

**Multi-Criteria Decision Making and Applications**  
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**Week 06**  
**Lecture 28**

Welcome back my dear friends and dear students participants for this course titled multi criteria decision making under the NPTEL MOOC series and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur in India. As you know this is the total duration of 60 lectures spread over 12 weeks and we are in the sixth week and each week we have five classes each class being for half an hour. This is lecture number 28 out of the 60 and the broader umbrella under which or the broader set of topics under which we are discussing these lectures at least for which has been going on for a long time for quite a few weeks. And the topics are multi criteria decision making, multi objective decision making which we are considering and we will continue discussing different type of problems. Multi attribute decision making which will come later on where attributes characteristics would be important and then combined with multi attribute utility theory and then consider those type of problems where different type of quality decision making are considered. The set of topics which we will consider is the scales of measurement which are under nominal scale, ordinary scale, interval scale, ratio scale and a big part would be about goal programming and how you can formulate different type of problems both linear as well as quadratic programming and see that how bi objective linear and bi objective quadratic programming can be solved in using very simple concepts.

Obviously, I would not discuss the algorithms based on that, but I will definitely discuss the results and all the coding have been done in our language and I will discuss the results in details. Continuing about multiple attribute decision making which we will discuss briefly and then again switch over to multi objective decision making and then again revisit the idea of multiple attribute decision making later on. As already mentioned each human being as a decision maker has before him or her a set of alternative choices, goals, decisions or equilibrium states to aim for and such that maintain the states at the respective desired equilibrium conditions such that the decision maker or the human being who is taking the decision gets the best benefit. Some alternatives, choices, goals, decisions are categorized by the fact that more is the better.

Obviously, more amount of utility, more amount of benefit is better, but obviously when the problem is trying to minimize like amount of pollution being the output when you are producing some product that has to be minimized, loss has to be minimized, cost of buying a house has to be minimized, maintenance of a car has to be minimized, cost for education has to be minimized. So, all the examples which I said about the cost of the house, maintenance of the car, cost for education would be respective to that problem where do

you want to consider respectively that you are buying a house, buying an apartment or you are buying a car or you are basically investing for higher education. Goal programming which will come later is basically the idea that goal seeking behavior is the action and adjustment levels are aimed at through proper adjustment by the decision maker to maintain this equilibrium such that he or she gets the best benefit what he or she thinks is best for him or her. Two concepts that define goal seeking behavior are the satisfying model and the compromise solutions example being goal programming which I mentioned. We will discuss in goal programming in details later on.

As multiple attribute decision making does not try to compute an optimum solution but rather determine either a ranking system of all the alternatives which are there in front of him or her. So if you remember we had discussed that I will use a different color. So the human being of the decision maker has a set  $A$  and there are alternatives marked  $A_1$  to  $A_M$ . These are the alternatives and the criteria are marked as  $C$  again a vector  $C_1$  to  $C_N$ . So as I said rather than ranking determine the optimum solution the ranking procedure or the alternatives are based on set of the criteria. So you would basically rank I am writing them so the ranking has to be done amongst them  $A_1, A_2, \dots, A_{M-1}, A_M$  and this is to be analyzed based on the criteria  $C_1$  to  $C_N$ .

So to try to find the optimum actions amongst the alternatives to arrive at that ranking. In accomplishing the ranking various ranking procedures are available and we will consider them one at a time later on. Now we will come to the scaling system and why scaling in system is important we will study that. So there are different type of scaling system the first being the nominal scale according to the nominal scale it is generally used to measure qualitative data information and the scaling system differentiates item based on their qualitative characteristics and qualitative classification. Like if I want to understand that whether the style of the car is good, it is super it is not that good so basically qualitative statement, qualitative characteristics attribute.

If I want to understand that how safe is that apartment in the locality or how safe is the locality where you want to buy a house that is basically a scaling system or you want to consider say for example the university or the college where you want to apply and how good the alumni network is such that they help their juniors either in their summer internship either in their job or final placement. So that would can also be classified as a qualitative statement. But say for example when you are trying to optimize or find out that what would be the profit what would be the revenue what would be the cost so these are quantitative values. Generally for the nominal scale numbers may be used to represent the characteristics of the items but these numbers do not convey any concept of numerical value relationship. To give an example say for example somebody is doing a marketing survey and the marketing survey is about the quality of a product a company has sold or

the services the bank is given.

So obviously customers would be asked to rank them on a scaling of 0 to 5, 0 being the worst and 5 being the best. Now this scaling system 0 to 5 which I mentioned like 0 as I mentioned few seconds back is the worst then a slightly better one level is 1 then 2, 3, 4 and 5 is the best. Now rather they have a scaling system from 0 to 1 for the numeric one for scaling or qualitatively and then quantitatively qualifying the services could also have been done on a scaling system of 0 to 100 like 0 being the worst, 10 being the next best then 20, 30, 40, 50, 60, 70, 80, 90, 100. Furthermore the scaling system could also have been done on a scaling system of 11, 12, 13, 14. So there is no sacrosanct concept of the numbering system which is being mapped on to the scaling system.

So that is why he says that it does not convey any numerical value relationship but only gives a relative position or the relative value. Now if you remember when we are doing the concept of utility theory, so there was one example where two teams X and Y were being analyzed based on the number of wins, draws and loss and they were case 1, case 2 of the ranking system based on the score. So say for example if a win gives the team 3 points, a draw gives the point of 2 and a loss gives the 0 and in case 2 say for example a win gives 5, a draw gives 2 and loss gives 0 then obviously we saw the ranking system based on this two different concepts of case 1, case 2 point system. The teams can be analyzed at different positions. Example of nominal scale can be the classification like gender, male, female, nationality.

He or she can be Indian, he or she can be from Bangladesh, he or she can be from Japan, he or she can be from the US. It can be blood group also A, A+, O so on and so forth. Nominal scales only classifies items into groups like all Indians in one group, all people from Japan in another group, all people who have a blood group of O+, all people with A+ blood group in another group. So classifies items into groups and the mathematical operations which is applicable for nominal scale is not equal to. So either you belong to the group or do not belong to the group.

While mode which is basically the value which divides the overall distribution into two equal proportions considering the probability on to the left hand side of the mode or the right hand side of the mode is 50% each is the only central tendency which can be used when you are using the concept of nominal scale. Next is the ordinary scale. Ordinal scale allows for rank orders using which data can be sorted, ranked without any relative degree of difference between them. So you can do the ranking and the differences or the degrees of differences between them would not be of relatively they would not have any significance. So relatively when you are trying to compare that would not come.

I will come to an example within few minutes. This scaling system only allows ranking from the highest to the lowest without any absolute value. So you are not trying to take the absolute value, you only rank them from the max to the min. The real difference between adjacent ranks may not be equal. Example one can rank students in the class of statistics course but the difference between the 7th and the 8th student may not be same as that between the 21st and the 22nd student.

So this is not an example this is a statement I will come to an example later on. Now when you are considering the difference between the marks between the 1st and the 2nd student or between the 2nd and the 3rd student the marking whatever scale you use the difference between first second and second third may not be equal but their relative positions are there. Examples of ordinary scale are somebody sick versus healthy. So you basically analyze that whether it is right or wrong, whether you agree or disagree. IQ scale is basically based on the concept of ordinary scale, emotional coefficient EQ scale is also based on the concept of ordinary scale.

Mathematical operations like  $>$ ,  $<$  are applicable for the ordinary scale while the actual one of the central tendencies which is the median quartiles is the central tendency which is considered under the ordinary scale. In the first scale nominal one it was the mode here is the median. My apologies mode would be the value which is basically maximum I am extremely sorry for that. So mode with the value which is the max and when I am considering the concept of median is basically which divides the overall distribution into two equal halves.

I am sorry for that. Coming back to the interval scale when the degree of difference but not the ratio between the items is applicable then the interval scale or the scale variable should be used. Examples being the temperature scale which is centigrade, degree centigrade, Fahrenheit which is degree Fahrenheit, location in the Cartesian coordinate which what is the x point, what is the y point. The direction measured in degrees from true north whether you are going to northeast direction or west direction so these would be considered under the interval scale. Further continuing the interval scale one should remember that for this scale ratios are not the same though it is possible to express the concept of the ratio differences. When you are taking the temperature and when you consider that centigrade and Fahrenheit, so in centigrade and Fahrenheit we know centigrade  $0^{\circ}$ ,  $100^{\circ}$  and when you consider the Fahrenheit scale it is 32, 212.

So the difference which is  $100 - 0$  and there  $212 - 32$  which is basically 180. So they have been scaled in such a way that if you remember the conversion scale from centigrade to Fahrenheit the formula which is basically used is one of the concept based on which the interval scale concept is being utilized to use those values. Mathematical statistical

measures like mean, median, mode and deviations can be used while the mathematical operations of plus and minus are allowed under the concept of interval scale. Coming to the ratio scale, measurement is the estimation of the ratio between a magnitude of a continuous quantity and an unit magnitude of the same kind is used. This concept can be expressed by the ratio scale.

A zero scale has a meaningful, unique and non-arbitrary zero value. In physical sciences and engineering almost all measures are done using the ratio scales. So you have basically a unique and non-arbitrary, no confusion zero value based on which you measure. So measurements like length you start from in centimeter you start from zero. So whatever is zero that is based on that you can measure anything when you consider in feet or in inch whatever it is.

The zero scale is based such that conversion between one scale to the other from inches to centimeters, centimeters to feet is possible. Consider the measurement of weight, pound and kgs. So zero value would be unique non-arbitrary without any confusion such that conversion from one measuring scale to the other is possible. Few examples which I mentioned in the ratio scale are mass, length, duration, angle in  $\theta$ , energy level, joules, electrical charge, coulomb and so on and so forth. Other examples being temperature measurement using the Kelvin scale because Kelvin has a zero fixed value.

Atmospheric mean, geometric mean, arithmetic mean are ratio scales used for measuring the central tendency. Well apart from variance coefficient of variance is also a concept which can be used under the concept of ratio scale. The mathematical operations which are allowed for the ratio scales are  $\times$  and  $\div$  in the other scale just the last one was plus and minus. If you see this  $+$  and  $-$  are the reciprocal of each other. If you consider multiplication and division they are reciprocal of each other or consider inverse.

Now to give the analysis of the mean, median, mode I have considered few of the distributions in order to basically give an understanding about the scaling concept. So this is the histogram and the theoretical distribution being drawn for the normal distribution with a mean of 3 and a variance of 2. This is the mean value and the variance is this. So definitely use a different color if possible. So this is the variance and this is the normal distribution.

So obviously for different mean and standard deviation you can have different distributions for the normal case only plotted. So say for example, the standard normal deviate is used. So this is the value of 0 which is the mean value 0 and the mean value of 0 and the standard deviate has a distribution as a standard deviation of 1. So let me draw it.

It will look like this. So with a standard deviation value being of 1. This is the histogram and theoretical distribution for the exponential one with a value of 2 and  $\lambda$  as 5. So it is not  $a$ , if it is  $a = 0$  so obviously the distribution will start from 0 for  $x = 0$  but it starts from 2. So the distribution would look like this and we know its mean value in this case would be calculated, the variance would remain fixed if  $a$  is positive or 0, variability remains fixed.

Only mean value will be shifting. On to the right if  $a$  is positive and correspondingly a value would be from 0 to technically infinity. This is the histogram and theoretical distribution considering the  $f$ -distribution with the degrees of freedom of  $m$  and  $n$  are considered as 25 and 15. So if you know the  $f$ -distribution has 2 degrees of freedom  $m$  and  $n$ . So this looks almost like the exponential distribution in the last slide but it is not the exponential, it is the  $f$ -distribution. So the mean values, standard deviation, mode can be calculated and we can pass comments about the different type of measurement scale.

The second last distribution which we will consider is the histogram and theoretical distribution for the generalized extreme value distribution and  $\alpha$ ,  $\beta$ ,  $\gamma$  as you know that for any distribution there is a scale, shape and location parameter. The values of  $\alpha$ ,  $\beta$ ,  $\gamma$  for this distribution GVD is respectively 4, 2 and 0.7. So you will basically have a distribution which slopes very slowly on to the left but peaks and falls very sharp on to the right. If you change the values  $\alpha$ ,  $\beta$  remains same and you take a  $\gamma$  value as -1, this is the generalized extreme value distribution how it would look like.

So coming back to the example which I said, consider the marks of 28 students who took one of the courses considers the stochastic calculus and finance course in 2023 and same one at IGIDR Mumbai in India and as I said there are 29 students, the marks for the students and without going to the actual marks I will draw the curve and without mentioning the roll number of the students, so this is the actual data. So the data points which you see are marked accordingly, one is red in color which is unsorted so the marks are just given randomly based on their roll number. So based on the roll number I will just hover this pointer here, so this is the marks how it is rooted and the marks are given in triangular red format. Basically this would be the maximum which I am now circling in blue. Now when I used them and sort them out from the minimum to the maximum or maximum to the minimum they are shown in green and the minimum marks which was actually here which I am just circling becomes the first point green, so the green circle and the maximum marks basically comes here.

Why I am doing that, that will be explained in the next slide. Now if you see the plot the difference of the consecutive students and the marks, so if you see so obviously they were

28 students so the difference would basically be 27 in number, 27 data points and if I see the difference of the students based on that, so the differences between two consecutive students are not equal. So this scaling system whether you scale up, scale down may change but the relative position of the students would continue to remain the same. Few concepts which I will just touch upon and I will highlight later when we come to the nonparametric of the multi attribute concepts of trying to use a multi criteria decision making. So consider an  $n$  dimensional space it can 2D, 3D, 2D is easy to understand Cartesian coordinate, 2D, 3D would you can also visualize but if I consider the  $n$  dimension one the formula I will state and then we can find out and definitely use those later on and as I said the concepts would be utilized later on also.

So consider  $x$  which is a vector or the points  $x_1$  to  $x_n$  in  $n$  dimension only one point is there with the coordinate system being  $x_1$  to  $x_n$ ,  $y$  being the other point with the coordinate system on the values of  $y_1$  to  $y_n$ ,  $x$  is another real line from 0 to  $\infty$ , all the values,  $y$  also being the real line. Real line in the sense obviously each point  $y_1, y_2, y_3$  till  $y_n$ . Then the LP norm distance measure is given by this we all know and why I am stating that it will be utilized. We all know this formula which is the L2 norm the distance measure measuring the distance of the coordinate system consider these two points there is  $x$ , there is  $y$ . So consider  $x$  is  $a_1, b_1$ ,  $y$  is  $a_2, b_2$  and I want to find out distance this one.

So obviously this will be given by the formula here. The L1 norm is basically known as the Manhattan distance is basically just the mod of the difference. So if you consider New York City the streets are run parallel to each other that is why it is known as the Manhattan norm. And  $L^\infty$  norm is basically when you find out the max of the differences between  $x_i$ 's and  $y_i$ 's and then take the maximum value. We will try to utilize the Manhattan norm, the L2 norm which is the simple distance measure  $L^\infty$  norm when we are trying to compare the liking and the disliking of different type of alternatives based on the criteria when the nonparametric or qualitative techniques are used.

The question would be does distance measure using this formula give a good picture the answer is no but this is a good methodology based on which we can analyze the problems. I will just touch upon that very briefly. Remember that concept of distance which is used as a similarity of familiarity may not be complementary but when you are trying to use the concept of distance as a measure or as a unit distance means as in the scaling system to give a concept that how familiar these two decisions are. They may not give the correct answer but they give you a lot of simile or complementary methodology of trying to do the work of trying to analyze similarity and dissimilarity concept. Affirmative or familiarity may be interpreted as a distance concept which has direction.

So consider very simply the distance between two points in the Cartesian coordinate

which I just drew which I will go back  $x$  to  $y$  and  $y$  to  $x$  are the same but when I am saying that the liking or the disliking of  $A$  and  $B$  which are decisions when you are trying to analyze from  $A$ 's perspective or the decision of the liking and disliking when you are trying to analyze from  $B$ 's perspective may not be same and I will come to that later on. Distance between two points in any direction considering the Cartesian coordinate system does not have that similarity familiarity concept attached. So two points in whichever direction you look it will give you the same measure and the units based on which the values would be fixed. So with this I will close this 28th lecture and consider more such concepts and consider the concepts of measurement scale distance measure in the nonparametric method of multi-criteria decision making and in the next lecture 29th, 30th and few more considering that we will consider the concept of goal programming and the utilization of as I said linear programming and quadratic programming in a much detailed manner to understand how this concept of goal programming can be utilized. Thank you very much. Thank you.