Multi-Criteria Decision Making and Applications

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Week 01
Lecture 02

Welcome back students and participants. Very good morning, good afternoon, good evening to all of you and as you see the course title is Multi-Criteria Decision Making in Application and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur in India. So this is the continuation of the introduction class and this is the second lecture out of the 60 lectures which is basically a 12 week course and each week we have 5 lectures, each lecture being for half an hour. What is the coverage in this set of lectures? We will try to, we will consider third, fourth, fifth and sixth example considering the concept of multi-criteria decision making which we have mentioned in the first set of slides in lecture 1. So consider the third example, an international construction company wants to take a decision to undertake laying crane tracks and Mr. Abhimanyu Dasgupta who is the general manager or department of civil engineering for projects for this company, big multinational company is in charge for building the crane tracks in three different remote corners of India which are the western ghats, the north-east India and central India.

So he and his team has to analyze each project because there are three projects in these three different parts of India and they have to consider based on numerous features namely the cost, total cost because the more the difficult the terrain is, more lengthy the track is, more safety features obviously it will increase the cost. The environment impact because all these three parts of India which I mentioned, have a lot of forests there and mainly in western ghats, central India and north-east India and there obviously would be environmental concerns. What is the time taken to complete the project? Longer the time would escalate the cost, longer the time the benefits to the whole population in those areas would be later on coming up later on but obviously the utilization has to be done as fast as possible because it will basically ease the mode of transportation. What is the ease of utilizing different new technologies trying to lay the track, safety features, whether you are going to use diesel locomotives, electricity locomotives and so on and so forth.

What is the availability of skilled labor? Because in the central India or in say for example, western ghats the availability of skilled labors people who work will be much easier than north-east India. So obviously that will escalate the cost in the part of India which is north-east India. What is the social relevance and impact? So whether it will have an environmental impact, whether it will increase the overall economy of that area, whether other modes of communication which is there should not be adversely affected but it will give a positive benefit for the society in that part. So all these have to be considered and the different type of criteria which I mentioned and as I was discussing some may be quantified, many of them may not be quantified. So Mr.

Dasgupta and his team has to analyze his project based on numerous features which I mentioned accordingly. As the team leader he is aware that the combined budget, total cost, total numbers of engineering experts, laborers who are there as also the total time available for all these three projects are already decided, pre-decided because if this is a big public project, railways, building a dam, hospital, roads they will come as a benefit to society and that benefit has to come as soon as possible. So shorter the time you have to basically expedite the work, do it fast and obviously it will entail cost also but delaying it also would have different cost implications also. Hence the total time availability and also in the initial stages the total cost for these projects are already decided and fixed. Therefore the different facets he, Mr.

Dasgupta and his team has to consider for all the projects would have distinct impact or importance for these three different projects in three different parts of India. Hence the team has to have weights or importance assigned to the above items under different type of criteria they have for the different projects such that his team that is Mr. Dasgupta's team can make judicious decision about how to finish all the three assignments given that the constraints are actually there and they would definitely constrain and have limitations on all these three projects separately. Let us consider the fourth example and here we will bring some data in with the consideration and try to basically give an implication of that. Consider the company Impala Gezima Group which is based in South Africa has a large production unit in Quaga industrial development zone at Port Elizabeth in South Africa.

So that is a big area of development for industrial projects in South Africa. So what does the company do? It procures three different type of items which are ethylene, propylene and industrial chemicals at cost price and the costs are given in South African Rand. They are per ton and the cost for these three atoms ethylene, propylene, industrial chemicals are respectively 10,000, 9,500 and 11,000. So what does that company do? The company procures these three items to make products and there are two products which is industrial rubber and industrial plastics and which is sold in the market to the different type of other industrial companies throughout the world at respective price again the prices are given in South African Rand which is 23,000 per ton and 17,000 per ton. But obviously there are some limitations and practical limitations which I will read accordingly.

To produce one ton of industrial rubber which is the first product which is produced by this company, they would need 1.25 ton of the first raw material which is ethylene, 0.5 ton of the second which is propylene and 0.25 ton of the third one which is industrial product. Similarly, if I go to the second output, the requirement of the raw materials respectively would be 0.25 ton, 0.5 and 1.25 ton, obviously the second output is industrial plastic. But obviously you cannot or the company cannot have infinite amount of raw materials and is not able to produce infinite amount of output also because there is limitation of resources, limitation of manpower, limitations per day, electricity and all these things. Based on that the monthly availability of ethylene, propylene and industrial chemical which are the three inputs raw materials are respectively 200 tons. So these are given in the monthly basis, it can be on the daily basis, yearly basis, quarterly basis but basically we are trying to consider the problem on a unit time scale which is per month.

So requirement let me read it for the three raw materials are respectively 200 tons, 250 tons and 300 tons. While the production constraints, because as I said they would be production constraints because you cannot produce infinite quantity, per month are maximum of 175 tons for each of these two outposts which is industrial rubber and industrial plastic. So it can be different also for product 1, product 2 but here we are considering same. Due to market demands the maximum combined production so obviously the company uses these three raw materials, produces the two outputs but they do not have separate production units. So the combined production per month for these two outputs which as I mentioned is as the industrial chemicals and industrial plastics total combined is 300 tons per month.

Now with this concept of the input requirement of three raw materials what is the quantum or raw materials utilized to produce these two outputs, what is the constraint of the requirement of the raw materials, what is the constraints for the output for the products, what is the combined constraint for the output for products. Based on that we first formulate the problem as a simple objective function which may be needed to maximize or minimize we will see that in the problem and what are the constraints. So if I consider only the revenue function, the revenue which is the overall input which is coming to the company is given by 23000 for x_1 and 17000 for x_2. Now the question would immediately come up what do I need as my x_1 and x_2, I did not mention that there is a reason for that. Now for any problem when you solve there are certain sets of decision variables which you need to optimize and here the decision variables are these two which is the industrial rubber which we denote by x_1 which is marked here and the second one is the industrial plastic which is denoted by x_2 which is marked here.

Now why are we trying to take only two decision variables because depending on the output which you want to maximize, maximize the revenues we will consider the decision variables accordingly. So the decision variables are two: x_1 and x_2 which I have denoted by the vector x_1 and x_2 and we want to optimize them. Now based on the revenues there is a cost function also so we will first define the revenue function, the cost function and later consider a combination of that we will come to that later. Now the cost function if you consider that you are utilizing the inputs, three inputs for both for product 1 and product 2, product 1 being industrial rubber which is denoted by variable x_1, product 2 is industrial plastic which is denoted by variable x_2. For the first product which is x_1 we require 1.25 for the first input which is raw material, 0.5, I am not mentioning the units they are in tons, 0.5 for the second input and 0.25 for the third input. Similarly for the x_2 which is the industrial plastic, the requirement for the raw materials are respectively for raw material 1, 2 and 3 are given by 0.25, 0.5 and 1.25. So, if I find out and these factors which I am mentioning this one, 10,000, 9500, 11,000 for the first one first product of output then 10000, 9500, and again 11,000 are based on the requirement which is there of the inputs. Now this is the cost function, so I will denote the revenue function as 1, cost function as 2. Now if I consider the obviously if I consider these two objectives revenues I want to increase to the maximum, cost I want to decrease to the minimum, but obviously this company which is in South Africa has to basically find out what is the overall net profit. So, profit I am considering if it is positive, if it is a loss it will be net loss. So, if I consider the net profit based on the revenues total, cost total, I basically find out the difference between the total revenues and total cost if I put it using 1 and 2 which is given I find out the net profit revenues which I denote by 3 as given which is (3000 x_1 - 4000 x_2) which is basically x_1 and x_2 you remember are the decision variables which you want to optimize.

Now if I bring the constraint, so based on the constraints or requirements which I want to produce the availability of raw materials, the total output which I can do maximum, what is the total combined of the outputs I can do maximum. If I consider this, then for all the constraints which is given, they can be analyzed as this. So, the first one is based on the requirement of constraint 1 which is for the first input which is the raw material. So, I will denote them as, say for example, A, not 1, 2, 3, 1, 2, 3 was basically for the objectives. B is basically for the constraint based on the second set, C is basically based on the third set, third set means raw materials 1, 2, 3.

Now if I come to the total allowable output for each of the products I cannot produce more than 175 for both of them individually, it can change also, but for the problem we have taken it simple. We will denote the fourth constraint as D, fifth constraint as E and the sixth one which is the total combined output for decision 1 and decision 2 we will denote it by F. So, for the objective function either denoted by 1 or 2 or 3, the sets of constraints which you can see are given I have denoted them as A, B, C, D, E, F. Now bringing that in the picture, I have the problems stated very clearly. First consider the maximization of the revenue and if you see I have highlighted the word revenue in bold blue colors because that will make sense as you proceed with the problem and why it will make sense you will see that very soon.

So, you want to maximize 1 which is for the revenues, so this is the revenue function which is given and obviously revenue as I said I want to increase and this set of constraint which you see which I am marking are basically sets A, B, C, D, E, F. So, 1, 2, 3, 4, 5, 6 as it is mentioned. So, the first three are based on the raw materials and this 300, 175 which you see on the right hand side are based on the total combined individual amount of output for product 1 and product 2. Now if we solve this problem and try to maximize the revenue the overall value which comes out is 135.7 for x_1 which is the first product these are in tons and for the second product again in tons is 112.5. Remember the units are in tons and the time frame is months. So, that part of units and time frame I have made it very simple in order us to understand the problem in the simple way. Now, given this output which is optimal, best one and I will come to that how the overall picture looks like. Why we are able to visualize that in a picture because that is a two dimensional problem because there are only two outputs x_1 and x_2. Obviously in the three dimensional case it will be a 3D figure which will consider for other problems later on and for other four or more we have to just solve the problem visualizing it and drawing it may not be easy for us.

So, considering x_1 and x_2 which is the optimal output for the product 1 and product 2 if I put in the revenue function which was here, which I am circling now. If I put that in the function the total revenue comes out to be 50,75,000 in South African Rand and very interestingly I want to mention that if I use these x_1 and x_2 and try to double check whether the constraints are being met, if you put in constraint A B I am just mentioning them in the sequence of A B C D E F. So, A B C then it was basically D E F, if you utilize those two outputs and then put the output means x_1 and x_2 and try to put them in the constraints, all the constraints are satisfied, that means they are not violated. For example, the first one would be what? I am just giving an example for the first one, first constraint which will be 5 which is x_1, x_1 is basically, this is the multiplication sign, this is not x this is the multiplication sign x 137.5 + 1 again the multiplication sign x 112.5. If I found find that value, it should definitely be ≤ 100, sorry 800 which is the first one. Similarly, if I put in the second one again the multiplication sign this starts in the multiplication sign this first value for the decision variable is 137.5 + 2 x 112.5 ≤ 500. Similarly, I can do it for the third one, the fourth one, fourth one is what, I will skip the third one for the timing because considering the first two is there you can definitely find out for the third one. For the fourth one which is given as $x_1 + x_2 \leq 300$ I will put it here 137.5 + 112.5 ≤ 300. So, if I add up, so let us consider only I am concentrating on the fourth one which is here I am highlighting. 137.5, 112.5 this becomes 718, 910, 0.13145 this would be basically 250. So, obviously 250 does not equal to 300 which means if I consider the last two where x_1 and x_2 each individual < 175 which is met there because the first decision variable is 137.5 second one is 112.5. So, having solved that and obviously as I mentioned these can be solved using very simple linear programming problem, the prerequisites which I mentioned when I was doing the introduction part. If I solve it and draw and visualize the output which is a two dimension is one easy. So, basically I will plot for convenience all of us know that I can use and plot the first decision variable x_1 along the x-axis, the second decision variable x_2 along the y-axis and obviously can be changed there is no problem in that, but there is a convenience based on which I am followig. If I see the overall feasible space, the overall feasible space would be given which I am hashing here. So, in the constraints which I draw in the boundary they would be based on the fact that there are if I remember I am just writing A, B, C, D, E, F. So, these six constraints are there and if you see the constraints would be giving you this feasible region. So, obviously there are infinite sets of points in the feasible region, but technically if I consider the concept of linear programming idea we have to only concentrate the solutions as the corner points we all know in simple linear programming concept. There is a corner point here also and there is a corner point here also and the objective function which you see which is dotted here this part which is the revenue function. This is tilted in a way based on what is the parameter based on which x_1 and x_2 are being multiplied. So, based on the parameters which was basically the revenue function if I plot it and then we all know we have to basically keep shifting the objective function. So, this is basically I am drawing a very random objective function, but the line tiltation is based exact on what are the variables which multiplies x_1 and x_2 in the objective function. So, if I keep moving it to the maximum point. So, this is the point which I have based on which I find out the maximum value. So, the maximum value if you see as I mentioned was 50,75,000 South African Rand based on the values of x_1 and x_2 which I mentioned. This is the point which I will have. So, this will give me the best solution optimum maximum point for the case when you are trying to maximize the revenue.

Now, consider the cost function. So, it is a totally different objective function. So, obviously if the revenues are to be increased obvious logic will say that we want to decrease the cost function. Hence, we use the word a concept of minimization. Again the constraints are same if you look at it the constraints are basically what we are denoted by A, B, C, D, E, F they are the same they are not changing. So, first three are based on the combinations of raw material 1, 2, 3 to produce the products which we have denoted the decision variables are x_1 and x_2.

And the last three which you see which I am putting a double tick mark here are based on the total quantum of output for x_1 for x_2 and the combination of x_1 and x_2 where x_1 and x_2 are the decision variables. So, if I want to find out the cost function it is given by 20000 x_1 x 21000 x_2 and we solve it again using the simple concept of linear programming because there are linear programming ideas only. When we find obviously the values are x_1 is 0 and x_2 is 0, that means we should not produce because we are not considering the revenue function at all. It is only cost. And if you put that 0, 0 for x_1 and x_2 in the objective function which is here the value will come out to be 0 as it should be.

Now, let us again draw it. If I draw that feasible space, again feasible space does not change because the constraints are same. And the cost function is given as the objective in the cost function in the second problem. Again if I take the cost function again I am drawing it very arbitrarily. But interestingly rather than increasing it the cost has to be decreased. So, we will move on to the bottom left corner and the point which gives us the minimum cost is this Z*, which is the origin which is 0,0.

So, this is basically the cost function not the revenue function. Based on that we basically try to find out the minimum point. See here just a note this point where we maximize the objective which was the revenue is this point. Again infinite sets of feasible points but based on the concept of linear programming we find out the corner points and the corner point which gives us the minimum cost is 0.

Finally, let us consider the profit function. Profit function if you remember is basically the difference between the revenues and the cost. So, profit is basically to be maximized and very interestingly note down the profit function has the parameters based on which we multiply x_1 and x_2 is 3000 - 4000 for x_2. Again the sets of variables are exactly the same A, B, C, D, E, F. When we solve it, the actual values based on which we get the best input for the net profit is (160, 0) that means 160 for x_1 and 0 for x_2.

Now, intuitively immediately you will ask why it is 0. Check here if x_2 was basically positive so obviously this value of -4000 x_2 would basically bring down the net profit. So, obviously it is much more intuitive to imagine that x_2 value would be negative will be 0 such that the value or the component which adds up to the net profit function would be 0. Based on that when we utilize this (160, 0) for x_1 and x_2, in the objective function which is here the overall net profit comes out to be 4,80,000. Again let us see the idea of the feasible set does not change because the constraints are same and if I take the revenue net profit function and that net profit function again can be plotted considering is a two dimensional one profit has to be increased. So, if I consider the profit function as this so it has to be go on to the right hand side increase. So, the overall best profit point comes out to be Z*, which is 4,80,000 as given. So, with this I will end the second set of slides and consider more examples such that we are in much more comfortable position to start of the ideas of MCDM. Thank you very much and have a nice day.