Similitude And Approximations In Engineering,
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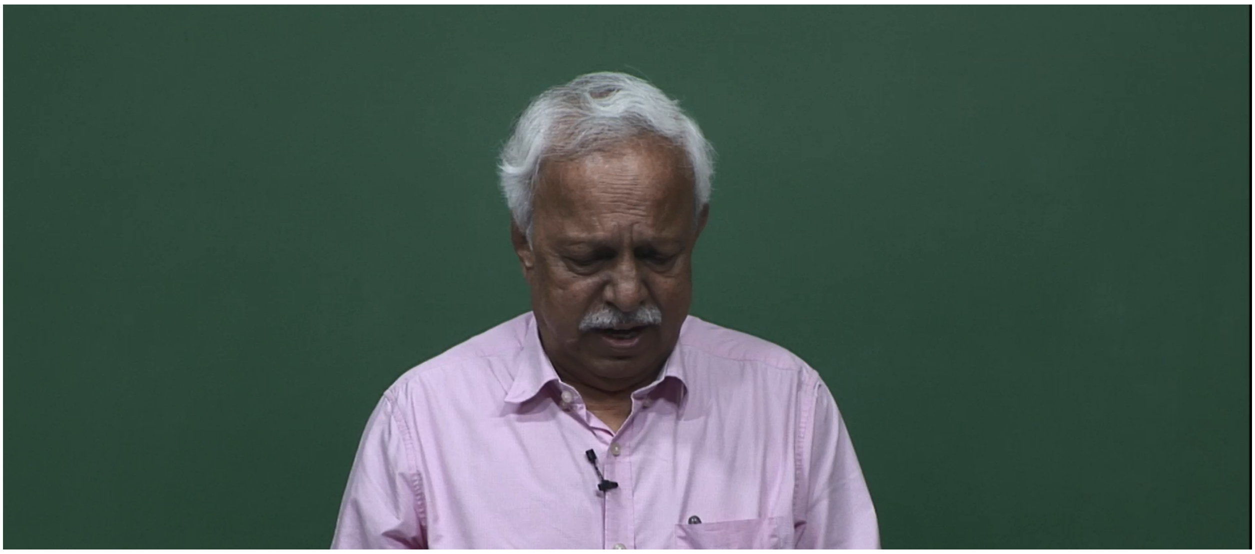
Week - 08
Lecture - 27

Welcome back.



This is the last lecture of this course of lectures and where we will discuss two case studies, longer examples with more details. Both these case studies are connected with the work that I had done in my career. The first study pertains to an inquiry that I received in late 80s from Maharashtra electricity supply company. They were building power plants or extending the power plants in Chandrapore super thermal power stations, and they were designing three chimneys each 200 meters tall and they are worried about their elastic properties. What happens to them when the wind blows past them? We know as the wind blows past a circular section, there are vortices that are shed.

These vortices that are shed result in transverse forces on the cylinder. And these transverse forces could result in transverse vibration of the chimney. And if the frequency of this vibration coincides with the natural frequency of the chimney, then the chimney could collapse. And so they wanted me to investigate this problem and see that the natural frequency of the chimney is far away from the frequency of the vortices being shed. The aeroelastic behavior of a tall structure like a tapered cylindrical chimney is governed principally by its inertia and elastic forces, and by the inertial forces of the wind that acts on it. The viscous effects are usually negligible because of a very large value of the Reynolds number involved. Since the chimney can be considered as a smooth cylinder, the vortex shedding from it are somewhat dependent on the Reynolds number. But this dependence is usually small at very large Reynolds number, of the order encountered in the practical cases. Further the platforms and this spiral staircases along the chimney would make the vortex shedding frequency quite independent of the Reynolds number.

So, therefore, it was quite reasonable to neglect the viscous effects completely. We allowed for different scaling factors for diameters and thickness and that was required because of the differences in the densities of the prototype and model material and the difficulty of fabricating the very small wall thickness required if the same scaling factor was used. The following scaling factors for the various forces can be obtained. For the inertial forces of the chimney, the scaling factor for the force $k_1 = k_{\rho_c} k_L^2 k_h \bullet k_\omega^2 k_L$, ρ_c is the density of the chimney, volume of the chimney which should vary like $k_L^2 k_h$ and the acceleration $k_\omega^2 k_L$ for vibratory mode of the chimney vibrating at a frequency omega. The inertial forces of the air, convective acceleration, $k_2 = k_{\rho_a} k_L^3 \bullet k_V^2 / k_L$ where k_L^3 is for volume, and k_V^2 / k_L is the scale factor for convective acceleration.

For the vibratory inertial forces of the air, unsteady inertial forces in the air, $k_3 = k_{\rho_a} k_L^3 \bullet k_\omega^2 k_L$. The elastic forces in the chimney wall $k_4 = k_E k_\varepsilon \bullet k_L k_h$. Cross sectional area is like $k_L k_h$, h is measured in the thickness direction and L is the length and the diameter scale. For identical curvature that is when $k_{radius} = k_L$ is 1. So, that $k_4 = kE\, kL\, kh$. And the last of all the viscous damping of the wall material we use the non dimensional logarithmic decrement $\delta = (1/n) \ln (a_0/a_n)$, where a_0 and a_n are the amplitudes at the beginning and end of n cycles of vibrations. Since this is non dimensional, this itself is a pi number, and has to be matched in the prototype in the model.

Dynamic similarity requires that the scale factors noted above for each of the force components be the same. This then gives us the following modeling rules. From equality of k_2 and k_3, that is of the forces relating to the convective acceleration of air and the unsteady acceleration of air, we get $k_{\rho_a} k_L^3 \bullet k_V^2/k_L = k_{\rho_a} k_L^3 \bullet k_\omega^2 k_L$, or $k_\omega k_L/k_V = 1$. This is this the Strouhal number, $St = \omega L/ V$ is invariant, should be identical both in prototype and the model flows.

Next, from the equality of k_1 and k_2. k_1 is the force of vibration in the chimney material in k_2 is the convective acceleration of the air. We get a relation $\frac{k_{\rho_a}}{k_{\rho_c}} = \frac{k_h}{k_L} * \left(\frac{k_L k_\omega}{k_V}\right)^2$. This is pertaining to the Strouhal number. Since $k_L k_\omega/k_V$ is 1, as already established and since k_{ρ_a} is 1, the model fluid is the same as the prototype fluid, both air, because we are testing in the atmospheric air. This means $\frac{k_h k_{\rho_c}}{k_L} = 1$. This is the second modeling rule.

So, though we are using the same density of air by using a different value of k_h from k_L, we can use $k_{rho,c}$, the density of the chimney material different in the two cases, the prototype and the model. So, this conditions establishes the need for a different value of thickness scaling factor in the model chimney density is different from that of the prototype.

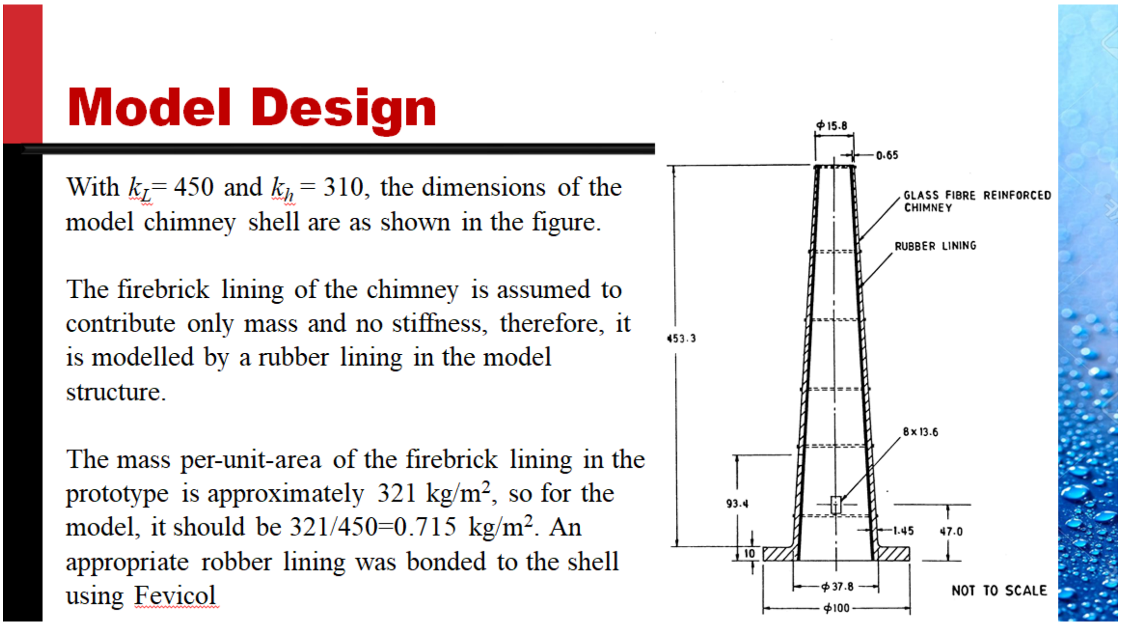
Next, the equality of k_4 and k_2, that is, the stress forces within the chimney material and the convective acceleration forces in air, gives you this relation: $k_E k_\varepsilon \bullet k_L k_h = k_{\rho_a} k_L^3 \bullet k_V^2/k_L$, or,

$$k_E k_h = k_{\rho_a} k_L \bullet k_V^2.$$

Since $k_{\rho_a} = 1$, same air, this gives you $k_V = \sqrt{k_E \frac{k_h}{k_L}}$, and establishes the velocity at which the test is to be conducted. Thus, with the choice of k_L, k_{ρ_c} and k_E made appropriately, we can first calculate k_h, the wall thickness scaling factor, and then the velocity scale factor k_V. This gives the velocity at which the test should be run to give dynamic similarity. Then this Strouhal number equality gives the prediction rule for the frequency. So, frequency of vibration, the natural frequency of chimney, $k_\omega = \frac{k_V}{k_L} = \frac{1}{k_L}\sqrt{k_E \frac{k_h}{k_L}}$. This can be used to predict the frequency of the prototype, once the model frequency of vibration is measured from the test.

To summarize: the process of modeling is to choose appropriate k_L. We ended up a value of k_L very large, because the test section of the wind tunnel available to us was quite small. It was 70 cm by 90 cm. Then, choosing modeling material, and obtain k_{ρ_c}, the density scale factor and k_E, the elasticity scale factor. Then obtain k_h from the modeling rule equation 3 above, and then obtain the velocity scale factor, k_V and the velocity to run the model test. If we do this, it ensures following the rules can be used to predict prototype quantities $k_{\omega} = \frac{k_V}{k_L} = \frac{1}{k_L}\sqrt{k_E \frac{k_h}{k_L}}$, $k_{amplitude} = k_L$. k_{ε} is 1, and k_{σ} is like $= k_E$.

Keeping in view the dimension of the test section in one experiment the length scale factor k_L equal to 450 was used. This would result in a wall thickness less than 1 mm. To facilitate fabrication of the model, it was decided to use structural grade glass fiber reinforced plastic. We chose ERALDITE LY 556 with hardener HY 951. The average density of whose composite with the glass fiber cloth was measured is 1,720 kg/m^3. Since the average density of RCC can be taken as 2,500 kg/m^3, the resulting scale factor for the chimney material density was 1.45. Then using Equation 3 above, the wall thickness scaling factor was obtained as 310, quite a bit smaller than 450 for the length and diameter.



This shows the dimensional sketch of the model chimney, which is not drawn to scale because the length is quite large. k_L= 450, k_h = 310. The dimension of the model chimney shell are as shown in the figure. The actual prototype chimney had a fire break lining. How to model that? It was assumed that the fire break lining of the chimney contributed only to the mass and no stiffness and no damping. Therefore it was modeled by a dummy mass. We used a rubber lining of the model stuck in the inside of the Eraldite fiber glass composite chimney. It was stuck loosely so that it did not contribute to any stiffness or damping, but only to the mass representing the mass of the fire break lining in the prototype chimney. The mass per unit area of the fire break lining in the prototype is approximately 321 kg/m^2. So, for the model it should be 321/450=0.715 kg/m^2. An appropriate rubber lining was bonded to the shell using Favicol. The mean elasticity E of RCC is 35.3 GPa while that of the RPA composite used is 4.5 GPa. This implies a scale factor for E k_E = 35.3/4.5 = 7.84. The required modeling velocity scale factor then is obtained as $k_V = \sqrt{k_E \frac{k_h}{k_L}} = \sqrt{7.84 \times \frac{310}{450}} = 2.32$, a very reasonable scale factor for velocity which could be easily implemented in the wind tunnel that we had.

The prediction rules are then obtained as $k_{\omega} = \frac{k_V}{k_L} = \frac{2.32}{450} = 0.0051$, $k_{ampl} = k_L = 450$, $k_{\varepsilon} =$ 1, and $k_{\sigma} = k_E = 7.84$. This summarizes the various scale factors for the model and for the results.

Scale Factors

Scale factor for	Symbol	Value
length and diameter	k_L	450
Wall thickness	k_h	310
chimney-shell density	$k_{\rho c}$	1.45
elasticity	k_E	7.84
velocity	k_v	2.32
frequency	k_w	0.0051
amplitude	k_x	450
strain	k_E	1
stress	k_σ	7.84

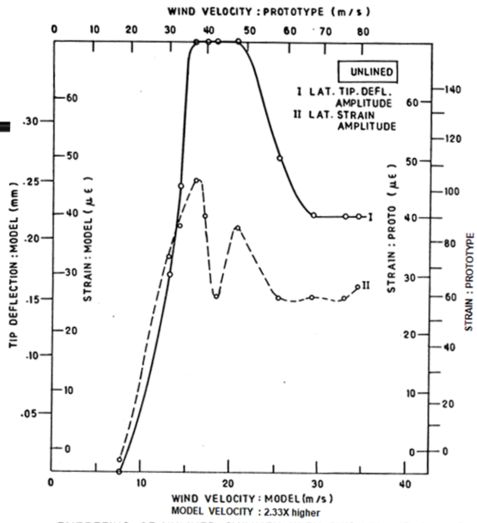
Three kinds of tests were conducted. In the first series of tests, the chimney model was mounted rigidly on a wind tunnel floor and was subjected to properly sheared wind profile with no terrain modeling upstream of the tunnel. The actual prototype chimneys were to have buildings to the north of the chimneys. So, when we use no terrain modeling upstream of the tunnel, this represents the condition when wind blows from any direction, but the north.

The strain and static deflection of the chimney were measured by 4 1.5 mm into 3 mm wire strain gauges mounted near the base of the chimney at 4 locations 90 degrees apart. The strain gauges were energized by V-shear strain indicator and signal displays on Textronic type oscilloscope. The dynamic displacement of tip was picked up by BNK type 4374 accelerometer energized by a charge amplifier BNK type 2635 and the dynamic signal displayed on this oscilloscope. The logarithmic decrement on the model was found by shaking the model and taking a polaroid photograph of the decay of the strain signals on the CRO.

Careful measurement gave a value of delta is equal to 0.18, the logarithmic decrement of vibrations which falls very much within the minimum and maximum limits for the concrete shell. Since, this was a modeling requirement, this was met. So, now after proving that the requirement for invariance of logarithmic decrement was met, a second series of experiment was conducted. In this properly scaled block representing the various structure of wind upstream of the chimney for the northerly wind was pasted on the floorboard and in-line deflection and strain, both static and dynamic, were recorded as also the lateral strain and tip deflection amplitude.

This represents the condition with wind blowing across the power house and other structures to the north of the chimney. The third series of experiments explored the effects due to wake buffeting. For this three chimneys model were mounted in line in the tunnel with the air blowing along the centers. The lateral vibration of the downstream chimneys was picked up by strain gauges and accelerometers. All these tests were repeated for a chimney without the inner lining representing the condition when the chimney shell has been constructed, but the brick lining work has not yet been assembled.

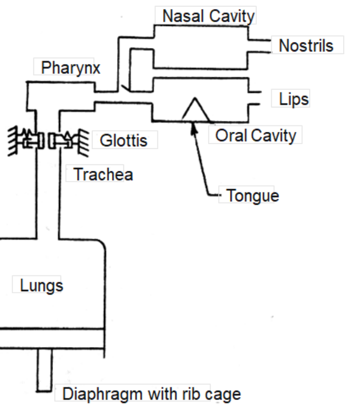
Results



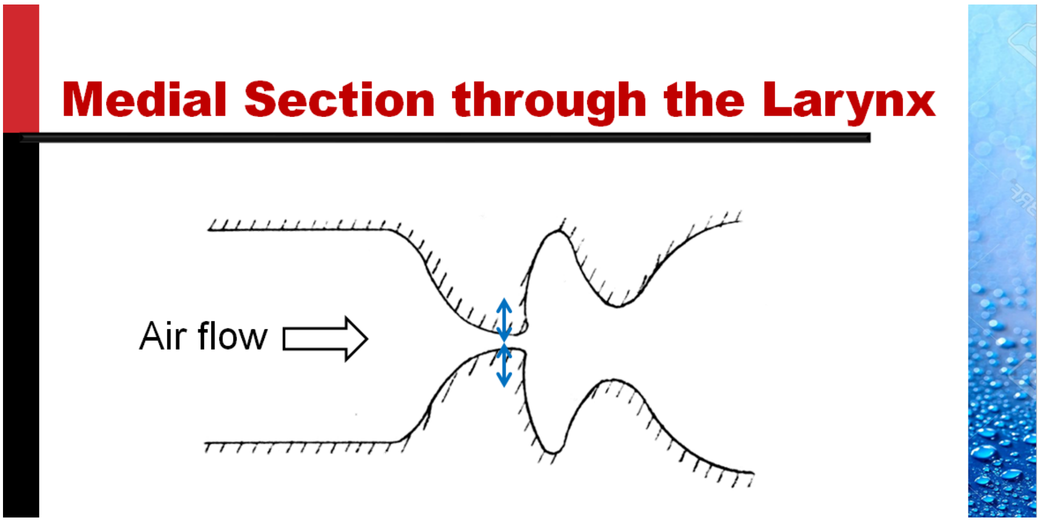
BUFFETING OF UNLINED CHIMNEY WITH TWO UPWIND CHIMNE
MODEL n = 95. 8 Hz ,Prototype n = 0.48 Hz , δ = 0.11

One of the results look like this. Wind velocity on this scale, model wind velocity was 2.33 times higher because k_V was 2.33. So, this is wind velocity of the prototype on this scale, same graph wind velocity for the model. Tip deflection of the model here, and in mm, and this gives you the strain of the prototype and strain of the model. The results were well within what was acceptable.

Functional components

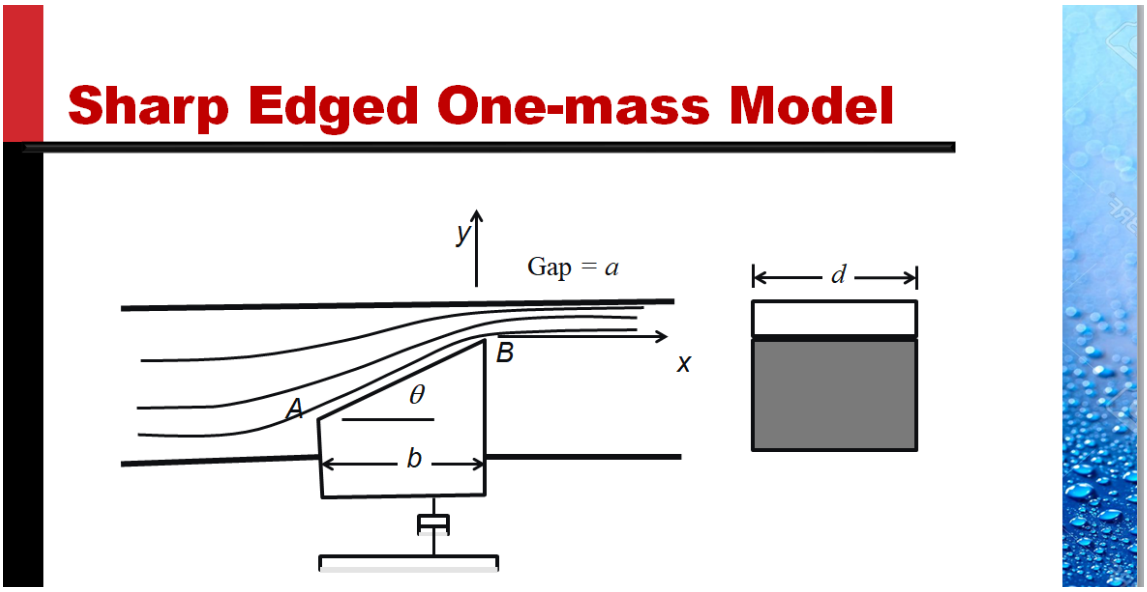


The next case study is a complex modeling exercise in which the mechanism of human voicing was explored. The human voicing apparatus consists of lungs which exhale air through trachea across the glottis, the glottis vibrate and produce sound. So, functional components are lungs and diaphragm which push air pass the glottis into pharynx and which is divided into passing through the oral cavity or through the nasal cavity. These glottis, the vocal cords vibrate when the flow goes past them.



What makes them vibrate? What is the mechanism? It was expected to be aerodynamic in nature is the that the pressure forces which oscillates make these larynx oscillate. So, if this is the medial section to the larynx, the air goes around this, this is towards the mouth, this is from the lungs. And as the air flows this, these two vocal cords which are mass and which are elastic vibrate. Why do they vibrate? If there is a mass spring dash board system, the equation of motion can be written as $m\ddot{x} = - c\dot{x} - kx$, where k is the spring constant, c is the damping coefficient and m is the mass, x is the displacement of the mass. In a vocal fold as the wind passes through this, the energy is fed into the vocal force.

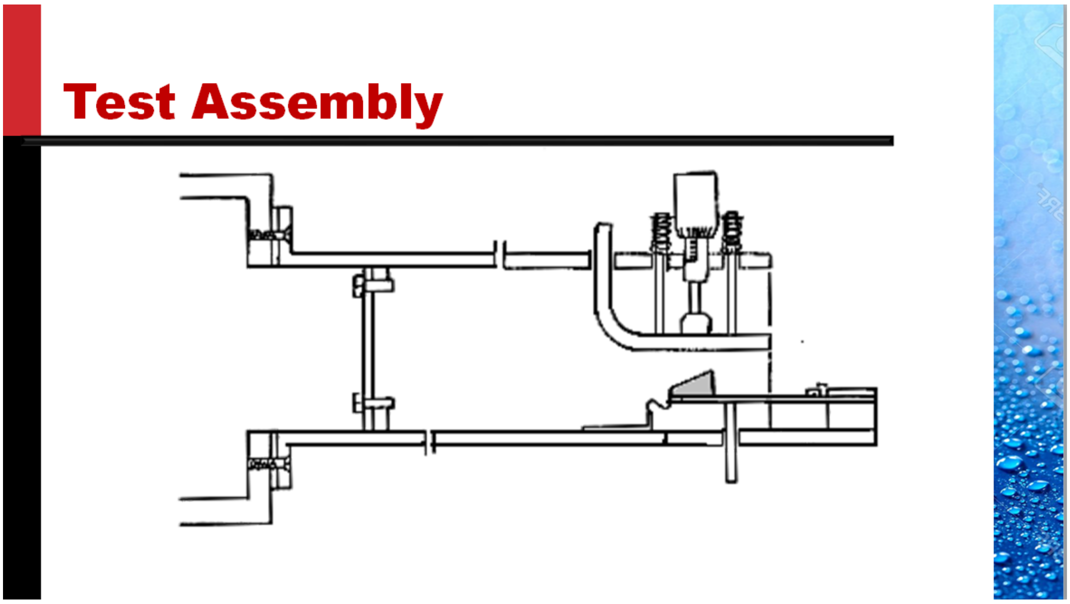
So, we are writing the equation of motion of the total force. This c should be negative that is the damping that is the result of aerodynamic forces, that is, the force component should be in phase with the velocity. So, that c is negative and we get energy fed into this vibrating system. So, if we make a model, let us represent this to be the center line of the glottis. So, there is only one vocal fold that we are shown.



We assume a simple model with this tapered at an angle theta of width b. So, when this is moving up, there should be a force here which is upward. And when it is moving down, there should be force which is downward. The force is applied by the pressure of air. So, the pressure on this face of the model vocal cord should be negative when it is moving up and should be positive when it is moving down. This results in feeding energy into the system.

The mechanism that we suspected at the beginning of the study was something like this. It was assumed this edge of the vocal fold is rounded, which really is. If you look at the vocal folds. And this roundness has something to do with why the energy was fed. The suspected mechanism was that as this vocal fold vibrates, the air coming in separates at this edge. It comes out as a jet. But the point of separation oscillates around this curved rounded edge of the vocal fold. When the vocal fold is moving up, the point of separation moves down so that the separation is a little below the top of this. Since the velocity of the flow here, in the absence of viscosity, is related to the pressure there, and so this velocity is constant, quite independent of the location of the vocal fold or the velocity. And since this area is larger, this velocity is same, this area here is smaller. So, the velocity is larger and the pressure here is smaller.

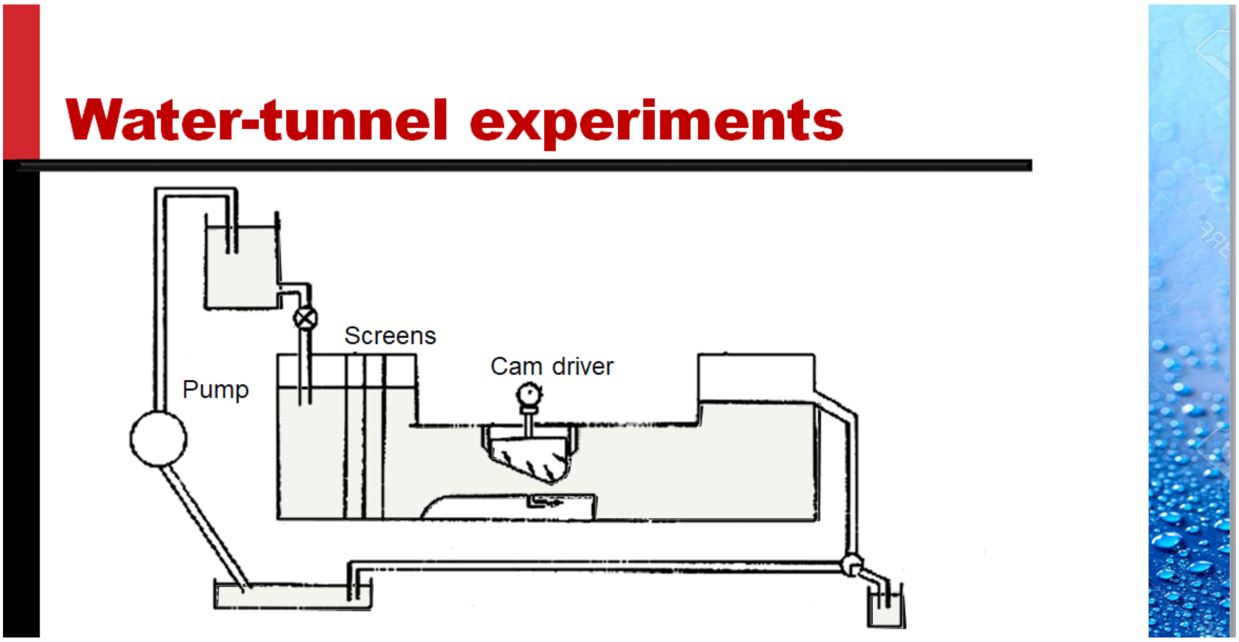
And so as the pressure on this is smaller, this means that there is a force upward. When this is moving up, there is a force upward. The right direction of force. Larger area, larger velocity and lower pressure. But when this is moving down, the point of separation moves back to here. Now this smaller area, smaller velocity and higher pressure. So while it is moving down, there is a higher pressure and this tends to push it back.



We started our investigation with this hypothesis. A model was constructed of plexiglass in which we used dummy springs. This represents the vocal fold mounted on a brass reed and this provided the springiness to this. As it move up and down, this brass reed bent and created stresses. String gauges at this location picked up the vibrations. We could adjust the opening by using this micrometer screw gauge. The wind moved in that direction and the results studied.

The signal from the strain gauges was sent to a CRO. And pictures taken. This is how the vibration grew when we started from rest. From this we could determine the logarithmic increment rate delta as discussed in the previous case study. And from this we could determine the rate at which the energy was fed. After doing lot of work with this setup, it was concluded that not enough energy was being fed.

That this mechanism was not explaining the mechanism for the actual human vocal cords or even for this. Where is that energy coming from? So with lack of any leads, It was decided to do some model test with a large model in a water tunnel. A 1 by 10 scale model that is 10 times the size of the vocal cords was constructed. It was driven by a cam driver so that it moved up and down. There was mechanism to inject dye at this so that while the experiment run we could see the movement of the separation point by noting down where the dye streak was separating from this model vocal fold.

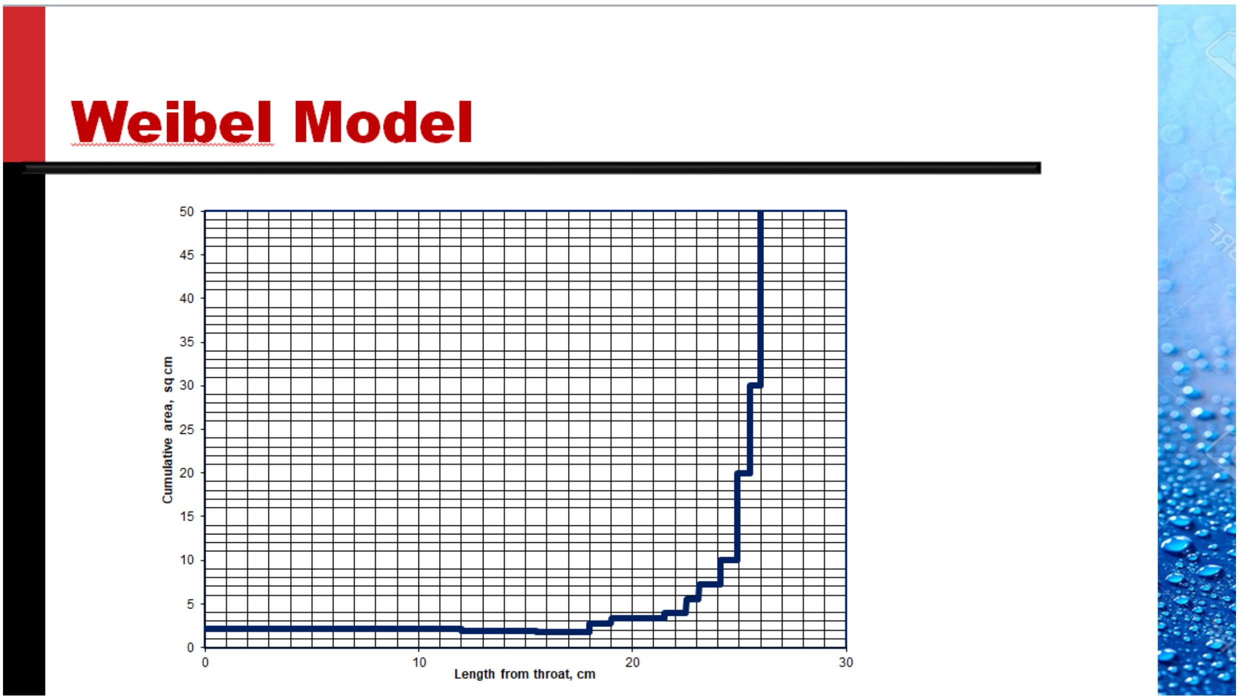


The initial forces in the water: scale factor for that is $k_{F,i} = k_\rho k_L^2 k_V^2$. The unsteady force scale factor $k_{F,u} = k_\rho k_L^4 k_\omega^2$, and so this gives you this Strouhal number. With $k_L = 1/10$; $k_f = 3/110$ for human vocal cords. The frequency was taken 110 and for this model was running at a frequency of 3 vibrations per second. So that gives you a $k_V = 1/366$. The velocity in the prototype is of the order of 100 m/s. So therefore, the velocity of the model was 2.7 cm/s. We ran the test at that scale. And when the test were run at that scale something interesting was seen.

Acually there was shifting of the separation point at this rounded edge, but this was very minor. Very interesting thing was seen here that the level of the fluid in the tank upstream was fluctuating at the same frequency and in phase with the motion of the cam driver. When it was moving up, the level was going up, when it was moving down the level was going down. In fact, this was the eureka moment. So it is the capacitance of this upstream system which is responsible for producing the force in the right phase, in the correct phase, to feed energy to the vibrating system.

Going back to the experiments that we had. In the same model as before we stuck in a microphone here to measure the fluctuations of pressure. And we found out that the pressure fluctuations were in the same direction as required for feeding in energy. We constructed an analytical model and we determined that this does feed the energy. We did experiments on a earlier air model and we find out the p_1/a_1 versus velocity.

The analytical model fitted very well. The dots are the measured points, the line is the calculated line. So this does explain the phenomena. How does it translate to human vocal cords? We looked at the model of the lungs.



This is throat. Distance from the throat in centimeters. You see the lungs are a series of branching tubes. Basic trachea, then the first generation, then the second generation up to about 29 cm. And they measure the total of the area for each generation. This is what the graph looks like. So when the diaphragm pushes air through the lungs, it is as if we are feeding air to the larynx through a tube which will open at this end.

So this is what the model is. This is the trachea and the lungs. This is the length effective length of that tube that we did found out in this model for about 29 centimeters. So this is how now this is a reactance model not a capacitance model, this is a tube. Air flows past this and then if this creates if this sets into vibrations, then the energy is fed into this and for particular frequencies not all frequencies. This could explain the mechanism. There was more work done which I am not presenting here because this permitted voicing only in certain range of frequencies.

So to cover the whole range of frequency, we have to construct what we call a two mass model and that two mass model would explain the energy feed into the vocal cords at all frequencies.

Thank you.