization problem. So, the sum of the weight is 1 and obviously the weights considering that we are not going to consider any short selling. This idea of short selling I am just mentioning it is not a finance course, so, I am not going to go into that. So, considering the ideas of short selling is not there the weights will all be between 0 and 1 as mentioned. So, if I consider weights sum of 1 weights are between 0 and 1 and weights are all between 0 and 1 inclusive and is a continuous set of variables. So, if I solve the problem for the first one the weights come out to be this for the six scripts and for convenience if you add up the weights will all add up to 1. Point one. The second one when I put these weights in back into the formula here, based on the standard deviation of i, standard deviation of j, which is square root of variances, for all the six scripts and also utilize the correlation coefficient then the overall portfolio variance comes out to be the value as given here.

So, this is the minimum variance which I can have for investing in this six different type of scripts which are there which was mentioned there in the other slides we have taken from NSE and based on that we can find out what is the overall risk. Now, if you remember, I have mentioned that in the problems for portfolio optimization the idea is always to reduce the variance (risk) and increase the return. So, obviously, in the first optimization problem the second idea of trying to increase the idea of the returns is not considered. So, if I want to find out what is the return for the minimum value of variance obviously, those values of weights which were found out could be replaced in trying to find out the overall portfolio returns. The portfolio return formula I am just writing it will be w_i*, why I am using the star (*) because the stars (*) are basically the weights which I found out multiplied by r_i^{--}, r_i^{--} is from where from this formula which I found out I can find it out and note down what is the return for the minimum risk. This is not calculated here.

Now, having said that let us consider the second problem which is optimization of the return and here we are trying to maximize the return not minimize the return here risk was to be minimized return was to be maximized. Again based on the concept the weights add up to 1, weights are always between 0 and 1 if I put it in the formula and solve it the simple in optimization 1 and by the way this is of 6 dimensions. so, it is not possible for us to visualize. If you put it in the problem solver we will get the weights as only for 1 it is total investment others are 0 and if I find it out the total returns comes out to be 0.0059. Again in this case, if I want to find out the corresponding risk for that optimum portfolio where we are trying to maximize return the variance for the second case, which I will denote as 2 and this was basically 1, where we wanted to minimize the risk. the risk for the second case can be found out utilizing w_i*, wj*, σ_{ij} and what is w_i* and wj*, would be found out from here and what is $\sigma_{ij\ j}$, $\sigma_{ij\ j}$ is basically if you know the formula, σ_{ij}, which is the covariance is basically correlation standard division 1, standard division 2 i and j. Sigma from this sigma from this correlation coefficient formula I have not given. So, obviously this the idea which we discussed in this problem is basically to optimize separate two different objective function one was basically for maximization one was for minimization, similar way for trying to increase the revenue decrease the cost. So, obviously the idea would be coming here that can we take a combination of that as we did for the net profit? Yes we can, but the idea for multi criteria decision making would be much more intuitive and much more practical in the sense rather than taking a very simple convex combination we will consider different ways of trying to optimize a multi criteria decision making problem. Now, this sixth example which we were little bit more longer one. So, considering that we are in the third lecture out of the 61, 60 such lectures we will start and slowly proceed as we consider this example in details.

So, now consider it is much more mathematical in nature, but why I have taking this one because it is very easy for us to visualize. It is a two dimensional problem. The problem is that we want to basically consider two functions and there are very simple problems in trying to depict this functions as simple circles. So, the variables are, if it is a sphere obviously it will be in a three dimension of x_1, x_2, x_3, the first circle which I am denoting by red color, why I will come to that later. So, the first the function is given by x_1 - 15 and x_2 – 13, both being whole square which is 15 and 30 are the centers for the first circle which I will denote by f_1. The second objective function which is the second circle which is denoted by f_2 is given by $(x – 30)^2 + (x\ 2 – 15)^2$.

So, it means the center for the second circle has changed from the first. So, first one was 15 30 now it has changed to 30 15. Now, the idea would be if there was no constraint you can increase and find out the maximum value. So, it would be infinite. Now, you want to basically minimize them separately maximize them separately and then take the combination of trying to minimize both of them together maximize both of them together.

So, that means in this whole problem we will basically concentrate on four parts maximizing 1 which is f_1 maximizing f_2 minimizing f_1 minimizing f_2 and also minimizing the combination of f_1 f_2 maximizing the combination of f_1 f_2. So, let us proceed one by one that is why I said the problem 5 would be as 6 as you see sorry my apologies the 6 which you see may basically spill over to the fourth lecture also. Now, what are the constraints? I made the constraints very simple. Why? If I consider non-linear constraints the problem can be solved, but trying to visualize them may be difficult for us. So, I have taken very simple linear constraints. By the way if you consider for all the problems till the fourth one, the first one were more of the intuitive field for the one where the South African country was combining different type of three inputs to make two products all the objective functions were linear all the constraints were linear. In the case when we consider the optimization problem for the finance one objective was linear constraints were linear second objective where it was basically to do with the variance the objective function was quadratic and the constraints were linear.

Now, here we are again considering two objectives, but both are non-linear constraints are linear. So, obviously we expand the ideas as we consider non-linear objective functions as well as non-linear constraints also. So, for problem 6 it is only non-linear objective functions and linear constraints. What are the constraints I will mark them and I will basically definitely highlight them using very simple diagram. It will be much more intuitive and easy for me to explain. So, the constraints are $x_1 + x_2 > 60$, $x_1 + x_2 < 90$, x_1 is between 15 and 90 and x_2 is between 15 and 90.

So, if you see all of them are straight lines: four constraints. Now, based on the objective function which are given colored one which I said the first one was red in color and the second one which you can see in the slide, but I did not mention is blue in color and why I said I will come to that. If I consider the single objective function when it is a minimization problem I want to find out the minimum value for f_1 based on the sets of constraints which are given. Similarly, I can have a second objective function again minimization for the f_2 based again on the same set of constraints as it is there and constraints if you remember are all linear. Now, let us spend some time in the overall diagram.

So, without going into the circles which are drawn first let us consider the constraints as such. So, if I consider the constraints so this is the Cartesian co-ordinate I am measuring x_1 along the X direction x_2 along the Y direction and what are the constraints I will mark them. I will mark them with dark blue. So, x_1 being >15 so obviously it will mean on the right hand side. So, the line is this one on the right it will go. The second one related to x_1 was $x_1 < 90$ so it will go on to the left hand side means on to the left.

So, these are based on x_1, x_2 as I have highlighted here. If I come to the for x_2 part, one is basically x_2 >15, so all the points which are greater than this and another part is $x_2 < 90$ which is this part. So, the overall set of constraints based on only the linear part > 15 < 90 for both x_1 and x_2, I will basically have the region which I will highlight using the panically this is the area square I have not finished yet. This is the rectangle. But now I have another two constraints and one is x_1 and $x_2 > 60$; x_1 and $x_2 < 90$. So, if I consider that, x_1 and $x_2 < 90$ is this one which I should highlight in a better color.

So, this is the line less, this part and if I consider the part related to x_1 and $x_2 > 60$ this is the part. So, if I consider the whole area, I use the yellow color to mark the total common area is this one. This is the area. So we have to basically find out the minimum point for function f_1 and f_2. So, I would only highlight f_1 and f_2 let us concentrate on the red circle that is why I mention the color and the blue circle. The red circle has center given, which is here which is 15 and 30 and x_2 which is blue one, f_2 is blue one is given by the coordinate centers are 30 and 15.

So, if I draw the concentric circles like ripples going out, if I concentrate only for the first one it will touch that line the feasible space which is given by the yellow line at some point which will find it out and keep increasing till it reaches for the maximum part. Maximum part I am not coming, but I am just mentioning it will reach it and beyond that it will go. Similarly, the blue one will also go here simply try to increase like a concentric circle touch that line x_1, $x_2 > 60$ and then increase. But if I consider the combination of that that will be the interesting part we will consider later on. So, with this I will close the third class I as I if you remember I mentioned that this discussion for the sixth example would be longer one it will give you a much more better and in depth idea about trying to visualize.

Because before solving it should basically able to visualize the ideas that how in a two dimension case and obviously, we will try to do that for the three dimension case and try to solve problems accordingly. Have a nice day and thank you very much for your attention. Thank you.