

Similitude And Approximations In Engineering,
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Week - 06
Lecture - 22

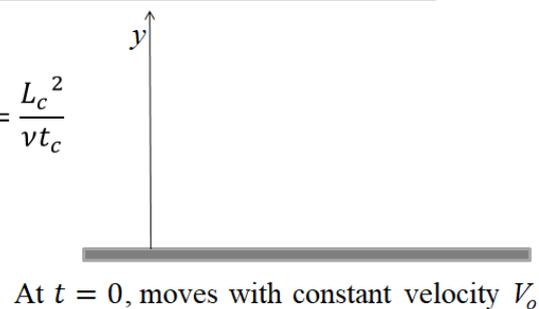
Welcome back. In this lecture, we will introduce the concept of self-similarity. Self-similarity is different from the similarity that we have been discussing so far. In the similarity that we have developed up till now, it is the number of the independent parameters in a problem that get reduced. And because of this, we are able to model with unicity parameters which are different from the original unicity parameters. But in self-similarity, we deal with the reduction in the number of independent variables rather than the number of independent parameters.

Typically, in many problems, we have two variables, one a space like variable and one a time like variable. We would see that there are cases where neither a characteristic length nor a characteristic time is specified in the problem. And because of this, it permits us to define a characteristic length in terms of a characteristic time or vice versa. We start with the example of the Stokes first problem.

Stokes first problem

$$\left. \begin{array}{l} \text{Unsteady forces: } \rho L_c^3 V_c / t_c \\ \text{Viscous forces: } \mu \frac{V_c}{L_c} L_c^2 \end{array} \right| \quad \Pi = \frac{\rho L_c^2}{\mu t_c} = \frac{L_c^2}{\nu t_c}$$

$$\frac{V(y, t)}{V_o} = \mathcal{F} \left(\frac{y}{L_c}, \frac{t}{t_c}; \frac{L_c}{\sqrt{\nu t_c}} \right)$$



$$\frac{V(y, t)}{V_o} = \mathcal{F} \left(\frac{y}{\sqrt{\nu t}} \right)$$

This is a problem which is related to the fluid in the half space above an infinite plate. The fluid is at rest originally, at time t is equal to 0 starts moving with a constant velocity V_o . In this situation, there are only two forces that are of interest. One is the unsteady force, the other the viscous force. The unsteady force, as usual, is estimated as mass times the acceleration, $\rho L_c^3 V_c / t_c$, where c stands for the quantity being a characteristic quantity.

Viscous forces on the other hand are like viscous stress μ times the velocity gradient V_c by L_c times the area which is L_c squared. These two estimates result in a pi number $\Pi = \frac{\rho L_c^2}{\mu t_c} = \frac{L_c^2}{\nu t_c}$. Now, we can write the dependent variable as normalized by V_o . So, that

$\frac{V(y,t)}{V_o} = F\left(\frac{y}{L_c}, \frac{t}{t_c}, \frac{L_c}{\sqrt{vt_c}}\right)$. Now, there is no characteristic length and there is no characteristic time in this problem.

If we take t_c equal to the current time t , then from this parameter, this parameter should not be too small and should not be too big because neither the viscous forces can be dropped from the equation nor the unsteady forces. This means this should be of order 1, and if t_c is like t , the current time, then L_c would be like $\sqrt{vt_c}$ where v is the kinematic viscosity. And so,

$\frac{V(y,t)}{V_o} = F\left(\frac{y}{L_c}, \frac{t}{t_c}, \frac{L_c}{\sqrt{vt_c}}\right)$ functional equation reduces to this $\frac{V(y,t)}{V_o} = F\left(\frac{y}{\sqrt{vt}}\right)$. We have reduced the equation to only one independent variable which is a combination of y and t . This represents considerable advance.

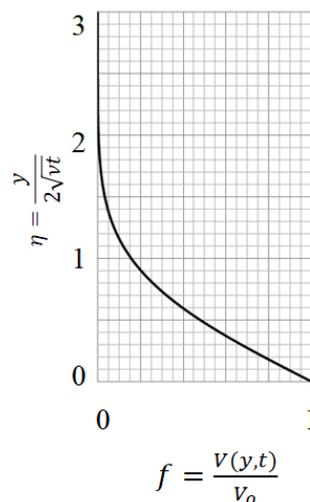
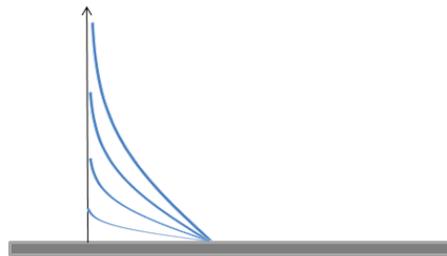
Stokes first problem

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \rightarrow f'' + 2\eta f' = 0,$$

with $f = 1$ at $\eta = 0$, and $f \rightarrow 0$ as $\eta \rightarrow \infty$

This is readily integrated to give

$$f = 1 - \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\zeta^2} d\zeta = \operatorname{erfc}(\eta)$$



So, we introduce η as a lone dimensionless variable is equal to $V^* = f(\eta)$. This factor of 2 is for the convenience in the sense that if we introduce this 2 here, then the resulting equation is easy to solve. The normalized velocity then is $V^* = f(\eta)$. If we make the requisite transitions, then the equation $\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$, the governing equation for this flow is reduced to $f'' + 2\eta f' = 0$. With the boundary condition that $f = 1$ at $\eta = 0$, and $f \rightarrow 0$ as $\eta \rightarrow \infty$.

This is far away the fluid is still at rest. This equation is readily integrated to give $f = 1 - \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\zeta^2} d\zeta$. And this is recognized as the error function, or rather, the complementary error function $\operatorname{erfc}(\eta)$. f is a complementary error function of η . The plot of this function is as shown. We have taken f on the horizontal axis and η on the vertical axis. f decreases very sharply from 1 to 0 over a range for something like 0 to less than 2. Now this is the normalized velocity as a function of η . So, if we choose a small time, then the velocity

profile in the flow would look something like this. For a little larger time this now is this and so on for still greater time.

I have drawn this profile by simply stretching the original profile in the y direction. As t increases the value of y for identical values η increases. That is accomplished by stretching the profile in the y direction. It is often convenient to talk about the penetration depth. The velocity jump at the plate reduces as you go up. At this value of y at this time the velocity is reduced to less than 1 percent of the velocity at the plate. We say that this is the penetration at this time. At a little larger time the penetration is more. We are now able to reduce a partial differential equation in two independent variables t and y to be an ordinary differential equation in a combined variable, independent variable, η. At η is equal to 1.82, the value of f is about 0.01. This is saying that at eta is equal to 1.82 the value of V has reduced to 1 percent of the original perturbation at the bottom plate. η is equal to $\frac{y}{2\sqrt{vt}}$ is equal to 1.82 at this location. This gives us penetration depth to be $3.64\sqrt{vt}$.

Stokes second problem

Flow in half space above an oscillating plate:

Unsteady forces: $\rho L_c^3 V_c / t_c$

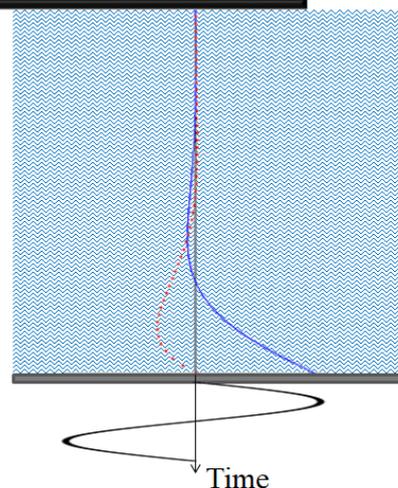
Viscous forces: $\mu \frac{V_c}{L_c} L_c^2$ $\Pi = \frac{\rho L_c^2}{\mu t_c} = \frac{L_c^2}{\nu t_c}$

$$\frac{V(y, t)}{V_o} = \mathcal{F} \left(\frac{y}{L_c}, \frac{t}{t_c}; \frac{L_c}{\sqrt{\nu t_c}} \right)$$

In this problem there is a characteristic time: $1/\omega$.

The value of L_c is obtained as $\sim \sqrt{\nu/\omega}$

$$\frac{V(y, t)}{V_o} = \mathcal{F} \left(\frac{y}{\sqrt{\nu/\omega}}, \omega t \right)$$



Let us consider one variation of this problem. This Stokes second problem. In this case the fluid is initially at rest and then the lower plate starts moving sinusoidally. So, that its velocity is given by $V_o \sin \omega t$. Here again, the unsteady forces and the viscous forces are of

significance and in the same manner as we did before we obtained the pi number $\frac{L_c^2}{\nu t_c}$. Now

since both kind of forces must be there in the flow, none of these forces can be neglected. This parameter $\frac{L_c}{\sqrt{\nu t_c}}$ is of order 1. The characteristic time can be taken as $1/\omega$, the circular frequency

of the velocities for oscillations of the lower plate. So, from this we can obtain the value of the characteristic length. As before we can write the normalized velocity $\frac{V(y, t)}{V_o}$ as a function of $\frac{y}{L_c}$,

$\frac{t}{t_c}$ and $\frac{L_c}{\sqrt{\nu t_c}}$. So, the value of L_c is obtained as $\sqrt{\nu/\omega}$ and we can write $\frac{V(y, t)}{V_o} = F \left(\frac{y}{\sqrt{\nu/\omega}}, \omega t \right)$

as a functional relationship for the normalized velocity.

This is not a self similar flow. There are both y and t in this equation as independent variables. But there is still progress, in the sense, we get to find out what is the penetration depth. This is the penetration depth, where this value is constant as ω changes. This value of the penetration depth changes. Larger the omega lesser is the value of y , lesser is the value of the penetration depth.

Here in this picture this blue curve represents the velocity of the flow over the plate, and the red dots represents the particle motion the location of the fluid particles at given time. We see a slight variation of this problem. In this case, the plate at rest, but the fluid above this is oscillating with a velocity $V = V_0 \sin \omega t$. The plate being at rest, tries to slow down the fluid near the plate. Same formulation, unsteady forces, viscous forces, and the same value of ν . The functional relationship is also same. Then again the characteristic time can be taken as $1/\omega$, but there is no characteristic length that is available in the problem statement. So, we can use $\frac{L_c}{\sqrt{\nu t_c}}$ for defining a characteristic length and that characteristic length would be clearly $\sqrt{\nu/\omega}$, t_c being $1/\omega$. This characteristic length is nothing, but equivalent to the penetration depth the distance above the plate in which the viscous effects can be felt. This is how the velocity profile and the particle motion would look like in this case. The blue line again the velocity profile is a distance like this. It would be L_c the effect within which the penetration of the presence of the lower plate is felt that is the viscous effects are felt. Above this the fluid motion does not recognize the presence of the stationary plate at the bottom.

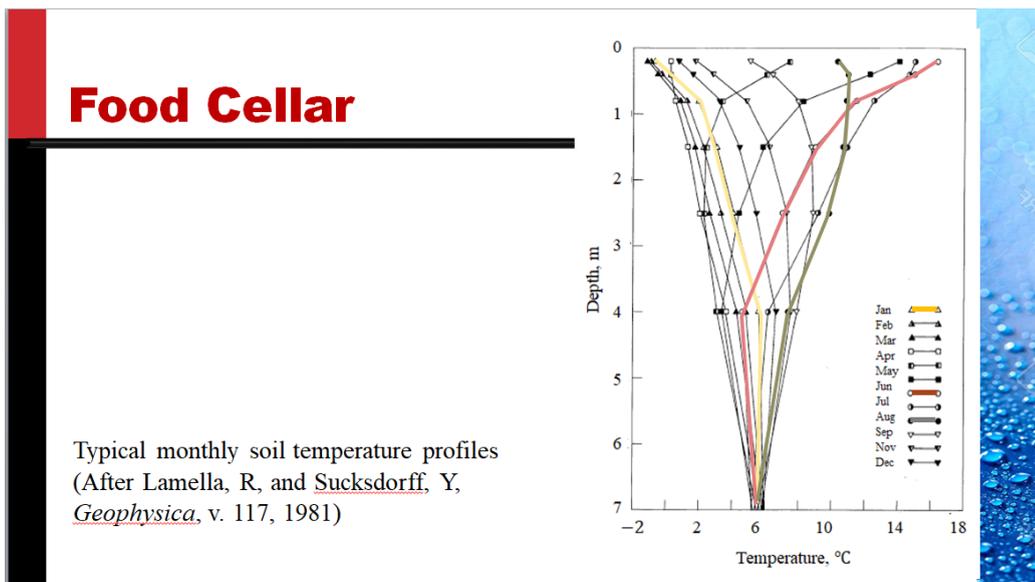
One of a very serious problem in many countries is spoiling of cooked food in the absence of refrigeration or even power. One solution has been suggested for the problem. If we go down and store the food items in a cellar which is sufficiently below the surface of the earth with the effect of the daily or yearly changes of temperatures are felt the temperature at some depth remains constant throughout the day or throughout the year. The temperature at the ground level can be seen as a superposition of two cyclic temperature variations. One the yearly cycle with the maximum temperature sometimes in June and the other the daily cycle of 24 hours with the maximum temperatures in late afternoon. This is for the northern hemisphere. We break the problems in these two parts and consider the two cyclic variations one at a time.

The unsteady heat conduction problem is exactly like the Stokes second problem, but here, instead of forces we work with the heat currents. The unsteady heat accumulation \dot{Q}_u is given as $\rho V C \frac{\partial \theta}{\partial t}$. $\frac{\partial \theta}{\partial t}$ is the rate of change of temperature. This gives us the scale factor k_{Q_u} is $k_\rho k_L^3$, which is k_M for the mass, k_c the specific heat, and $k_\theta k_t^{-1}$ for the temperature gradient. Similarly, heat current through conduction is given as A times the heat flux, which your Fourier law, is $k \frac{\partial \theta}{\partial x}$. So, that k_{Q_k} for conduction is k_k , the scale factor for thermal conductivity times the area k_L^2 , times the temperature gradient $k_\theta k_L^{-1}$, that gives you $k_k k_L k_\theta$.

And from this, this should both scale factor should be same. We get $\frac{\rho L_c^2 C}{k t_c} = \frac{L_c^2}{\alpha t_c}$ as a pi number. Exactly similar formulation as in the last problem, except that ν , the kinematic viscosity here has been replaced by α , the thermal diffusivity, which is $k/\rho C$, thermal conductivity divided by density times the specific heat. A , the thermal diffusivity of the

material. With this then, the normalized temperature differences $\frac{\theta(x, t)}{\theta_o}$ where θ_o is the amplitude of the temperature fluctuation at the surface, is $F\left(\frac{x}{L_c}, \frac{t}{t_c}; \frac{L_c}{\sqrt{\alpha t_c}}\right)$.

Here again we are given the characteristic time. There are two characteristic times in the problem. One is for the daily variations, 24 hour time period, and the other is for the yearly variation, 365 days as a time period. But there is no characteristic length, but as we have seen in the last examples, we can use this to find out the characteristic length. And what will be the interpretation of the characteristic length? The same as before. The distance through which the variation of temperature at the surface penetrates. The penetration depth and the value of penetration depth is like $\sqrt{\alpha\tau}$. Then, this is the functional relationship for normalized temperature difference: $\frac{\theta(y, t)}{\theta_o} = F\left(\frac{x}{\sqrt{\alpha\tau}}, t^*\right)$. For soil the thermal diffusivity is like $\alpha = k/\rho C = 1.38 \times 10^{-7} \text{ m}^2/\text{s}$. For the daily variations, the penetration depth is $3.6\sqrt{\alpha\tau} = 0.39 \text{ m}$ where τ is $1/\omega$, which is related to 24 hours daily variation. This gives you about 0.4 meters or 40 centimeters. So the temperature below 40 centimeters would not change within a day. On a daily basis they would be constant. And for the annual variations, the penetration depth is obtained as 7.56 meters, 7 and a half meters. So below 7 and a half meters there are no temperature variations. Throughout the temperature remains constant.



This is some data I could find in literature. This shows the typical monthly soil temperature profiles. This is in the cold climate, so temperature is varying between minus 2 degree Celsius and about 16 degree Celsius. This yellow line represents the average temperatures in the month of January. This red line the average temperatures in the month of June. The surface temperatures have changed that much. At a depth of 4 meters, the temperature in June is lower than the temperature in January. And about 7 meters they are the same. 7 meters about the same distance we saw in the last slide. The penetration depth of the yearly variation of temperature. And this shows the temperature variations, this gray line, the temperature variations in the month of August. The temperature has come down from the June temperature at the surface, but it is warmer than June at the depth of 4 meters. As 7 meters all temperature variations are almost absent. So these measurements confirm the development that we did in the last slide. The above analysis has practical implications for construction of buildings in Siberia or in Alaska. With the temperature throughout the year are very low.

Permafrost

If at the onset of summer the ground can be assumed to be at a uniform temperature of $-20\text{ }^{\circ}\text{C}$, and if the summer lasts for 100 days when the average temperature can be assumed to be $5\text{ }^{\circ}\text{C}$, find the depth to which the ground thaws.

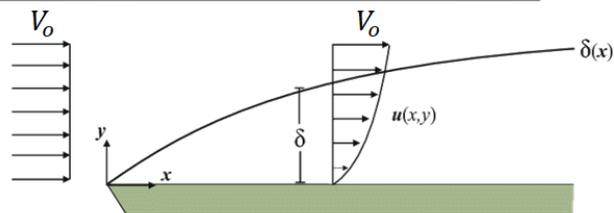


The ground is frozen in the winters but when the sun shines in the summers, the temperature of air at the surface changes. Becomes warmer, and so the soil begins to thaw. It gets hotter and it thaws, and it can no longer support structures. So foundations of the building must be done to below the level till which the effect of the summer thaw penetrates. The layer beyond which the soil does not thaw is called permafrost. So buildings must be anchored in the permafrost. And in a calculation similar to that done in the last slide we can find the depth of the permafrost for any given location. If at the onset of summer the ground can be assumed to be at a uniform temperature minus 20 degrees Celsius and if the summer lasts 100 days when the average temperature can be assumed to be 5 degrees centigrade, find the depth to which the ground thaws. Given that the characteristic time is now 100 days, we can find out the penetration depth in the development that we did in the last problem.

Let us now employ the concepts that we have developed to the development of boundary layer on a semi-infinite plate. A plate that starts at x is equal to 0 and goes up to infinity to the right.

Boundary layer on a semi-infinite plate

$$u = \mathcal{F}(x, y; V_o, \rho, \mu)$$



It is convenient to work with stream function ψ which automatically satisfies the continuity equation

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}$$

$$\psi = \mathcal{F}(x, y; V_o, \rho, \mu)$$

Clearly there is no characteristic length in the problem. There is no characteristic time in the problem. So there is a definite possibility that a self similar solution may exist. Functionally, the velocity u over the plate is a function of the two independent variables x and y , and of the parameters V_o , the upstream velocity, ρ the density of the fluid, and μ the viscosity. It is convenient to work with stream function ψ which automatically satisfies the continuity equation.

If we do not work with stream function ψ we need to deal with two dependent variables u and v , that complicates thing. So we introduce stream functions $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$. This definition of ψ automatically satisfies the continuity equation. So we only have to work with the momentum equation. Here again we can write $= F(x, y; V_o, \rho, \mu)$.

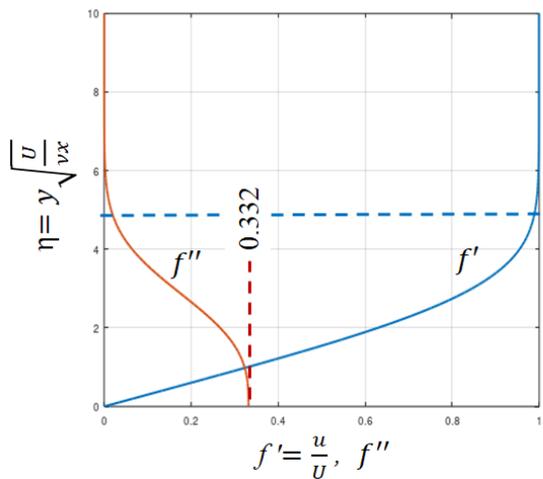
Within the boundary layer, viscous forces cannot be neglected as postulated with Prandtl. This means the inertial and viscous forces are important. So we consider these two forces. Inertial forces are like mass and the convective acceleration. Now we have used after Prandtl, a different length scale in the x , and a different length scale in the y direction.

In the x and in the z direction which is normal to paper, that is, into the plate, the characteristic length is L_c , and we use δ , the boundary layer thickness, as yet unknown, to be the characteristic length in the y direction. So the volume is now like $L_c^2 \delta$, and the convective acceleration which is like V_c^2/L_c . Viscous forces are like μ times $\frac{V_c}{\delta}$. So $\partial V/\partial y$ is estimated as $\frac{V_c}{\delta}$. The area which is not in the y direction which is x and z direction, is L_c^2 . The resulting pi number is $\frac{\rho \delta^2 V_c}{\mu L_c} = \frac{L_c V_c}{\nu} \cdot \frac{\delta^2}{L_c^2}$. By equating the two estimates, both the forces are important. So they should be of the same order.

So we can equate the estimates and obtain this pi number. Now this pi number should be of the order 1, and if this is order 1, we can obtain δ/L_c in terms of $L_c \frac{V_c}{\nu} V$, which is nothing but Reynolds number. The normalized stream function is $\frac{\psi}{V_o \delta}$, since u is like $\frac{\partial\psi}{\partial y}$. So $\frac{\psi}{V_o \delta} = F\left(\frac{x}{L_c}, \frac{y}{\delta}, \frac{\rho L_c V_c}{\mu} \cdot \frac{\delta^2}{L_c^2}\right), \frac{\rho L_c V_c}{\mu} \cdot \frac{\delta^2}{L_c^2}$ is nothing but Reynolds number times $\frac{\delta^2}{L_c^2}$.

There is no independent L_c , and we do not know δ . Let us use x , the local x location, as L_c . Then $\frac{\delta}{x} = \frac{1}{\sqrt{Re_x}}$. This gives you an estimate of the boundary layer thickness as a function of Reynolds number and x . And then the resulting function relationship is $\frac{\psi}{V_o \delta} = f\left(\frac{y}{\delta}\right)$. Using this in the momentum equation we get this an ordinary differential equation in terms of $\eta = \frac{y}{\delta}$: $f f'' + 2f''' = 0$. f'' is the second derivative f with respect to η and f''' is the third derivative f with respect to η . And this is to be solved with the boundary condition that $f = f' = 0$ at $\eta = 0$, and $f' \rightarrow 1$ as $\eta \rightarrow \infty$. The stream function as well as the velocity at the plate must be 0. And from far away the velocity must tends to V_o .

Blasius solution



We arbitrarily define the boundary layer thickness δ as the location where the value of u/U is 0.99

At $\eta = 4.91$ $f' = 0.99$

$$\text{This gives } \delta = \frac{4.91x}{\sqrt{Re_x}}$$

This is valid as long as the boundary layer flow is laminar, which on a flat plate is up to a $Re = 4 \times 10^5$

So, $\frac{u}{v_0} = f'$ tends to 1 as η tends to infinity. Basic equation is not easy to solve, but the famous scientist Blasius had obtained numerical solution of this equation using hand calculators almost 100 years ago. Today using MATLAB the solution can be obtained in a fraction of seconds with a small program a program of less than 10 lines you can solve this and this is the solution that we get. f' which represents the velocity $\frac{u}{v_0}$ is a function of η , and η was y/δ , and δ was like $\frac{x}{\sqrt{Re_x}}$. So, simplifying we get $\eta = y\sqrt{\frac{U}{vx}}$. ν is the kinematic viscosity.

Blue curve is f' . This is a solution independent of x . So, this is the velocity profile at any x , if we reduce the y coordinates and transform into η by this transformation. We arbitrarily define the boundary layer thickness δ as a function where the value of $\frac{u}{v_0}$ is 0.99. At η is equal to 4.91, f' is 0.99. And so we define the boundary layer thickness $\delta = \frac{4.91x}{\sqrt{Re_x}}$. Many textbook gives a further approximation to this as $\delta = \frac{5x}{\sqrt{Re_x}}$.

The second derivative of f , which would be the first derivative of u , which will related to the viscous stresses at the plate, the value of this is seen to be 0.332. From this we can find out that the shear stress at the plate can be converted into a skin friction coefficient which is equal to $\frac{0.662}{\sqrt{Re_x}}$. This is valid as long as the boundary layer flow is laminar, which on the flat plate is up to Reynolds number is equal to 4×10^5 . Where have we made the assumption that the flow is laminar? The assumption is made when we wrote the viscous forces by the Newton's law of viscosity, μ times the velocity gradient which is valid only for the laminar flows. Note that in the boundary layer flow in a semi-infinite flat plate there is no characteristic length in the x direction.

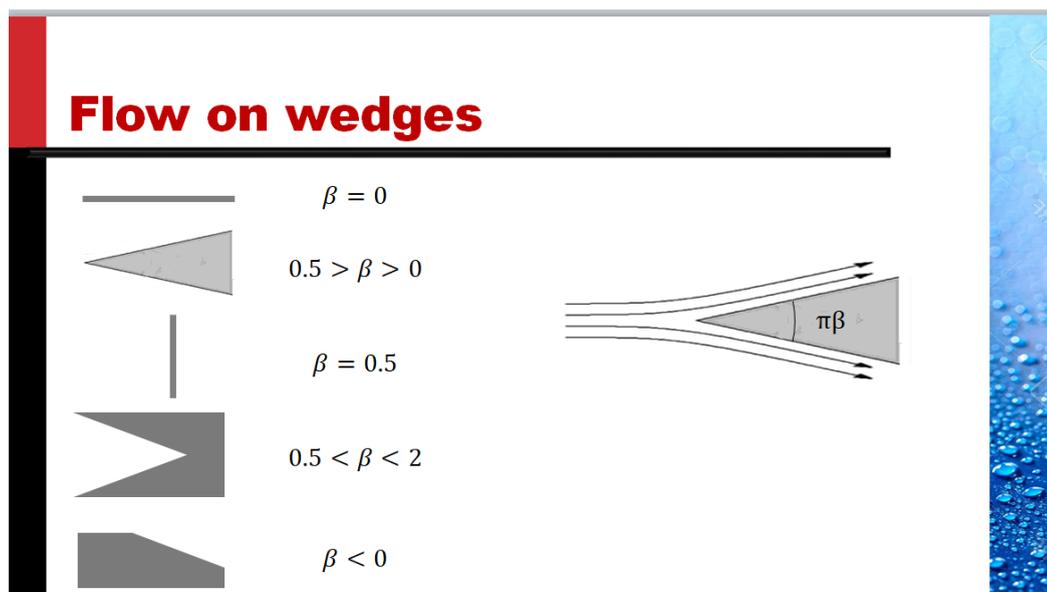
So, it is reasonable to suppose that the velocity profiles at various values of x are similar. If I want to find out if this is the velocity profile at this value of x , I can find out the velocity profile at a different value of x by simply squashing this in that direction. So, that this length

the boundary layer thickness is now reduced to this length. There is no change in length scale in the velocity, $\frac{u}{V_0}$ same, but the y is scaled by delta.

This is nothing but the manifestation of self similarity. The velocity profile at a given location can be found out from the velocity profile at another location by simply scaling the velocity profile. In this case, only in the y direction. And when the value of x the velocity profile is obtained by further scaling. These three profiles are drawn by simply stretching or collapsing in the y direction.

There is another way to interpret self similarity. We can consider the velocity profile at this location and the velocity profile at this location as two different experiments on the same plate with the same unicity parameters. And if we do this the velocity profiles are similar. Location exchanges. So, this scale in the y direction changes.

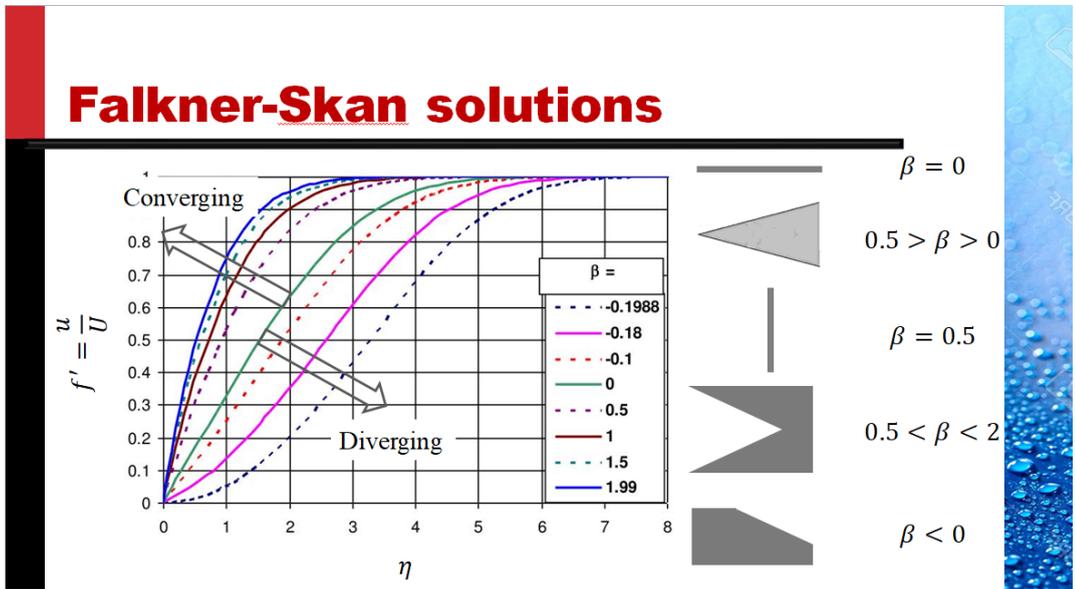
Self similar solutions are of great significance in many applications. In almost all such applications the common factor is that there are no natural characteristic quantities of the two variables involved. And so, the characteristic quantity of one variable is determined in terms of the value of the other variable. It is not just on the flat plate, but flow over wedges are also self similar. Falkner Skan obtained the conditions under which the flow could be similar, and he said flow over wedges is one case where the flows are similar, that is, the profiles that we get at any x is similar to the profile that we get at another x.



Flat plate correspond to the case β is equal to 0. This wedge with β between 0.5 and 0 is a flow in which the fluid is accelerating. This is for β is equal to 0.5. This is the case where the flow is against a flat plate. The value of β between 0.5 and 2, the flow reverses and this is also a flow in which along this surface the flow would be accelerating. This diverging flow on the other hand, for β negative, is the case where the flow is decelerating. And as we studied in the last lecture, the flow tends to separate and β is when the flow is decelerating. Let us see can we get this information here. So, Falkner Skan says that if there is a wedge with angle $\pi\beta$ in there, then the velocity as a function of x is can be written as $U = V_0 x^m$ with $m = \frac{\beta}{2-\beta}$.

So, for m positive the velocity is increasing in x direction for β less than 2. We introduce a variable η is equal to $y\sqrt{\frac{m+1}{2}\frac{U}{\nu x}} = y\sqrt{\frac{m+1}{2}\frac{V_0}{\nu}x^{\frac{m-1}{2}}}$. We get this equation

$f''' + ff'' + \beta(1 - f'^2) = 0$, an ordinary differential equation of one dependent variable f and one independent variable η . And this has to be solved with three boundary condition f and f' is equal to 0 at η is equal to 0 and f' tends to infinity as η tends to infinity. This is the kind of profile that we get. This is the edge of the boundary layer on the upper surface.



This is the velocity profile within the boundary layer. And these are those profiles for different values of β . This green line is for a flat plate when β is equal to 0, and we have discussed that case just now. This is for the converging flows where the flow is accelerating. These are the cases where the flow is accelerating. This is also a case where flow is accelerating, for β positive. We see the gradient of f at the wall changes. It increases. But this set of curves represent the negative value of beta.

This is for β is equal to -0.1 . This line is for β is equal to -0.18 , and this line is for the value of β as minus 0.1981 . What is the significance of this value? This is the value when the shear stress at the wall reduces to 0. That means, the gradient of velocity at the wall becomes 0.

Any further decrease in the value of beta would result in the reverse flow. So, this line, this last line here, is what represents separation on the surface. The boundary layer beyond this separates. So, what is self similar about it? Let this represents the growth of boundary layer. If this is the velocity profile at this location then this velocity profile at this location. It has been obtained by simply stretching this velocity profile both in this direction as well as this direction.

Stretching in this direction to match the boundary layer thickness here, and stretching in this direction to make the velocity at the edge of the boundary layer match with the inviscid solution at this location. Similarly, at this point this velocity profile, this velocity profile, and this velocity profile, all velocity profiles are self similar. Anyone can be obtained by scaling another profile linearly.

Trinity Explosion

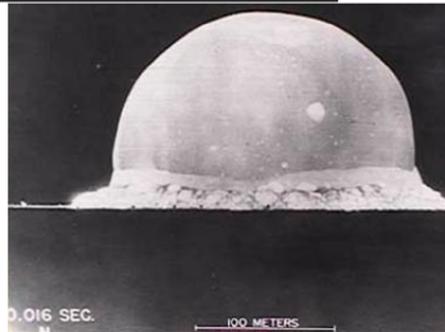
Two assumptions need to be made:

1. The energy (E) was released in a small space.
2. The shock wave was spherical.

We have the size of the fire ball (R as a function of t) at several different times.

How does the radius (R) depend on:

- energy (E)
- time (t)
- density of the surrounding medium ρ (initial density of air)



Let us give one more example, the example of Trinity explosion.

Trinity was the first test of a nuclear device conducted by the USA. In slightly after the bombing Hiroshima the US authorities released a sequence of still shots which contained both the time stamp as well as the scale. So, you could measure from this there are time t equal to 0.016 seconds after the explosion. This is the diameter of the fire ball, and this diameter can be scaled in terms of meter using this mark of the scale.

After the war a famous British physicist Professor G.I. Taylor estimated, using this picture, the yield of the bomb. There was no other data that was used. He did not have the information of what the yield is. Using self similarity alone he estimated the yield. Two assumptions needed to be made that the energy E of the explosion was released in a very small scale and that the shock wave that came out the fireball was spherical.

Now from this picture we have size of the fireball as a function of time at several different times. The radius of the fireball should depend upon the energy E , the time t , after the explosion, and the density of the surrounding medium, which is air. The initial density of air. Since there are no length scales and time scales involved in the problem there is no parameter which contains any length scale or time scale. We may take the current time t as a characteristic time, and express the characteristic length in terms of this time t .

k_E , the scale factor energy would be like $k_m k_v^2 = k_\rho k_L^3 k_v^2 = k_\rho k_L^5 k_t^{-2}$.. Visualize a picture at two different times to as to be as if there are two different experiments, in a fashion similar to what we discussed in the last example.

And since it is the same explosion $k_E = 1$. From this we get $k_L^5 k_t^{-2} = 1$. That is, R should vary like $t^{2/5}$. At time t is equal to 0.006 seconds, the radius of the shock wave was approximately 80 meters. This is the diameter, so radius is reduced to radius 80 meters. And at time t is equal to 0.016 seconds radius 105 meters. From this Professor Taylor estimated the yield to be $10^{14} J$. 1 kilogram of TNT is energy equivalent to about 4 mega joules. So, that $10^{14} J$ is about 25 kiloton of TNT. The actual yield was 22 kiloton. So, that should convince you of the power of similitude analysis and self-similitude.

Thank you.