Multi-Criteria Decision Making and Applications
Prof. Raghu Nandan Sengupta
Industrial Engineering and Management Department
Indian Institute of Technology, Kanpur
Week 01
Lecture 01

Hello, good morning, good afternoon and good evening to all of you and let me wish all these students and participants a very happy new year of 2024 and welcome to all the participants and the students for this MOOC course titled Multicriteria Decision Making and Application and my good name is Raghunandan Sengupta from the IME department at IIT Kanpur in India and let me also wish all the participants, all the students a very happy new year of 2024. I am sure everything would be perfect and fantastic for all of you. So, as you are aware this course would be for 60 lectures which will be spanning over 12 weeks and each week we will have 5 lectures and each lecture would be for half an hour. So this is the background with that I will start the general discussion about the course, the content and some pre-requisites would be involved and as we go on with the course you will understand the overall nuances and the detailed coverage of the course as required. So why this course? The question would be and if I look at the objectives and if the students look at the objectives it is basically in all domain of multiple levels decision-making, we consider numerous alternatives and conflicting criteria. So conflicting criteria when I say that profit and cost are two conflicting ideas; in the sense that we would like to reduce cost and we like to basically increase the profit. Or if you want to basically buy a car, we want to buy the least expensive car and the idea would be we want to basically get the car or purchase the car which gives us the maximum mileage. So, these what I mean by conflicting criteria. So hence it becomes imperative for the decision maker or the set of decision makers. Why I mean by the set of decision makers we will come to that later on to come up with the best set of alternatives or best solution and that is basically would be the main thrust focus of the course. Ideas of multi criteria decision making, multi objective decision making, multiple attribute decision making helps and why I mention the word of multi-criteria decision making, multi-objective decision making, multi attribute decision making that would also become clear as we proceed with the course.

 It helps the decision maker to make rational as well as well-founded mathematical and well-grounded decision to solve these sets of problems. Problems which I gave for the example of buying a car or profit and loss may be quantitative, but there are many examples which will be very qualitative in nature, we will also consider that. In many cases when stakes are very high and one has non-commensurable units of measurements; what I mean by non-commensurable units of measurements would be like say for example, when you are buying a car, cost would be in rupees or dollars or yen or euros whatever it is, but if I want to find out the safety features that would be very qualitative in nature. Non-

commensurable units of measurements as well as multiple conflicting objectives such that MCDM definitely would aid us in better decision making. This course will benefit the students and the participants in the masters and doctoral programs and working in variety of areas like engineering, electrical engineering, mechanical, civil, chemical, in the area of mathematics and statistics in multiple decision making in the area of economics, in management, in supply chain management, in quantitative finance etc. to tackle and solve very interesting problems, both from the theoretical as well as from the practical point of view. What are the key learning take aways from this course? It will help students master the rich repertoire of tools for scientifically rational analyzing multi-criteria decision making. So, it will definitely, we will try to cover all the simple and exhaustive quantitative, as well as qualitative techniques with examples. It will facilitate students with both basic and advanced theoretical background, which is necessary for MCDM and it will help them to analyze the problems in a very practical manner. It will definitely equip the learners, participants and students with the requisite skills in utilizing different techniques through practical examples which you will solve.

It will also improve the decision making in myriad of decision processes be it in the area as I told in engineering, management science, psychology, biological science, environment economics and so on and so forth. Now before I start the course there are some pre-requisites which I will request if the students have, it will be really be helpful for them and even if they do not have still I will try to give the simple background which will be required. Now the two courses which are definitely required obviously there are other courses also which I will mention as we proceed through the courses through this course but I will request if the students can brush up this general concept on two basic fundamental areas of statistics and operational research. First is statistics. So the course number which I have mentioned is IME 602 is the course which I teach at IIT Kanpur in the IME department.

So basically they cover axioms of probability, conditional probability, they cover discrete and continuous random variables and when I mention about the courses it is not from the theoretical point of view is basically from the practical point of view somebody should definitely have the knowledge. The course also which is IME 602 covers functions of random variables, what are moments of random variables like the first moment which is expected value, second moment which is the variance, third moment, fourth moment, skewness, kurtosis so on and so forth and what is the concept of the moments. It also covers generative concept of generating functions and also in a way try to basically cover the concept of probability generating functions, moment generating functions, what are the significance of the generating functions for the distribution as such. Some very basic concept of limit theorems and what are the uses of limit theorems. It considers joint distribution random variables, but mainly from the bivariate cases.

Considers very simply sufficiency and completeness concepts and why it is important. Then it goes in the concept of inference statistics and inference statistics basically consists of three areas which is point estimation, interval estimation and hypothesis testing. So considering all these three broad areas of inference testing the course covers, course means the IME 602 covers sampling theory and sampling distribution, different distributions and mainly being the three distributions, which is required for that course is basically the chi square, the t distribution and the f distribution. Obviously all these three comes from the z distribution with the standard normal distribution. Generally covers methods of statistical inference, theory of point estimation only from the practical point of view, no theory and no   theoretical   proofs. Then estimation of parameters, theory of interval estimation, theory of hypothesis testing. Then it briefly covers analysis of variance or ANOVA not MANOVA not for the multivariate analysis of variance. A very brief introduction of multivariate analysis and linear and multiple linear regression. So this is the one big part which may be required as we cover this course of multi-criteria decision making. So these are the books which are generally cover and it will be put up on the net, people can use these books in order if they want   to   basically   cover   this   area   of   probability   and   statistics.

The next course which is important is basically as a prerequisite is IME 605 which is operation research for management which is also a course under the IME department at IIT Kanpur. Basically it covers introduction of mathematical modeling, linear programming, the very simple concept of set theory, linear algebra, matrix algebra and how you formulate the problems. It considers the concept of duality, sensitivity analysis, robustness, what are the applications, the simple applications or linear programming, big M method and so on and so forth. It also covers that course of IME 605 considers network methods, max flow, min cut, what is the shortest path concepts. Then the very simple concept of dynamic programming, sequential decision making, how they are made in stages say for example for the areas of inventory management and so on and so forth, production management.

Then the very simple concept of optimal or optimality is considered. Then after the linear programming it goes into area of integer programming, its formulation, its problem solution, simple concept of quadratic programming even though the word quadratic programming is not mentioned in the slides but it does cover that very simply without any theory. The concept of non-linear programming, applications and solution methods. Few of the basic books, I am not saying they are absolutely the best or exhaustive but few of the basic books for IME 605 as I mentioned also for IME 602 is also mentioned here in the slides. Now coming back having said for IME 602 and 605, what is the coverage we will do   for   the   multi-criteria   decision   making,   the   syllabus.

So the syllabus is broken down into three parts. Part one which we will start with is

basically the concept of choice theory, ideas of rational and irrational choice theory and why people make a decision accordingly. We will also go into details about the concept of utility theory, what we mean by utility theory and why utility theory is important. We will consider the concept of non-association, risk characteristics, expected utility and the risks and how expected value of the utility risk value of the utility can be utilized to analyze different problems. We will also consider very simple examples of utility functions and what they mean actually in practicality.

We will also consider two important concepts which is not mentioned as you see in the slides the concept of absolute risk aversion and relative risk aversion and why they are important. We will consider in very simple sense the concept of stochastic dominance both the first order and the second order stochastic dominance. We will go into the areas of loss functions, and what we mean by loss functions like quadratic loss functions, zero one loss functions, linear-linear loss functions, linear loss functions and so on and so forth. After this concept of utility theory, we will go into one very interesting area of preference theories and dwell into in a very simple sense what is the difference between utility theory and preference theory. We will consider consumers choice behavior, what is the concept of concave and convex preferences and compare as I said few seconds back, compare the concept of the difference between utility theory and preference theory and why it makes sense.

We will also consider the concept of different scales of measurements and when I mean measure or I mentioned the concept of the scales of measurements, it is not only length, breadth, height it can be related to volume, it can be related to say for example the decision you are making, it can be related to say for example temperature, there can be different ways of scaling and why scaling is important, I will mention is very briefly as we proceed because those relevance will come out later on as we solve different type of problems using the concept of MCDM techniques. We will consider the normative, prescriptive and descriptive decision concepts and the choice under uncertainty and also consider very simply different type of heuristics and Bayesian decision analysis. So, this is the first part.

The second part which will be a little bit more mathematical than with respect to the first part would be we will cover the basic concept of MCDM, the concept of multi objective decision making, multi attribute decision making. So, having said that we will cover in the step 5 process as we proceed, concept of multi-objective optimization, Lagrangian multiplier the concept in the case of multi criteria decision making because Lagrangian concept is definitely used in the simple case of when there is only one objective function to be optimized.

We will consider the Karush-Kon-Tucker condition KKT conditions for the multiple objective problems. We will consider concept of dominated and non dominated solutions

and that will lead us to the very interesting area of Pareto principle, Pareto optimality. We will consider the concept of goal programming, what is the concept of no preference method, priori method, scalarization. We will also consider posterior methods and having said that we will cover later on, the concept of vector optimization, weighted sum, epsilon constraint, weighted matrix, Benson's value functions and so on and so forth. We will also go through the concept of interactive surrogate worth trade-off, then we will cover the areas on non-differentiable interactive multi-objective bundle optimization method. We will consider the light beam search method and under the different type of heuristic methods there are different type of heuristic methods. We will consider very simply the genetic algorithm and colony optimization, taboo search, then we will consider the particle swarm optimization and we will also consider the artificial immune system. There are different methods but we will very simply consider these methods in a very general method way such that we understand the logic, understand the pseudo code and try to solve it in a very simple way for very simple cases of MCDM techniques.

The third part will be much more qualitative in nature based on the first part and we will consider the concept of multi attribute utility theory, multi attribute value theory, the concept of out ranking method and there we will see why the scaling system, the different type of scale methods are important. We will consider the weighted sum method, weighted product method, we will consider the analytical network process and that will lead us to the analytical hierarchy process because analytical network process is the super set under which the AHP which is the analytical hierarchy process falls.

We will consider the Elektra method, the Topsis method, the Vicor method, the Promethee method, the Macbeth method, Paprika and few other interesting qualitative techniques. Few of the text books, now the text books which are mentioned in the slides and which you will be seeing on the slides is definitely quite huge. I am not saying there is only one text book which will solve all the different type of problems you want, you face or cover in this course or the concept which will cover all the theoretical concept but there are very good books which are available if you are able to get them in the library or in the net or you are able to purchase. So, few of the reference lists are given. I am not going to read each and every name because there is exhaustive list. So basically if you see in each slide I have given 5 names and by the way the text books are not as I mentioned are not any particular area of which is covers all the 3 parts.

They may only concentrate on part 1 or part 2 or part 3 or some combinations of part 1, part 2 or part 3 which is the 3 main areas which I covered as I was discussing the syllabus. So, this is the second slide based on the text books or the text. This is the third slide this is the fourth slide, this is the fifth slide which is there for this course. I amnot saying again they are not exhaustive but they definitely cover a huge portion on the coverage for this course. This is the last slide which is basically mentioned for the text.

Now having said that I will come back sorry that I took about 20 minutes to give a detailed background for this course. So, the first set of slides would be based on as I mentioned that we will have half an hour for each day, 5 days in a week we will have such classes or video lecturing and this course will continue for 12 weeks which will basically cover 60 such lectures. So, this is the first lecture out of the 60. So, we will basically start this course by considering only simple 3 different type of examples. So if you consider the first idea and what I mean by the idea of MCDM is that and if you read this the slide which mentions is men follow their sentiments and their self-interest based on what decision they want to make but it pleases them to imagine that they follow reasons and make the best decision.

And this quote is from Pareto and why I am using this quote is because Pareto concept and Pareto optimality and the ideas would be utilized time and again when we encounter different type of multi-criteria decision making not only for the bi-objective that means there are two objectives to be optimized but it can be case for the ideas when there are more than two different objectives to be optimized. Now many of the participants and many of the students would have heard the name of Peter Ferdinand Drucker. So, Drucker was one of the stalwarts in the area of management science in the 1900s. So according to him to manage a business is to balance a variety of needs and goals and this requires multiple objectives as I said multi objectives needs to be optimized and the word optimization can have different connotations which will visit very soon. In psychology decision making is regarded as the cognitive process resulting in the selection of a belief or a course of action among several possible alternatives.

It could be either rational or irrational. So, when I use the word rational irrational it will become very evident as we solve the problems. What is rational for one person may be irrational for the second person and vice versa and we will discuss that. The decision making process is a reasoning process based on assumption of values. Now the values need not all be quantitative it can be qualitative in nature also.

There are preferences and beliefs of the decision maker and when I use the word decision maker for the time being we will only consider one decision maker. It can be extended to more than one decision maker also which we will see in the third part of the syllabus as we discussed where we covered the concept of ANP, AHP, Electra, VIKOR, Macbeth and so on and so forth. Every decision maker produces a final choice which may or may not prompt any action because the decision maker makes a decision and the end result is such that the decision make is not finding any benefit from the decision he or she would not take that decision in order to proceed with the decision making process. Now when making a decision, a decision maker has in front of him or her a finite set of alternatives and what I mean by the world of alternatives as you see in the slides is basically highlighted with the

dark blue color we will come to that and each of these alternatives we will denote the alternatives with the capital A and I will come to that later on. They are evaluated based on set of terms of some evaluation criteria or sets.

When I mean by the word criteria we will be denoting that with the capital C. So there are different type of alternatives based on different type of criteria and based on the combinations of the criteria for each and every alternative the decision maker would rank them. When I am using the word ranking there is where the concept of different type of quantitative and qualitative techniques will be coming up very soon. The decision maker processes the set of information which is provided to him or her or based on belief which he or she has in his or her mind to rank the alternatives because the alternatives are what he wants to take the decision based on his or her practical implicit as well as explicit judgment beliefs, predilections and knowledge as well as information provided by the external environment.

So there are two things. One is the set of beliefs which the decision maker has in him or her and one is the set of information which is being provided by the environment. So based on that the decision maker makes the decision based on the criteria for all the alternatives and then ranks them. The decision maker may either be interested to rank the alternatives to find the best alternative as I said ranking and determine the relative total priority for each alternative. Now when I am using the concept of ranking using the concept of priority it does not mean that they are very simple to understand because as I said for one decision maker the ranking process would be totally different from the second decision maker in spite of the fact the set of alternatives may be same in front of him or her. Like if I am buying a car and if a second person is buying a car our total set of criteria and the weights I am using the concept of weights for the first time it will be coming up later on time and again.

It will be based on the fact that the person will be taking the decision accordingly. So the decision maker may be either be interested to rank the alternatives to find the best alternative based on their level of attractiveness for the decision maker. So the level of attractiveness is very qualitative in nature is not very objective it may be subjective. In the process the decision maker contemplates using all the criteria which I mentioned few minutes back, we will use the word C to denote the criteria for each of the alternatives which I said we will denote by A and this is what we will define as the concept of multi criteria decision making. Now when I mention the concept of alternatives, when I mention the concept of criteria there is not only one alternative and there is not only one criteria.

So there are multiple criteria's multiple alternatives and we will denote the alternatives as by the suffix i, i going from 1 to M capital M and criteria will be denoted by j, j going from

1 to N, N as in Nagpur, M as in Mangalore. Now this concept of number of alternatives, number of criteria may be equal or the number of criteria may be more than number of alternatives or vice versa we will tackle those problems accordingly. Now let us consider the first example. The first example is to do with making a choice of selecting a particular shade of color for decorating room. Consider this lady Shirin Amarpur is purchasing an apartment in the city of Isfahan which is in Iran and she stays in Iran. Her apartment interior designer has suggested to this lady to color her dining space with a shade of green and Miss Amarpur has been given a shade card color shade card from which she can choose the particular color of green and samples are given. So if you can see the slide there are different types of green. So let me read the different types of shades of green which is light green, mint green, malachite, neon green, spring green, lime green, kelly green and emerald green. So even if the color concept is given here which may not make much sense from the quantitative decision but when you consider and different type of upholstery, different the curtains she is going to put up, different type of cutlery sets and the overall ambience in the room there the concept of green may be very important for her because she wants to put the decor in her room accordingly. So it may not be a very quantitative decision but is definitely a qualitative decision where different concept of multi criteria concepts may be utilized.

The second example let us consider Miss Katryzyna is basically an is a student in the University of Warsaw in Poland. She has a bachelor's degree in statistics and econometrics and she has different offers from four different universities which are Rotman School of Management in Canada, INSEAD in Paris, France, Robert H. Smith School of Business which is at University of Maryland at College Park and the last one is Carey Business School and John Hopkins. So obviously she has to make a decision accordingly. Her choice of university will depend on many factors like cost of tuition along with the overall expenses, the courses offered and the specialization offered, types of companies which come for the summer and the final internship, the alumni network and the help she will get and the ease of getting educational loan to study in these universities in USA.

For each of these criteria which I mentioned some of them are qualitative, some of them are quantitative. Getting a loan amount of money she has to pay is quantitative, it can be visualized but amount at the level of alumni network, type of company she will get, the overall courses specialization would definitely depend on the student who is going to take the decision. For each of these criteria Katryzyna has assigned points, obviously the points she has assigned will depend on her and she has assigned points depending on their importance and hence the decision to choose the university will depend on the points she would assign to the several different factors which are there in front of her. For example, she may think that course and specialization offered, types of companies for summer and final internship are the most important component for her. While cost of tuition along with

overall expenses, she has to basically spend in trying to stay in that foreign country, the alumni network could come next which is the second level of priority for her and finally she would consider the point related to ease of getting an education loan to study in the university would be third priority. How she would assign the weights to these factors or on what basis she would do that would be utilized by her to facilitate the decision of choosing the university. With this I will close this first lecture and continue with the examples later on as they are required. Have a nice day and thank you very much.