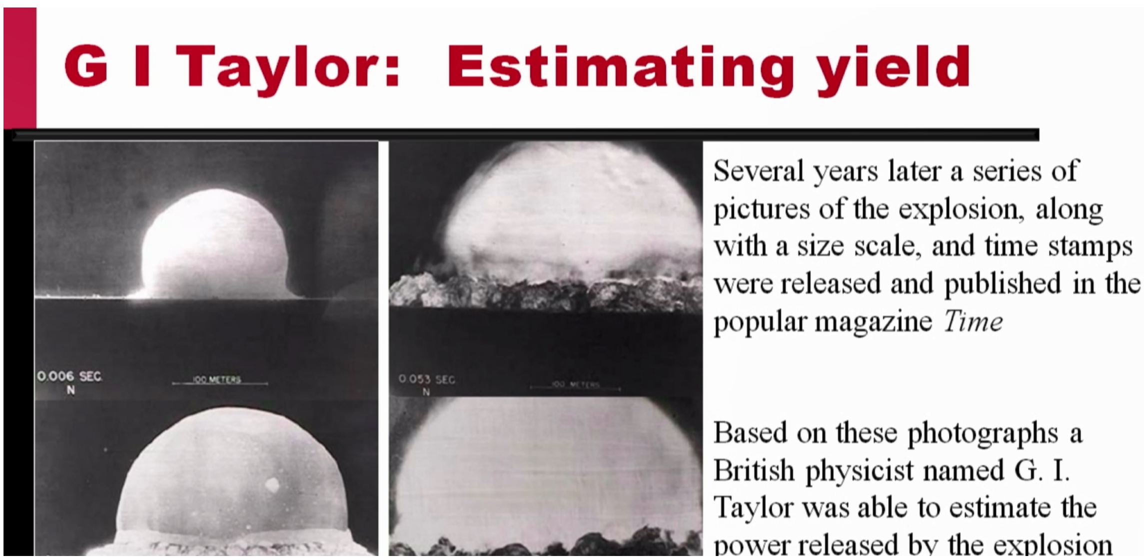
Similitude And Approximations In Engineering,
Vijay Gupta
Applied Mechanics
Indian Institute of Technology Delhi
Week - 01
Lecture – 01

Welcome, in this course we will introduce an important tool which engineers use, the tool is Similitude and the basis of approximations while doing calculations. In this first lecture we will introduce the power of the Similitude studies. Perhaps the most dramatic use of Similitude was when Sir G.I. Taylor of Cambridge University predicted the yield of the nuclear explosion. The first explosion of an atomic bomb was a Trinity test in New Mexico, USA on July 16, 1945.



It used an implosion device named the gadget. Several years later a series of picture with explosion along with a size scale and the time stamps were released and published in the popular magazine Time. Based on these photographs the British physicist named G.I.

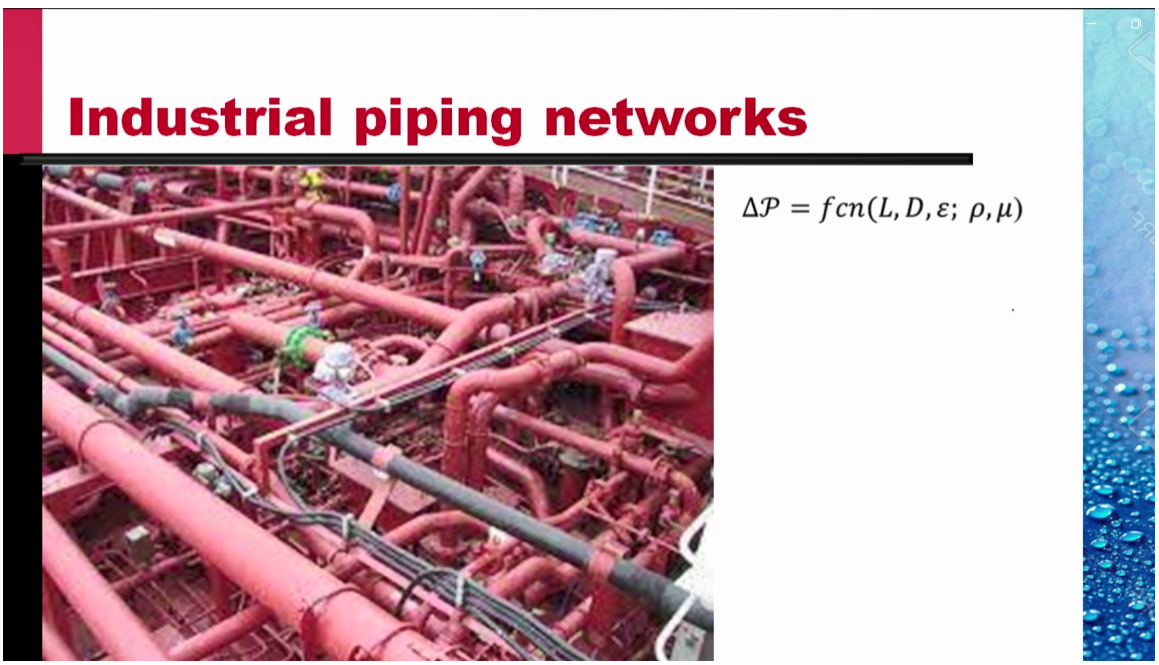
Taylor was able to estimate the power released by the explosion which was still classified at that time. Using purely dimensional arguments he showed that the radius of the fireball should be given by $R = \left(\frac{Et^2}{\rho}\right)^{1/5}$. From the release pictures at time t is equal to 0.06 second the radius of the shock wave was approximately 80 meters. Using the density of air as 1.

2 kilogram per meter cube this result gave the value of e the yield of the nuclear weapon

as 100 terajoule which is approximately 25 kilotons of TNT. This was very close to the actual yield of the atomic bomb and caused a controversy. We will talk about that a little later. Other applications of similitude. Here is shown the Saturn V rocket which was used in the Apollo program to land the man on the moon.

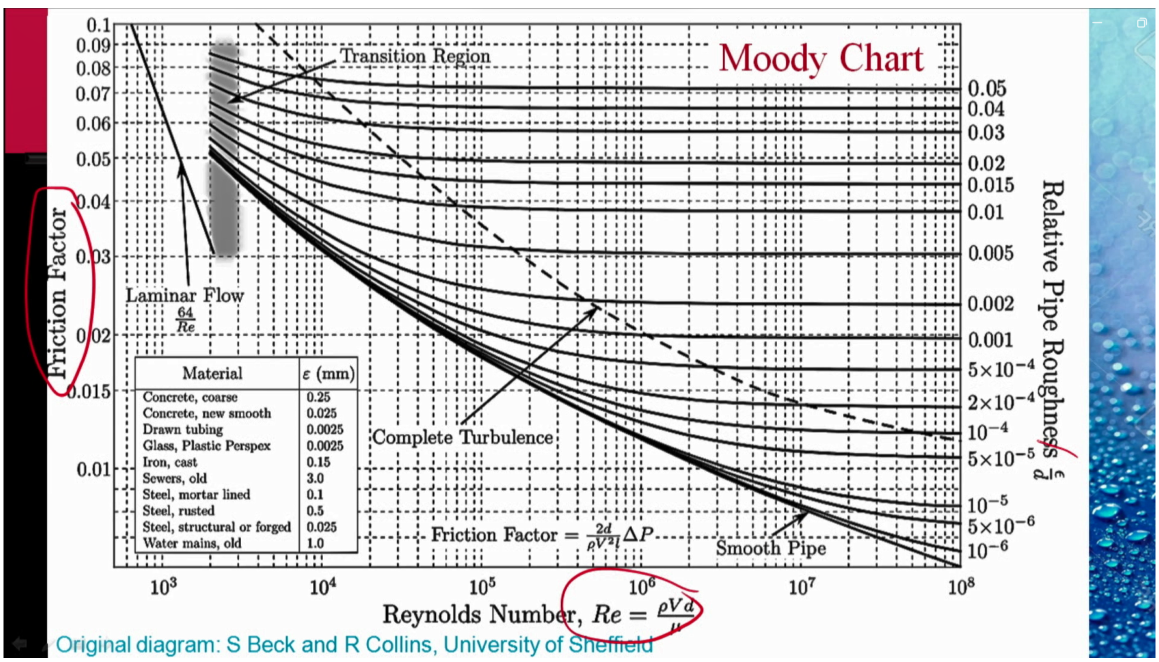
The actual rocket was 111 meter tall with 2800 ton and the value of this was 1.5 billion dollars in 2022 value. We had to be very sure that this rocket worked perfectly, and to ensure that this rocket worked perfectly we could not afford to make a real rocket. So, a lot of work was put into scaled models. A 1 is to 50 scale model was made and was used in a wind tunnel to study the aerodynamics of this completely.

There was also a larger model used to study the sloshing of fuel within the rocket. All these studies proved that the actual rocket when built would perform beautifully and only then the rocket was made. Another application of Similitude is in the industrial piping network. In any plant of making chemicals there are lot of pipes that are used to make the ingredients flow from one side to another. We worry about the pumping power required for pumping the fluid in these pipes.



The pumping power required depends upon various parameter. The pressure that needs to be applied to a length of the pipe is a function of the length, the diameter of the pipe, the roughness of the walls, the density and viscosity of the fluid. Since we use pipes of various diameters and various materials and various thicknesses, do we do experiments on each different pipe to find out the pressure losses? No. The dimensional analysis comes to help. It can be shown that rather than ΔP, the pressure difference, being function of 5 parameters L, D, epsilon, rho and mu, we could non-dimensionalize and determine that this parameter is a function of this parameter, this parameter and this parameter.

So instead of one dependent parameter being dependent upon 5 independent parameters, now we notice that one non-dimensional dependent parameters is a function of 3 independent non-dimensional parameters. This results in a lot of savings of effort. We can further show $\frac{\Delta P}{\rho g} = f\frac{V_{av}^2}{2g} \bullet \frac{L}{D}$, where f is a function of the first parameter, which those of you who have studied fluid mechanics would realize is the Reynolds number, and the roughness parameter $\frac{\varepsilon}{D}$. Now, the scientists discovered since f is a function only 2 parameters, we could do the experiments and determine how does the function f vary with the parameters Reynolds number and $\frac{\varepsilon}{D}$, and produce one chart. Now this chart can be used to predict the pressure differences required to pump the fluid in any pipe of any size at any flow rate.



You calculate the Reynolds number, enter from here, go to the relevant ε/D and on this scale you read the value of the friction factor. This results in a decrease in number of independent parameters, which represents a big saving. The above examples give a sample of applications of the first part of the course that is the similitude. In the second part of the course, we will discuss how approximation are made to obtain solutions of complex problems in engineering. Let us introduce a similitude a little further.

Similitude refers to the property of two distinct sets such that from the experiments or analysis conducted on one system, we are able to predict results for the other system. The two systems could be, one a model and the other a prototype. So, by doing experiments on model which could be small or big, fast or slow, we can find out the result or we can predict the result that will be obtained on the prototype. The two systems may be geometrically similar systems of different scales or they may represent two entirely different phenomena. Mathematically, similarity refers to a transformation of variables that leads to a reduction in the number of independent variables that specify the problem.

The beginning of the similitude is credited largely to William Froude, a British admiralty officer who was charged with designing the ships with low resistance and they devised rules for conducting the studies on scaled models of a smaller size and then predict from them the actual resistance of the full prototype when it is built. These are the two models that are in a museum now on which William Froude worked, and they give beautiful result. In fact, the technique that the device is still used in the present day naval engineering. This is a large scale towing tank where a model boat is being towed in a channel. The force required to tow it is red and then from this the prediction is made of what the actual drag would be on the prototype.

Another milestone in model studies is attributed to the Wright brothers who invented the aircraft. Frustrated by the failure of the earlier glider experiments, they chose to develop their own data in a wind tunnel. They designed an apparatus which is known as a wind tunnel. In this a fan here sucks the air through a rectangular cross section one foot by one foot. A scaled model of the wing is placed within this section.

The forces on these are measured and from this the prediction is made on what the lift and the drag would be on the full scale wings. The propellers that the Wright brothers used was also designed based on the data developed in the wind tunnel. From this small beginning wind tunnel grew in size and complexity. This picture shows the Langley full scale tunnel with a test section of 30 feet by 60 feet in which an actual airplane could be tested. There is no need to put the full scale aircraft in a wind tunnel.

So, this full scale tunnel has now been decommissioned and we can work with scale models or smaller scale aircraft components in aircraft are tested on those wind tunnels. The development of argument justifying the similarity analysis lies in the nature of physical quantities. We therefore begin this course of lectures by visiting these fundamentals. The equation governing the flow of fluids and or deformation of solids both form systems of simultaneous second-order non-linear partial differential equations which are formidably complex. Closed-from analytical solutions to these have been obtained for some various simple geometries and for very restrictive conditions only.

In such cases one or more of the terms from the equations drop out because of some or the other simplifying features like full development or the steady flow. But such features are not encountered in general. The solutions obtained through these simplifications are so cherished that we like to extend them to the realistic situation where these conditions are not met exactly. That is we resort to approximations. We obtain approximate solutions.

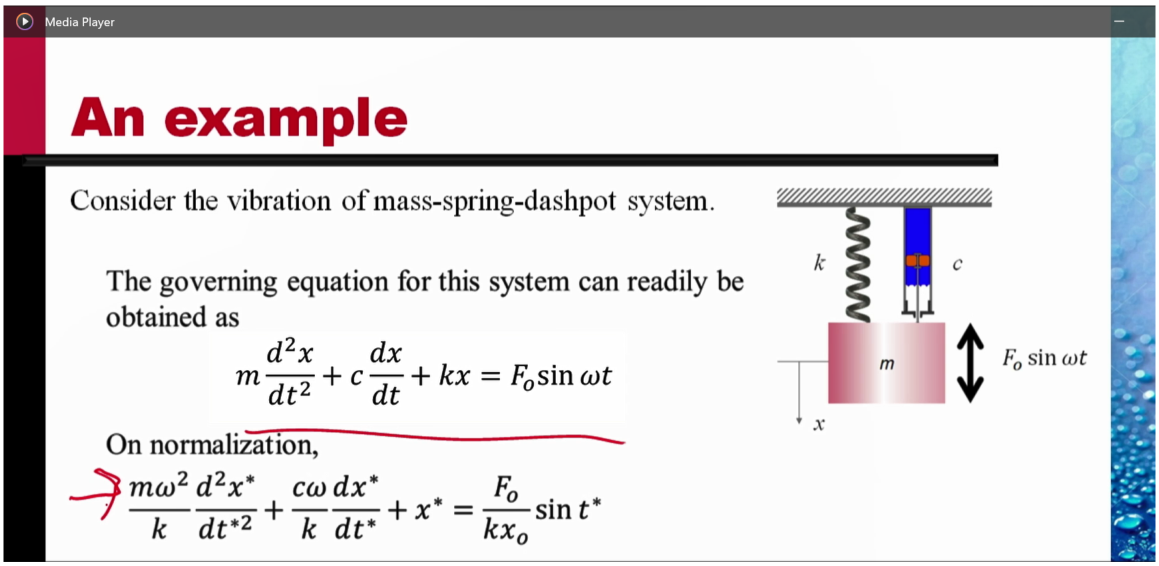
If we are predicting the global weather using thousands of equations on a super computer and someone suggests that we are making a mistake by not including the effects of a flapping of the wings of a distant butterfly several weeks earlier, we would simply have laughed it off. That is really not true. We know that a gentleman had a bat soup in a Chinese restaurant and that resulted and that small incident resulted in a large pandemic across the world where the medical facilities ran short, where the cities have to be locked down, schools had to be closed and the economy suffered drastically. So small causes not necessarily produce small effect. This picture here shows the vibrations of a double pendulum and it randomly produces patterns and these patterns can differ widely if the initial conditions change only a little bit.

This is known as the butterfly effect. A small butterfly can cause widespread havocs, but this is a special case. Edward Lawrence in 1961 found that a very simple rounding off of initial conditions while predicting the weather through a numerical model resulted in a massive deviation in prediction of weather about 2 months down the line. Here we show in this picture the prediction of an event in which the initial conditions change only slightly, the three different colors that you see. So the first part, the three solutions are identical, but that later on they start deviating, deviating very largely.

He named it the butterfly effect and it led to the development of theory deterministic chaos. We will not worry about this in this course. The practice of engineering proceeds on the principle that small causes produce small effects, small causes produce small effect. And we shall persist with this in this course here till something breaks down and we will see within this course that at times something will break down and we will have to step in to correct this. But till this happens, we will work with the principle that small causes produce small effects.

So if we can show a cause to be small, then we can neglect it because it will produce small effects. If viscosity is small, viscosity could be neglected and the fluid could be treated as inviscid. With the assumption, the results that we will get would approximate the result for the actual fluid with very small viscosity. In order to establish which terms in the governing questions are negligible, the magnitudes need to be estimated. So if we estimate the magnitude of each term, then the magnitude that is small could be neglected.

One very full, very powerful method of making such estimate is to normalize each variable by scaling it with its characteristic value. We will spend considerable amount of time in explaining what the characteristic value is all about. If the characterizing values of the quantities are chosen properly, the normalized variable are then expected to be order 1, that is except at certain isolated points. The value of the transformed variables are expected to be neither very large nor very small compared to unity. As an example, consider the vibration of mass-spring-dashpot system.



Those who have studied vibrations even at high school level would know that the governing equation for this system can readily be obtained as this. Here the first term on the left is the inertial force: mass times acceleration. The second term represents the viscous forces of the dashpot, which is c, the damping constant times the velocity, and the third term is the spring force: spring constant times the displacement. And on the right is the forcing sinusoidal force sin $F_o sin\ \omega t$. If we change these variables and make them normalized that is make them order 1.

How do we make them order 1? You divide them by their characteristic value. Let the solution of this system be x is equal to sin $x_o sin\ \omega t$. Let us normalize x as x/x_o and let us normalize time as t by t_o and then on normalization the equations become $\frac{m\omega^2}{k}\frac{d^2x^*}{dt^{*2}} + \frac{c\omega}{k}\frac{dx^*}{dt^*} + x^* = \frac{F_o}{kx_o}\sin sin\ t^*$. Now, if we have chosen our characteristic quantity properly, it is hoped that the value of x star and t star as well as the dimensionless derivative of the various physical quantities appearing the equation are all of order 1. So, this is of order 1, this is of order 1, this is of order 1.

If these are of order 1, then these coefficients represents the relative magnitude of the inertia and the viscous term compared to the spring force term. If this is small, we could neglect the first term. Small compared to what? Small compared to the coefficient of the spring term which is 1. So, if this small compared to 1, then we could neglect the first term, the mass effect. If $\frac{c\omega}{k}$, the coefficient of the second term for small, then we could neglect the second term.

That is we could neglect the damping term altogether and this would then act as a simple mass spring system with no damping. This is the basis on which the theory of approximation is developed. In the latter part of the course, we will spend considerable time in developing this further.

Thank you.