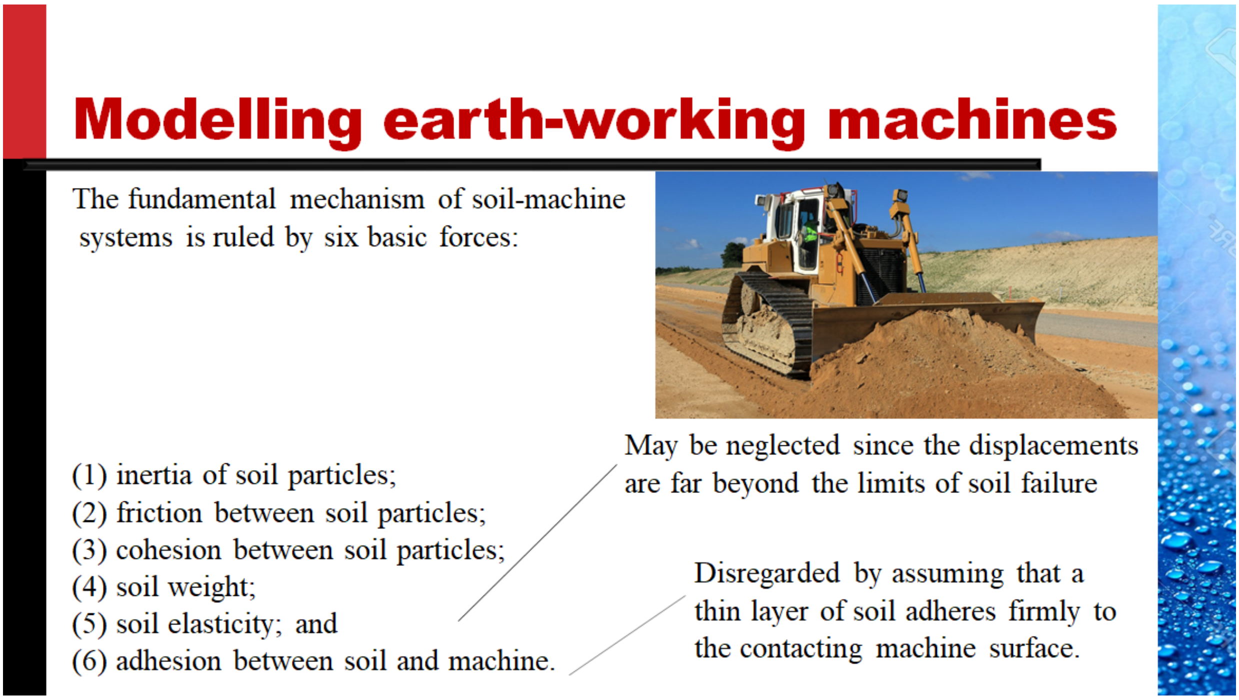
Similitude And Approximations In Engineering,
Vijay Gupta
Applied Mechanics
Indian Institute of Technology Delhi
Week - 08
Lecture - 26

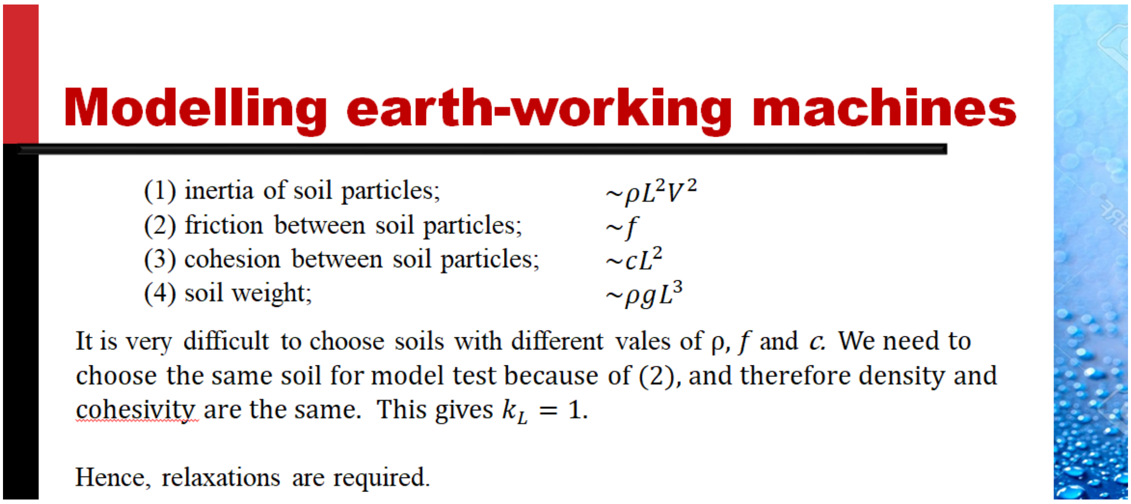
Welcome back. We continue in this lecture the justification and process of relaxing the modeling rules. We had discussed in earlier lectures that if there are weak laws and they are conflicting with some strong laws then we can neglect the weak laws and we learnt the techniques of doing so. But if the none of the conflicting laws can be disregarded, we must abandon the idea of modeling the whole phenomena. Instead, we can try to circumvent the most troubling law by any one of the methods that we discuss today. The first strategy is to restrict the generality of applications.

The phenomena is broken up in a number of special cases each governed by fewer laws than the entire phenomenon. If enough special cases are investigated and if they are all relatively independent of each other an approximation of the total phenomenon can be obtained by superimposing results of these special cases. Let us consider a ship in a severe storm at sea. Since the ship is operating in two distinctly different environments the air and water we can test two special cases.

First the ship is in still water, but exposed to wind forces and second the ship is in still air, but exposed to forces of water. Another special case that can be investigated independently of water and wind races is the hulls structural response to wave impact. Each of these special cases allow more scaling freedom than is allowed by the phenomenon its entirety.



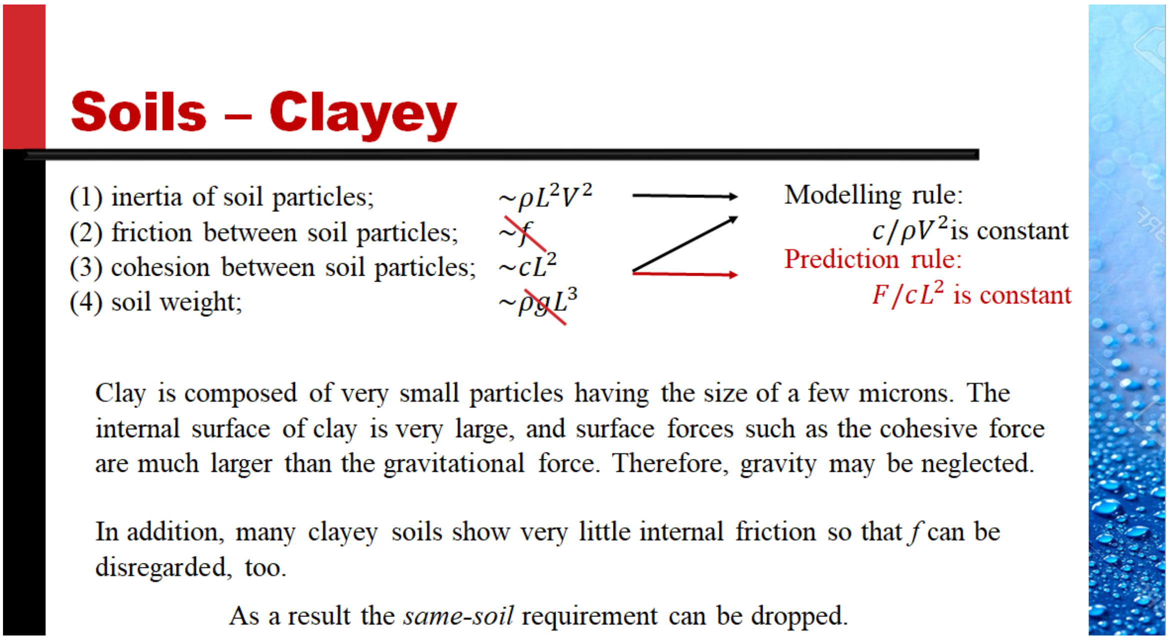
Let us do in detail an example of modeling of earth working machines. The soil machine interaction system is ruled by six basic forces. First, the inertia of soil particles. Second, friction between soil particles. Third, cohesive force between soil particles. Fourth, the weight of the soil. Fifth, the elasticity of the soil, and sixth, the adhesion between soil and the machine.



Since this soil is being worked to this extreme, and the soil failure takes place in such machine interactions, we may neglect soil elasticity completely since the displacement are far beyond the limit of soil failure. Similarly, the adhesion between soil and machine can be disregarded by assuming that a thin layer of soil adheres firmly to the contact machine surface. So, any further interaction of soil with this is only with the soil not with the machine surface. Thus there are four laws that need to be modeled. The inertia of the solid particle is like $\rho L^2 V^2$.

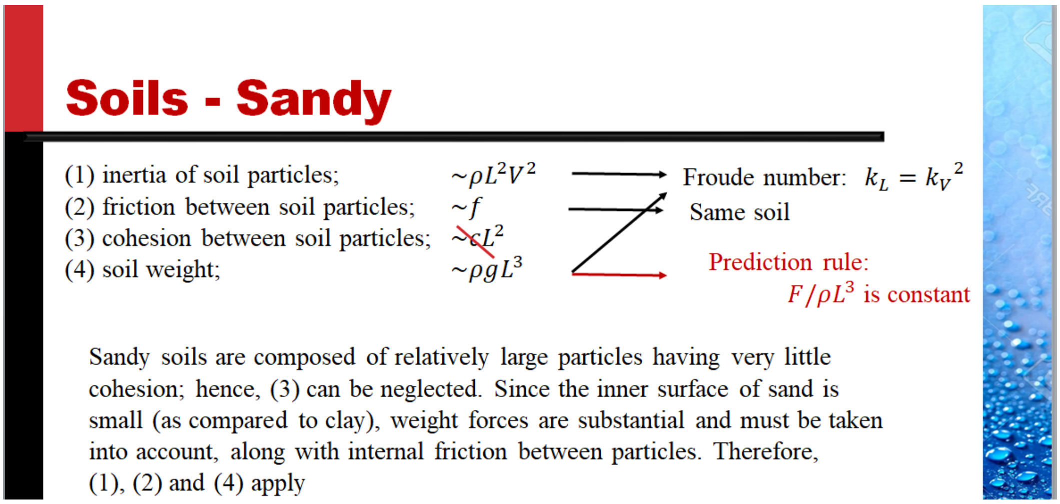
The friction between soil particles is modeled by f the coefficient of friction. The coefficient between soil particles is like cL^2 where c is the cohesive coefficient and the soil weight which is like $\rho g L^3$. It is very difficult to choose soils with different values of rho f and c. They go always as a package. We need to choose the same soil for the model test because of the second requirement above. The f itself is the pi number and invariance of that would require that the value of f be same. And if the value of f is same and we cannot choose ρ, f and c except as a package. Therefore, the density and cohesivity are also same. This requires that k_L be 1. No scaling is possible.

Hence, if we want to scale, the relaxations are required. So, let us see how we go about this. One of the methods of taking care of this problem is to classify soils as sandy or clayey. Clayey soils are entirely different from sandy soils. Normally, a soil has mixed properties, some clayey and some sandy, but let us separate these in two classes.



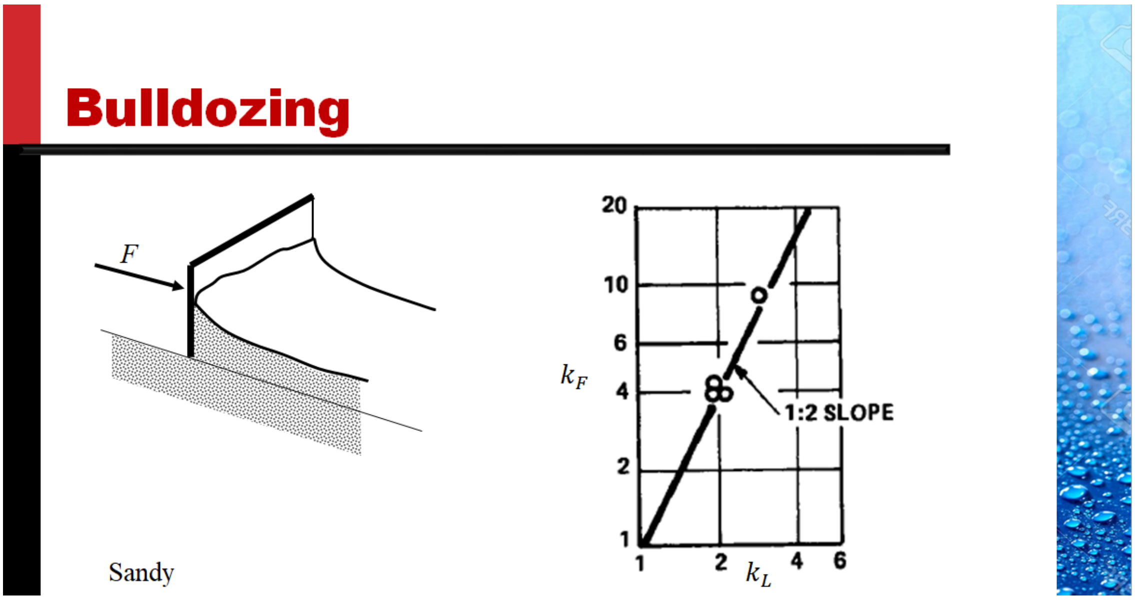
The clay is composed of very small particles having the size of few microns. The internal surface of clay is very large and surface forces such as cohesive forces are, therefore, much larger than the gravitational forces. Therefore, the gravity may be neglected. In addition, many clayey soils show very little internal friction. There is largely cohesion.

So, that *f* can be disregarded too. So, now only the first and the third laws need to be modeled. So, the same soil requirement can be dropped. We can choose different soil since we no longer need to keep f constant. The first and the third give the modeling rule that $c/\rho V^2$ is constant. So, the modeling rule is $c/\rho V^2$ is constant, and from the third requirement, we get the prediction rule that F/cL^2 is a constant.

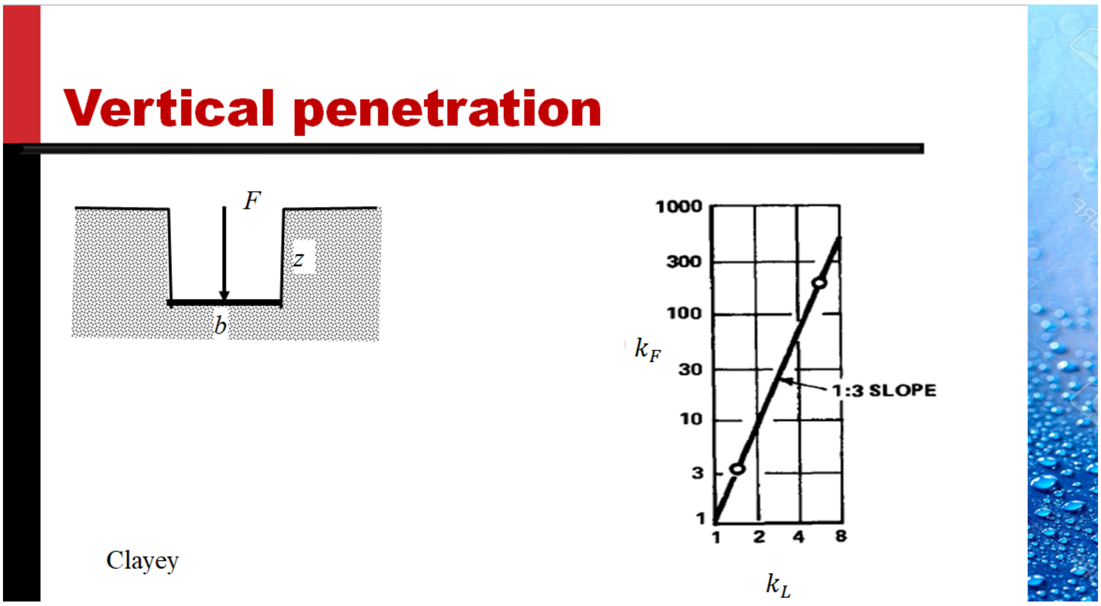


On the other hand, if we deal with the sandy soils, the sandy soils are composed of relatively large particles having very little cohesion. Hence, the third requirement can be neglected, since the inner surface of sand is small as compared to clay. The weight forces are substantial, and must be taken into account along with the internal friction between particles. Therefore, laws number 1, 2 and 4 above apply. From the first and the fourth, we get Froude number, and Froude number requires that $k_L = k_V^2$, since k_g is 1. This is the modeling requirement. We also need the modeling requirement that *f* is invariant, this requires that we use the same soil. And then we get the prediction rule from the last law that is the soil weight and that is $F/\rho L^3$ is constant for sandy soils. Most natural soils possess characteristics of both clay and sand, that is they are cohesive as well as there is internal friction.

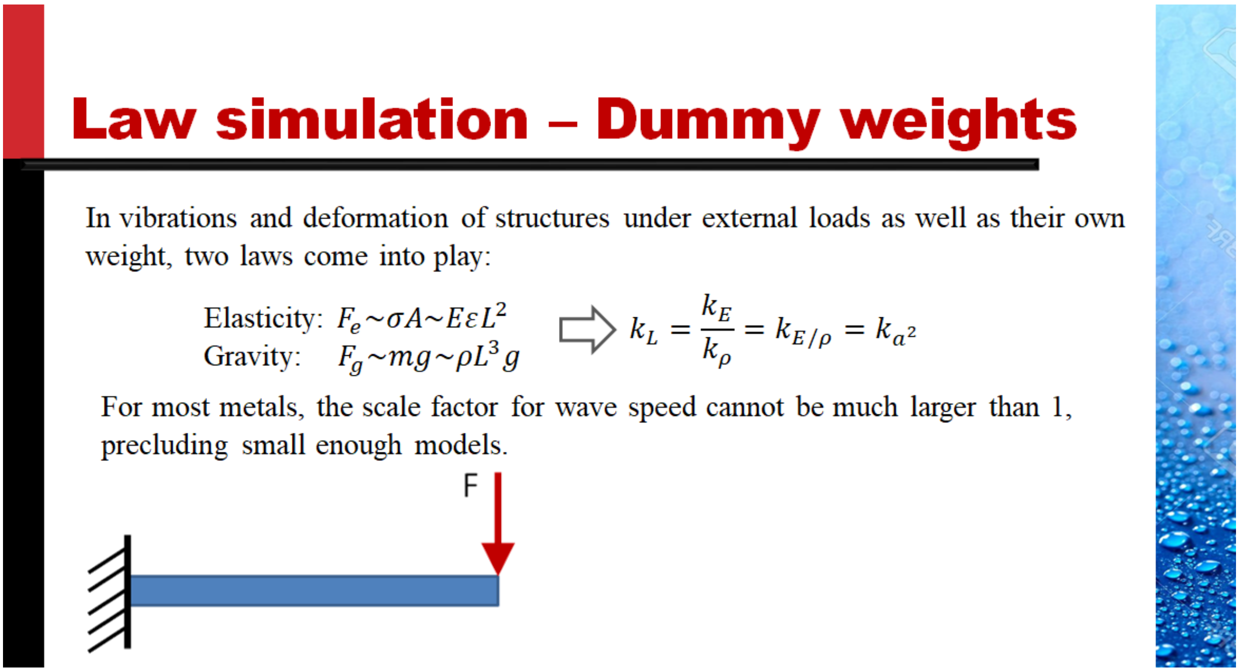
These characteristics cannot be modeled simultaneously, but the special soils of either dry sand or soft clay may be modeled separately because dry sand lacks cohesion and the soft clay lacks internal friction. Hence, by testing the model of an earth working device first in dry sand and then in soft clay we can estimate with some confidence its performance in mixed soil.



Let us consider the process of bulldozing. Here the blade of the bulldozer is pressing the soil in that direction with the force F. If we do these experiments then by changing the length scale k_L, this dimension, represents a length dimension. So, by using different length scales we determine the force required to move the sand and in one experiment we get these results. The slope of the line on log log plot is 1 is to 2. This indicates that k_F varies like k_L^2 and, as discussed earlier, this means that this is a sandy soil.

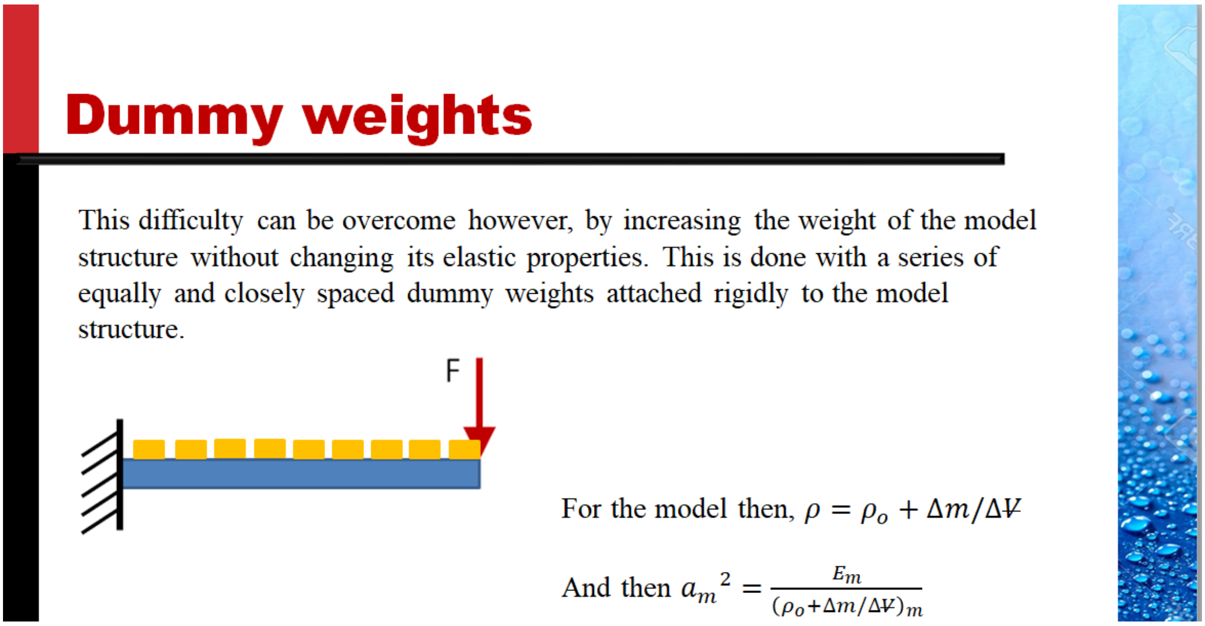


In the other experiment, we are considering the vertical penetration of a slab b. As the soil gets compacted, the required force F increases as z the penetration distance increases. So, in our experiments we measure the forces for the various values of b by z ratios and in two different experiments we use the same values of b divided by z. So, that the scale factor for b as well as of z is the same, k_L. And if we determine the force, the variation of force by k_L on a log log plot is with a slope of 1 is to 3. This means that k_F varies like k_L^3. This we saw is valid for clayey soils. So, if we do this experiment we get data for clay soils. How does the force of penetration vary with penetration?



Another class of techniques involve simulation of law, and this is a very interesting class of techniques. In vibrations and deformation of structures under external loads as well as their own weight, two laws come into play: elasticity, The elastic force is like s$F_e \sim \sigma A \sim E\varepsilon L^2$ where E is the elasticity, ε is the strain, and gravity force is like mg or $\rho L^3 g$, which gives you

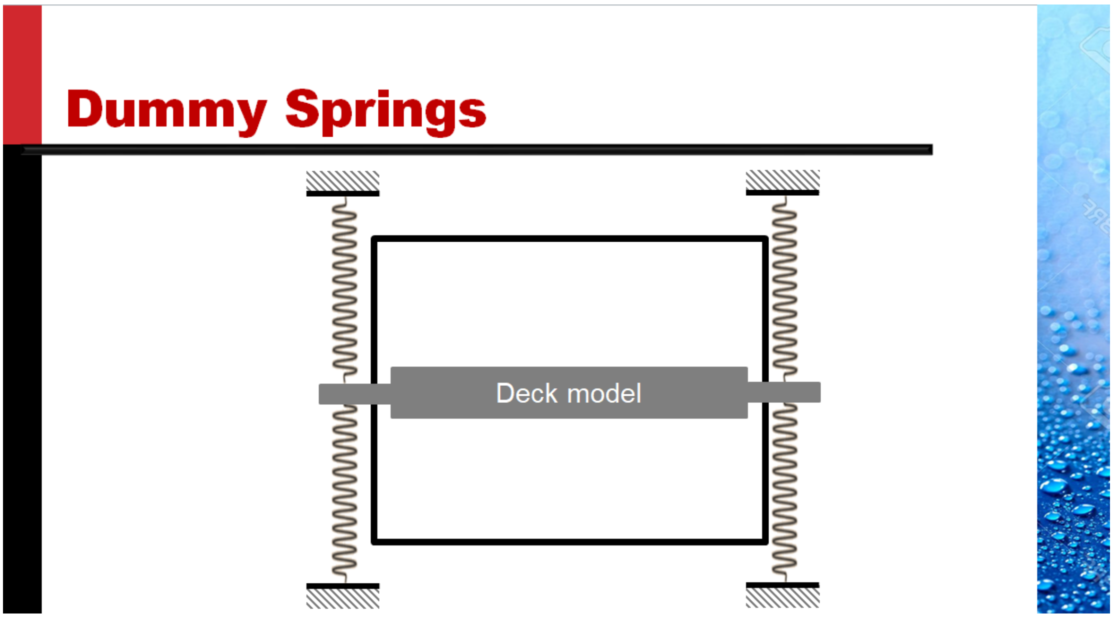
$k_L = \frac{k_E}{k_\rho}$, a single pi number which is $k_{E/\rho}$, and E/ρ is nothing, but the wave speed in the material, the speed of the longitudinal waves in the material. So, we must keep the length scale as k_{a^2}, a is the wave speed. Now for metals, the range of speeds, the range of wave speeds is very limited. We compare steel to an exotic material like silver, a squared changes by factor of 3 only. So, the maximum value of k_L that we can use is 3, if the model is made of silver. Obviously that is not going to fetch us much returns.



So, we need to simulate the law of elasticity. This difficulty can be overcome however by increasing the weight of the model structure without changing its elastic properties. That is why we use the term dummy weights. This is done with a series equally and closely spaced dummy weights attached rigidly to the model structure. Thus to this catalytic lever beam we could attach a series of equally and closely spaced dummy weights. Since these dummy weights are all not connected together they contribute very little to the elastic modulus, but increase the effective density. So, for the model the density ρ can be written as ρ_o, the density of the base material plus $\frac{\Delta m}{\Delta V}$, where Δm is the dummy weight per unit volume of the basic beam. And then $a_m{}^2$ for the model is nothing, but $\frac{E_m}{(\rho_o + \Delta m/\Delta V)_m}$ for the model. Thus, we are able to change E_m for the model drastically.

There was a bridge a cable state bridge several weeks after being open to traffic the bridge was observed to vibrate vertically in the winds between 40 and 50 kilometers per hour at a frequency of 0.6 Hertz. Only in a small window of velocities 40 to 50 kilometers per hour the bridge vibrated and the frequency was about 0.6 Hertz. Now why would the bridge deck vibrate? The deck of the bridge vibrates because of the vertices that are shed from this when the wind blows past this. If I have a rectangular section as shown here representing the section of the deck of the bridge as the wind blows past this a series of vortices are shed. These vertices, when they are shed alternately from top and bottom of the cylinder, cause to apply a vibratory force on the cylinder, A force up and down.

And if the frequency of this vortex shedding matches with the natural frequency the resonant frequency or the bridge deck the bridge could develop very large vibrations. The frequency of shedding of these vortices is governed by a constant value of the Strouhal number fL/V. So, given a natural frequency of the bridge and the length that in this case would be the height of the deck of the bridge. So, this would be resonant only at a particular velocity, and the velocity is between 40 to 50 kilometer per hour. We need to make modifications to the design of the deck so that the frequency of this changes.



After we change the design we need to test it in a wind tunnel. And one of the method is to use dummy springs. It is very difficult to match the springiness and the mass of the whole bridge. So, we take a section of the deck of the bridge, put it inside the cross section of a wind tunnel, and we suspend this model on a set of linear springs outside of the wind tunnel as shown. And the strength of the springs, the spring constants, are adjusted so that they give a frequency which is equal to the natural frequency of the actual deck. And so, with using these dummy springs, we could model the bridge in the wind tunnel.

Let us do in some details the modeling of a structural failure experienced by an aircraft carrier. This was an American carrier which broke up in a storm. The ship's behavior is assumed to be governed by inertial forces of the ship and of water governed by Newton's law of inertia, and by the weight of the ship and the water and the elastic forces of the ship. Since the propulsion is of no immediate interest, resistance due to the fluid viscosity is neglected.

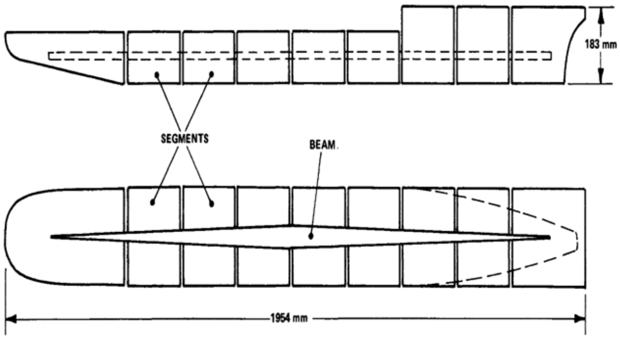
Inertial forces or the law of inertia, F varies like ma gives us a pi number, $\Pi = \frac{F}{\rho L^2 V^2}$, in line with similar expressions obtained earlier. The law of gravitation gives a pi number, $\Pi = \frac{FL^2}{EI}$. And the elasticity gives a pi number $\frac{F}{E\varepsilon L^2}$, as was done in the previous example of the vibrating beam. Inertia and the gravitational law gives you a pi number as Froude number $\frac{V}{\sqrt{gL}}$, And the elasticity and inertia give you Cauchy number $\frac{\rho V^2}{\varepsilon E}$. Since epsilon being non-dimensional itself, k_ε is treated as 1.

So, we get $k_L = k_E / k_\rho$, same as we obtained for before. Since the model tests are usually performed in water, k_ρ is 1. Density being a representative quantity stands not only for the density of water, but also for the density of ship's structure. The ship's model therefore is to be constructed for the material of the density of steel, but with the modulus E vastly smaller than that of steel. Such a material does not exist. So, it is not possible to model the phenomena fully.

Therefore, we reduce the scope of the study and restrict the model study to the bending vibrations of the ship. And here we infuse analytical knowledge of the bending vibrations in determining the laws of modeling. The law of bending vibrations are governed by equation that the bending moment $M = EI\frac{d^2y}{dL^2}$. This is a moment.

So, moment varies like E I divided by L. So, the force would vary like $\frac{EI}{L^2}$. So, the relevant pi number is $\Pi = \frac{FL^2}{EI}$. This is if we consider only the bending vibrations of the beam. So, we start with this and then the Cauchy number becomes $Ca = \frac{\rho V^2 L^2}{EI/L^2}$, or $\frac{L^4 \rho V^2}{EI}$. Recall that I varies like L^4 if the beam of the ship is geometrically similar. So, we will distort the similarity of the beam such that I does not vary like L^4. If we use the same material, that is, if we use ρ and E as same, k_ρ and k_E to be 1, then $k_V = \sqrt{k_I / k_L^4}$. And the Froude number which requires that $k_V = \sqrt{k_L}$, results in $k_I = k_L^5$. In one study the selected length scale factor was 136. Therefore, a model had to be built of the same material as the prototype whose area moment of inertia, that is, I was 136^5 times smaller than that of the prototype. k_I was 136^5.

Carrier model



The area moment, I, of the prototype was calculated in a separate computer study. The model's beam was varied to make it conform to the model rule of $I' = I/136^5$. This model was then used to study the ship's vibratory response at regular and irregular seas

Such a model was built by composing the hull of 9 segments, all joined by a continuous beam to allow flexures. Thus, the hull of the ship consists of 9 segments all connected by a beam inside, and the design of the beam was such that the I was scaled as k_L^5. The area moment I of the prototype was calculated by a computer study. The model beam was varied to make it conform to the model rule of I prime, that is I of the model is equal to I of the prototype divided by 136^5. This model was then used to study the ships vibratory response at regular and irregular seas.

Ramming mode performance of an icebreaker

The ramming mode (as distinguished from the continuous mode) of an icebreaker can be divided into three phases:

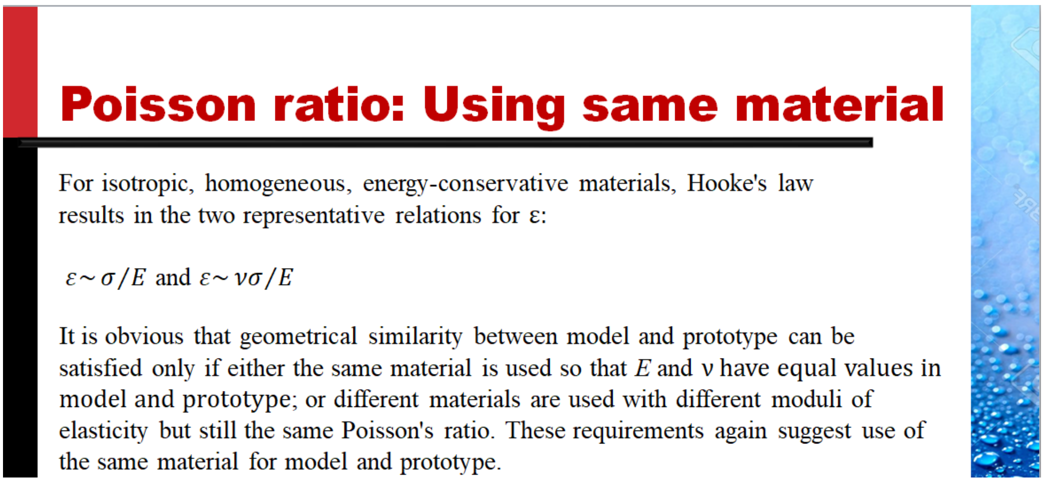
(1) the ship impacts and crushes the ice,
(2) the ship slides upon the ice;
(3) the weight of the ship upon the ice causes it to bend and finally to break.

Let us consider how to model the performance of an icebreaker. Icebreaker works in generally two modes, one is the continuous mode where the nose of the icebreaker ship penetrates the layer of ice on the sea. And the other method is the ramming mode which can be divided into three phases. The ship impacts and crushes the ice and it slides upon the ice. The weight of the ship now acting on this ice causes the ice to bend and finally, to break. The ship again advances crushes more ice slides further on the ice and then breaks the layer of the ice because of its own weight the ice cracks around the ship. In this process three forces dominate the inertial forces of the ship, the gravitational forces of the ship and the ultimate stress forces of the ice the ice is breaking. So, we it is controlled by what is the ultimate stress force of the ice. All other forces due to the acceleration and submersion of broken ice and to friction and wave making play only minor roles and can be neglected.

The Newton's law of inertia gives a pi number $\frac{F}{\rho V^2 L^2}$. The law of gravitation gives pi number $\frac{F}{\rho L^3 g}$ and the ultimate stress relation gives a pi number $\frac{F}{\sigma_u L^2}$ where σ_u is the ultimate stress of ice. The last two gives a pi number $\Pi = \frac{\sigma_u}{\rho L g}$. The model also operates in water. So, that ρ, g and σ_u are all same: their scale factors being 1. And so this leads to $k_L = 1$, that is, no scale model is possible.

We will have to work with the full scale model. In situation like this we infuse some analytical knowledge and this often proves useful. Here it is assumed that because of the failure of ice is mainly ascribed to its being bent, the ice will follow the theory of flexure. The bending moment is expressed as the bending moment M is due to force $\sigma_u \cdot LZ$. We use a different length scale in the vertical direction, in the direction of the thickness of the ice layer. So, if the thickness of the ice layer is Z and L is the horizontal length scale, then the area of the ice layer is L times Z. And the moment when we apply you will determine the moment by multiplying the force with Z. So, that the moment M varies like $\sigma_u \cdot LZ \cdot Z$, rather than $\sigma_u \bullet L^3$, if we had used Z as the same scale as L. So, the stress relation, the last relation here changes to $\Pi = \frac{F}{M/L} = \frac{F}{\sigma_u Z^2}$ converting moment into force. And if we use this, the invariant pi numbers become $\Pi = \frac{\sigma_u Z^2}{\rho L^3 g}$. For $k\sigma_u$, k_ρ and k_g all equal to 1, that is, if we work with water this leads to $k_Z = k_L^{3/2}$.

We can choose any k_L provided we scale the thickness of the ice layer differently according to $k_Z = k_L^{3/2}$. With k_L is equal to 50, k_Z is 354, a much thinner layer of ice then would be dictated by strict geometrical similarity. This is a result that could be easily implemented in the controlled environment of an arctic pool where experiments are done.



For isotropic homogeneous energy conservative materials, Hooke's law results in two representative relations for ε. Epsilon varies as $\frac{\sigma}{E}$ in the longitudinal direction and epsilon varies like ν the Poisson ratio times $\frac{\sigma}{E}$ in the transverse direction. It is obvious that geometrical similarity between the model and the prototype can be satisfied only if either the same material is used so that E and $\frac{\sigma}{E}$ have the same values in the model and the prototype or different material are used with different moduli for elasticity, but still the same Poisson's ratio. These requirements again suggest use of the same material for model and prototype. The condition of same material can be relaxed only if the transverse deformations are disregarded. Then the influence of Poisson's ratio becomes unsubstantial and permits the use of different material for model and prototype. This disregard of Poisson's ratio is an important relaxation in almost all structural problems.

Thank you.