Multi-Criteria Decision Making and Applications
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Week 05
Lecture 21

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you who are participants and students for this course titled multi criteria decision making under MOOC NPTEL series and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur and if you do remember we have covered the few examples in details about the analysis, we have covered the concept of utility theory, many different descriptions about the ideas of what is utility function, the concepts and the properties of utility theory, the ideas about different utility functions in general like the quadratic, logarithmic, then the power function, exponential function, then this ideas and concept of absolute risk aversion, relative risk aversion, certainty equivalent. Then we considered the different other methods of trying to analyze decisions like geometric mean method, safety first principle, the three different concepts of safety first principle how they can be analyzed pictorially. We also considered the concept of stochastic dominance, there are other different parts, but generally what we have covered as required. The discussion which we are going to have is basically we are going to start the 21st lecture under the NPTEL MOOC series for this   multi criteria decision making course which is for 60 lectures and the overall coverage is overall umbrella coverage which we will be doing through different lectures would be about MCDM multi objective decision making, multi attribute decision making and multi attribute utility theory. What would be the coverage specific for this lecture? Obviously, there are many points written and as you know they will be kept updating, I will be keep updating them and putting there exactly what we are going to cover each lecture wise and depending on that and after listening to the videos and the overall inside discussion we will have about different topics and going through the slides and obviously the text books which I mentioned are quite vast in number after going through that you can do definitely each week they would be assignments and after assign each week assignments obviously you take the final examinations. So, the coverage here I am reading many of them some would be discussed later on would be MCDM ideas and concepts multi attribute decision making, multi objective decision making, multi attribute utility theory, MADM versus MODM which is multi attribute decision making versus multi objective decision making where MADM is more qualitative in nature, MODM is more quantitative in nature and there are other differences also.

 We will consider Pareto optimality, property of dominance, strong Pareto optimality, weak Pareto optimality and further on go into concepts of Pareto optimality in two dimensional space effects versus inefficient solutions, Karusch Kuhn-Tucker conditions,

then scales of measurements nominal scale, ordinal scale, interval scale, ratio scale with examples and then go into goal programming. The ideas of multi criteria decision making which is a broader area MCDM whose aim is to facilitate the decision maker who is faced with numerous and conflicting alternatives and choices. When we mean by alternative choices we have mentioned that I will always try to highlight the concept of alternatives and choices by using the set A which I am marking. So, this **A** means a set and there are different type of alternatives which we denote by **A** from starting from 1 to M, and each would be based each of these alternatives would be based on set of criteria or characteristics So, buying a car, the car we would have red color, mileage, then the safety features, then the maintenance cost all these would be characteristics or the criteria and the car which would basically or the set of cars which we will consider as alternative would be say for example can be Volkswagen, can be Kia motors, can be Maruti Suzuki, can be Tata motors, can   be   Honda,   Hyundai   whatever   it   is.

 So, based on this criteria you make a decision for the alternatives. Buying a house, so what is the price of the house, if you consider one of the examples we have considered in the first set of classes like what are the safety features of the house, what is the carpet space, whether the schools are nearby, the hospitals are nearby, whether it is a gated community or how far is the metro. So, those are the criteria and the houses you can consider like Mr. Murthy had considered different localities in Vizag. So, those are the alternatives   which   you   have.

  If you want to go for higher studies, so if it is say for example higher studies in India consider NITs, IITs if somebody is interested to go for the engineering studies, somebody wants to go for the management studies or somebody wants to go to social sciences, there are very good universities there. So, those universities would be the alternatives and the alternatives would be based on criteria may be type of subjects which are taught, what are the job opportunities, what is the quality of faculty members, how far it is from the home, what is the hostel facilities, how is the library, so those would be the criteria. And this criteria will denote by **C** and **C** is from 1 to N and based on this set of criteria as I said you choose the alternative, this is one decision maker who is choosing. In order to arrive at the decision that what is the final goal, the decision maker is required number 1 to identify the problem, what is the problem, what does he or she want to do, is it to choose a house, is it to buy the best house, is it to choose by the most safe most like if you want to buy a car, is the car based on the fact that it consumes the least amount of fuel. Then based on that he or she constructs the set of preferences, preferences are basically what are the different type   of   choices   which   are   there   for   him   or   her.

 He or she then evaluates the alternatives, evaluates the choices and based on the criteria, based on the ranking on the criteria he or she basically determines the best alternative and

the choices and ranks them. For the later two steps which is basically choosing and ranking, three kind of formal analysis are there and those are descriptive in nature, prescriptive in nature and normative in nature. The descriptive analysis basically also is termed as the behavioral decision research and is qualitative in nature while prescriptive and normative are more quantitative in nature. So we will try to analyze both of these type of problems where quantitative techniques as well as qualitative techniques would be utilized to make a decision of the alternatives based on the different type of criteria which is there for each alternative. The relevant questions pertinent in MCDM, MCDM again I am saying is the broader umbrella which need to be answered are, what is are the preference structure based on the decision makers choices, what are the weights he or she is going to give like relative weights, relative importance which the decision maker will assign and if there are conflicts among the different type of criteria and characteristics how they would be solved by the decision maker.

Conflict can be safety features is very good which which I like but price is very high which I want to avoid, I want to go for a lower priced or lower budget car or else consider the flat is in excellent location which has got very good hospitals nearby, it is safe, good transportation nearby but the flat size is very small as per my requirement or consider that I want to go to one of the IITs or IIMs or one of the best universities may be Delhi, Bombay, Madras, Calcutta whichever university to study for higher studies. For IITs it can be engineering related based studies, IIMs and other schools would be for MBA related, those universities may be for social sciences, basic sciences whatever it is. So consider the person wants to go to say for example Calcutta University but it may be too far from home or there is a lack of accommodation or say for example, Delhi University is the first choice of the person to go for higher studies, masters but maybe it is difficult to get admissions there considering the cut off. So those different ways of trying to analyze the alternatives based on the conflicting one and weights would be assigned. Conflicting concepts are the examples which I gave.

Weights may be considered for me if I consider a college for me the level of courses and the level of faculty members and the rigor of the courses would be more important than trying to understand what is the job features, what is the internship, what is basically the final placement. For some other person it may be just the other way round for him or her job facilities, internship facilities then long term employment would be much more important than the rigor of the course. So the weights would be assigned accordingly. MCDM may be classified into two main categories which is as I said MCDM is basically the broader umbrella, the super set under which you have the multiple attribute decision making, attribute means characteristic based which has limited number of predetermined alternatives and choices or the criterion characteristics that there is a discrete decision space from which the person has to choose with preference ratings being discrete and one

uses discrete mathematics to solve such problems. So discrete set they are may be easy to solve, but trying to analyze the output may be difficult while the second set which is multi objective decision making which is much vast in this application is based on concept of mathematical programming and is applied on a continuous solution space and consist of set of quantifiable objectives.

They are well defined and there are well defined constraints as the main concept considers a huge variety of interactions within the given constraints to arrive at the final decision under the concept of MODM. So MODM may be quite big, may be there are different type of mathematical models, but they are easy to analyze in the sense trying to basically bifurcate which is good, which is bad, which should be the top priority, which should be the bottom priority is much easier to do than with respect to the concept which are under multiple attribute decision making. Thus it is easier to arrive at the best solution using sophisticated mathematical programming techniques which is under the purview of multi objective decision making. Multi attribute decision making or MADM as I was discussing the first subset under MCDM can also be given the name of multi attribute utility theory. So there are multiple attributes or characteristic based on which the utility theory or the preferences is analyzed.

And also under that you have the out ranking method which will be one main part of our discussion. MAUT which is multi attribute utility theory represents the problem as an hierarchy structure utilizing different concepts of utility theory where the decision maker chooses the best alternative based on the highest utility value or the net value or the value which is good for him or her the decision maker. While on the other hand the out ranking method which basically utilizes the relationship of out ranking how is one alternative better than the other, what are the good features of the alternative than the other. So basically one wants to out rank and make the decisions accordingly, utilizes the relationship of out ranking to order the alternatives and the choices on a functional relationship where the functional relationships are basically denoted by this which I am highlighting.

So when you are considering two alternatives A_{i1} and A_{i2} where i1 and i2 are all elements of M, M is basically the number of alternatives which you have and J is the characteristics which is J is equal to 1 to N. So you will basically for each criteria you will try to basically compare the alternatives and basically make a ranking such that in case A_{i1} is better than A_{i2} we will say that A_{i1} which is alternative A_{i1} would out rank the other alternative A_{i2} based on the characteristics, based on the criteria. May be for the second set of criteria the ranking may change. So what you do is basically you consider each criteria, compare the alternatives and basically rank them. Then move to the second criteria again compare them in doing so you will basically have a set of ranking based on the criteria for all alternatives and then combine them accordingly.

Now there is one important thing which should be remembered that when you are going to compare the alternatives based on any one particular criteria you are very simplistically considering which may not be true in actual practical purposes that you are very simply considering that the alternatives or the characteristics are independent of each other which may not be true. So that may be one of the important points or which can be considered an important lacunae considering the different types of multi criteria decision making, multi attribute decision making we will be basically considering. In multi objective decision making which is the second sub topic under the multi criteria decision making, a decision maker again as I said faces a list of characteristics and criteria which we have already denoted by C_1 to C_N, N is capital based on which she or she wants to choose from a set of alternatives A, A_1 to A_M, i = 1 to M which may have conflicting outcomes. Mathematically, MODM which is multi objective decision making can be written as follows. So you have different multi objectives, so the first multi objective or the objective is f_1, next you will basically have which is not written here you will basically have f_2, I am not writing x I am just denoting the functions as f(x) the suffix, then you will basically have the third objective function f_3 so on and so forth till you have the N.

So this N should not be confused with the N which is there for the criteria. So actually when you are trying to solve the problem the number of characteristics and the number of optimizing objectives may be different. So the N which is shown in criteria and N which is shown in the objective function f_1 to f_N are different. Now you want to optimize these different type of objective functions collectively multi objective based on some criteria and the such then constraints are given as $g_k(x)$ which is any constraint $\leq b_k$ and k =1 to K again this K, M, N are different and x we will consider is always on the real line and there are basically L is of a dimension L. So again do not confuse this L with N which is the number of objective function.

Here x which is the vector x_1 to x_L is the vector of decision variables total ln number which I mentioned, M is the number of objective functions each being denoted by f_1 to f_M. So this actually is N and K denotes the number of constraints of the form given as $g_k(x) \leq b_k$ and k = 1 to K and we will consider this problems accordingly. Now one may solve these type of problems in variety of ways. So one of the simple way to solve this problem would be why not consider them individually as single objective function and once you solve the single objective functions you know what is actually the maxima or the minima for each of these objectives. So then what you do is that you put the first objective as in level 2 you put the first objective to be optimized and consider the other objectives are based on the constraints such that the boundary of the constraints are taken as the objective values which you based on which the second objective, the third objective, the fourth objective were basically considered we will see that later.

Then in the same step which is case 2 you basically solve the objective function as the second one but now considering objective 1, objective 3 and so on and so forth as the constraints. So simply we first again I am repeating we first consider single objective for all these objectives solve them and in the next step considering the objective functions one by one but now considering the other objectives which were left out are part of the constraints accordingly and then we solve it. So this is one method but there are problems here which we will discuss. The second way of trying to solve the problem is that we give some weightages and weightages on the objective functions such that we will combine this objective function all separately multiply individually by the weights and there are some restrictions of the weights and then basically solve the problem as a single objective function such that the weights can be changed in order to find out what is the importance we are going to give for each and every objectives accordingly. There are other methods also so first we will start with these two methods and solve and proceed accordingly.

One may solve the optimization problem first by formulating in two different ways which I mentioned one is optimize fi, fi means any objective function you optimize but what you do is that you consider the constraints as they are so $g_k(x) \leq b_k$ remains as it is but interestingly all the other objectives which you have which I will denote now. So consider the same problem consider f_1 to consider f_4. So what you do is that you basically and consider you want to optimize I will use the word optimize it can be optimum, maximum, minimum whatever it is such then conditions are I will consider $g_1(x)$ I am not writing $g_1(x) \leq b_1$ and the second of constraint is $g_2(x) \leq b_2$. So what I will do is that I will use a different color in order to explain that so consider the blue one I will solve f_1 first and what are the constraints? The constraint of $g_1(x) \leq b_1$ remains, $g_2(x) \leq b_2$ remains and I will put some of these other objective functions which are f_2, f_3, f_4 and bring them into the constraint. So I will write down as I am using different color I will put f_3 as now this f_3 is a part and parcel of the constraint is greater than equal to or less than equal to even though I have written greater than equal to that depending on optimization whether maximum, minima or I can use less than equal to say for example a_3 and this a_3 has to be decided this is what I said this can be a little bit tedious work.

The second the other objective being added again greater than equal to less than equal to would be a_4 again a_4 has to be found out such that this is met and another one which I have missed is basically f_2 greater than equal to less than equal to a_2. So now the first and so I am trying to optimize f_1. So I will basically I have converted this multi-objective into a single objective and considered the constraints and added the constraints of this the objective functions which have been left out and solve it. Now the question would be if this is basically for objective function 1 can we formulate a similar set of problems based on only objective function 2 the answer is yes. So again I will use a different color now

green one so now you will have objective optimize f2 now we are only considering f2 nothing of the other.

So subject to constraints g_1 g_2 remains so they will be copied and now f_1 f_3 and f_4 will come as part and parcel of the constraint. So $f_1 \leq \geq a_1$, f_2 is already there in objective function so it would not be considered. So this is $f_3 >< a_3$ and the last constraint to be there is $f_4 >< a_4$. Now this would be I would name it as 2 in step 2 this is the second model you will solve in step 1 this blue one in step 2 also this is the first model you will solve and if I consider the f_3 and f_4 the corresponding part would be I will erase one by one and write it so it is easier for us so I will erase the blue one which was for f_1 and use a different color let me use the color red. So you will have optimize f_3 now because f_1 already considered f_2 already considered, constraint g_1 remains as it is g_2 remains as it is now f_1 is not there in objective so it will come so it will be $\geq \leq a_1$, a_1 as exactly in somewhere a_1 is basically a constraint I want to put for the value, f_2 is not there in objective function so it will be considered, f_3 is not being considered so it will be removed not be considered here only f_4 will come.

So this would be so called problem 3 or a subpart for the step 2. Finally I know I am going a little bit slow but please bear with me, if I consider f_4 so you will optimize F_4 subject to $g_1 \leq b_1$ same thing is being copied, $g_2 \leq b_2$ then f_1 is not there so f_1 would be $\geq \leq a_1$, f_2 is not there in objective function so it is obviously $\geq \leq a_2$. I will write f_3 here because f_3 is not there in objective function is $\geq \leq a_3$ but f_4 already considered so this would basically formulate the subpart 4 of second step. So this is what is written here I will read it, so if I consider the first method which we are discussing so I will erase all this thing and not make it cluttered. So this was optimized based on the fact that you will individually consider and obviously there is a other way of to the second method is also there which we will discuss.

So here a_s are the predetermined respective thresholds for the s-th objective function which I mentioned and based on that we will solve the problems. Then we will consider first we will consider the other concept which I am putting a tick mark here in the next lecture and continue discussing very simple ways of trying to solve multi objective problem for problem formation. Have a nice day and thank you very much. .