Evolutionary Dynamics

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Week 06

Lecture 30

Hi, welcome everyone to the next lecture. So, we started our discussion of chance events dictating evolutionary trajectories, and we will continue to build on that theme in this video. To fully understand, we saw how in a chemostat experiment, the arrival of a beneficial mutant—the blue individual that we talked about in the last video—may not lead to blue individuals replacing all the green ones or the less fit ones. It is quite probable that the blue individual itself is lost from the population because it is being washed away by the exit stream. Hence, evolution is dictated not just by natural selection but also by chance trajectories.

To understand the role of chance events in dictating evolutionary trajectories better, a useful game to think about is called marbles in a jar. So, in this game, we have the following rules: we have a jar. And in this jar are n marbles. Every marble is of a unique color, and all n are of different colors. So, let us just say this is a marble.

And in all, there are n. Each is of a different color. Red, orange, and so on and so forth. So, everyone is a different color. That is our starting point. At the starting point, we will define a quantity called the frequency of each color.

So, if I write what is the frequency of each color? For example, frequency of red colored marbles is simply equal to number of marbles, number of marbles of color red divided by total number of marbles. Which in this case because every marble there is only one marble of every color in this jar. So total number of marbles of color red is simply 1 and total number of marbles is n. So frequency of red colored marbles is 1 by n.

Similarly, if I were to look at frequency of any other color, that frequency will also be equal to 1 upon n. This is how many blue marbles are there and this is how many total marbles are there. That defines the frequency of particular type of marbles. This is the starting point. Now let's try and write the rules of this game. One, we draw a marble from this jar.

take a look at its color its color put a same color marble Let's write these and I'll explain once I have written down the rules. We'll do a toy demonstration of this and then it'll make more sense. Put a same color marble in a next jar. Place the marble drawn.

Back in the first one and repeat until the number of marbles in the second jar equals n. Okay, so those are the rules that we are going to operate with, and let us draw the second jar that the rules speak about. So, what is happening in this game is that let us call this jar number 1, and let the second one be jar number 2. What is happening in this game is Rule 1 says we draw a marble from jar 1. So we draw a marble from jar 1, obviously without looking at its color.

So this drawing is done. So Rule 1 is done. Second, we take a look at its color, and let us say this is a blue marble. So we take a look at its color; that is also done. Third, put a same-colored marble in the next jar.

So since we drew a blue marble, we will place a blue marble in the second jar. That's the third one also done. Fourth says place the marble drawn back in the first jar. So when we were drawing from the first jar, we drew out a blue marble. We noted its color.

We kept an identical blue marble in the first second jar. And this one that we had drawn, we put it back in the first jar. So the first the number of marbles available in the first jar is still equal to n and not has not become n minus 1. When we draw when we drew the blue one, the number was temporarily n minus 1. But we looked at its color and placed it back.

So the number is again n. So place the marble back. And then repeat this process till the number of marbles in the second jar reaches n. So we keep doing this over and over again till the number of marbles in this jar reaches n. That's it. So the number here is also equal to n. One more thing we should note here is that all marbles are identical except their color. What that means is that when I am drawing a marble without looking at the color of a marble, every marble has an equal chance of being drawn from the pool.

And since I am drawing one and there is only one marble of every kind, there is only one by n chance that I will draw a particular marble. So, for instance, because they are all identical, what is the probability that I will draw a red marble? what is the probability that red is drawn that is just equal to 1 upon n because there are n options available to me to draw any marble and I am drawing any what is the probability that it is going to be red one since there is only one of red one and there are total there are n the probability that I am going to draw the red marble is 1 by n think of this as a dice problem where I am going to roll the dice and any number is going to come up what is the probability that the number is going to be 3 Since there are 6 options and 3 is only one of the 6 options, the probability that I will see a 3 is 1 upon 6.

Similarly, there are n marbles, each of different color, but they are identical in every other way. What is the probability that I am going to draw the one marble which is red in color? It is just equal to 1 by n. And this is why we are writing this statement that all marbles are identical in color, identical in every respect except in color. Their sizes are same, their shapes are same, their weights are same.

So when I have my hand in the jar and I'm pulling out a marble, I don't know which color I'm pulling out. So once, let's add one last rule, number six, that when So we repeated this till jar number two had n entries in it. After that, repeat the whole process. But now jar number two is drawn from and jar three is added to.

So, we forget about jar number 1 after the jar number 2 reaches a population size of n or the number of marble count equal to n. We forget about jar number 1 and now we start doing the same process except that we are drawing from jar number 2 and looking at the color and placing equivalent marbles in jar number 3 and so on and so forth. That is the game and this is called marbles in a jar. So let us see what is going to happen here. What sort of questions and what sort of understanding do we develop about this game? And then what are the principles that we can carry from there to our understanding of evolution?

First, we should learn this graphical representation. On the x-axis is the number of jars. So, I started with the first jar, and the next jar was jar number 2, then jar number 3, and so on and so forth. The y-axis is the number of marbles. Marbles.

Marbles. And this goes from 0 to n. The position of the representation that we are going to use will be the following. What is the number of red-colored lines? The marbles we had were just one, so 0 to 1 is red because there was one red-colored marble in the jar. That's 0 to 1. How many orange ones were there? 1 to 2 is represented as the orange bar because this represents that the distance from 1 to 2 represents the orange bar for the orange marble in the jar. Similarly, I will have a green bar, which represents the green marble in the jar, and so on and so forth.

So, every color will have its own bar, ending with this purple one, which goes from n minus 1 to n. So, that is the starting position that we have, and in this starting position, it's easy to see that every color will have exactly one by n available to it because we are starting with a situation where each one is equally present in quantity one. Each color is represented by only one marble. But play this game in your head and try to see what will happen as we transition from the first jar to the second one.

And to make the question more precise, the question we want to ask is what is let's start with a simpler question about what is the state in jar number one? What is the diversity in jar number one? And by diversity, we mean the number of colors that are present in jar number one. And that is easy because we were told at the start of the experiment that there are N marbles. Each one is of a different color.

So this is simply equal to N. Then the question that we want to ask at that stage is what happens to this diversity? What happens to diversity? as we move from jar one to two. So what I would like you to do is sort of play this game in your head for maybe 15, 20, let's take 15, 20 seconds of a pause here. And what I would like you to do in that pause is imagine yourself playing this game with, let's say, 10 colors.

So you're starting out with 10 colors, 10 marbles, one of one marble of each color, and you play this game. What do you think is going to happen to the diversity of colors in the second jar when you started with this initial condition in the first jar? So let's take a 20 second pause while you think about this question. So what's happening here is that once I start drawing these marbles from the jar, it's possible that some colors are going to get drawn repeatedly. For instance, I draw a blue marble at the first attempt and I place the blue marble back in.

As a result of that, the blue marble may get picked up again in one of the later draws, which means I am going to have two blue marbles in the next jar. But having two marbles of the same color in the next jar automatically implies that at least one of the colors is going to miss out. Because in the initial jar, we only had one marble of each color. And now if I have two blue marbles in the second jar, so blue is now represented twice. It automatically means that one of the colors is going to be missing from the second jar.

That could be red, yellow, green. It could be any color, but one of these colors is going to miss out. So what this means is that some colors are going to be drawn more than once. And hence these are going to be over represented as compared to their frequency in the first char. And some other colors as a result of this some colors will not be drawn at all.

So maybe in this example that we did, maybe the red color completely missed out and it's not represented in jar 2 at all. So red color has gone extinct as we transitioned from jar 1 to jar 2. As a result of this, the diversity, even though the population size, the number of marbles in the jar remains the same, the diversity in the jar decreases. What is important to note here that this diversity, so I hope it's not too difficult to see that this diversity is going to decrease with time. But what we also want to understand is that what caused this change?

Because the rules of the game, according to the rules of the game, everybody had a fair chance of being drawn. Everybody had an equal chance of being drawn. Despite that, some colors, if I'm the red color, I had the same chance of being drawn, but yet I went extinct. I wasn't picked. So this change in frequency of different colors in the jar is just caused by chance events.

And hence, chance events are bringing about a change in the color frequency in the jars that I am playing. To realize this again, maybe if I play this game once and the red color doesn't make it to jar 2. Maybe when I play this game tomorrow, the red color is drawn three times. In which case, the red color is now massively overrepresented. It was 1 by n frequency in jar 1.

But by the time it got to jar 2, the red color is 3 by n frequency. Its frequency has tripled. And that has also happened because of chance events. Because every marble had an equal chance of being drawn. So chance events are causing a decrease in diversity and different frequency changes for different colors.

That's one. Moreover, if we keep playing this game, let's let's say that at some point we have so we keep playing this game for a long time. Let's say at some point this is the state of the game. So, we have 2 blue, 3 red, 1 dark blue, 2 purple, 3 orangish and 2 black. This is our total population size.

So, N is 5, 6, 8, 10, 13. That is N. The next question we so so this this question we answered that as the game moves forward, diversity decreases. For instance, if this is what the game looks like in jar two, maybe when we go to the next jar, maybe this color is lost, which leads to even even lesser diversity. So it will keep on going.

Next question is, what is the ultimate end? this game where does this game end because as we keep moving forward in time what's going to happen is that loss of diversity will keep on taking place Maybe it won't happen from jar 2 to 3. Maybe each of these colors will be represented there. But eventually there will come a point when one of these colors was not drawn among the 13 draws that I did.

As a result, the diversity will keep on decreasing with time. Eventually, we will reach a stage where only one color is present. It's easier to see the loss of colors here, but let's imagine that at some future point in time, only two colors are left: blue, five, six, seven, and red, eight, nine, 10, 11, 12, 13. So there are six red individuals, marbles, and seven blue ones. Maybe at a future time, this is at some jar number I, there are only blue and red left.

In such a scenario, it's not impossible. Maybe if we go one step forward, this number becomes 8 and 5. And then we go forward, this number becomes 9 and 4, and so on and so forth. And gradually, it takes a while, but eventually there may come a time when all 13 are blue. That doesn't mean that the game can't go in the other direction.

That could totally happen where it might happen that when I go to the next jar, just by chance events, I draw eight of these and five of red. And then this becomes 10 and this becomes three, and so on and so forth. And it's actually the red one which reaches and completely replaces all other colors in the jar. So these are stochastic events. But the point is that the game will only end when one color is present.

Only one color is present in the jar. The identity of that color will be different in every possible run of the game. If I play this today, green might win. If I play this tomorrow, yellow might win, and so on and so forth. So that's the marbles-in-a-jar game.

Next, we want to draw some parallels between this game and how an evolutionary process proceeds. But before we do that, let's go back to our game. Let's go back to our representation and see how this looks. I started with one color, one marble of each color. Let me quickly draw a few.

That was the starting point. Two more. This was the starting point in jar number 1. Now, as we move forward, maybe this light red color reaches extinction in the next jar. So this is how I am going to represent it.

What that means is that in the next jar, this guy is present twice, the red one is gone, and the orange one is also present twice. So this expanded; this section expanded from one marble to two marbles. This light red contracted from one marble to zero marbles. This orange expanded from one marble to two marbles. Maybe yellow was an extremely lucky color.

It expanded to three. So this expanded massively. It expanded from one to three, and so on and so forth. But eventually, these contractions and expansions are going to keep on playing, and we can represent these trajectories like this—eventually, till only one color remains, which in this case is yellow. All other colors are gone.

And we can trace the history of what happened to each color. By just tracing these lines, maybe in the next generation, this came back to one again. So this expanded from one to two but then contracted from two to one again, and in the next one, it was gone because there was no representation. So if I look at the history of this particular color, it expanded and then contracted, and so on and so forth. This one was lost immediately. And so on and so forth. Maybe this orange one was never picked again, and then it went to zero. So by going back in time, I can trace the trajectory of each color even after knowing this final position.

I can record this history and see what happened in the process. And it is this process of marbles in a jar that we will try to build an analogy with evolutionary processes in the next video. Thank you.