Engineering Statistics
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Lecture No. 9
Expectation and Variance

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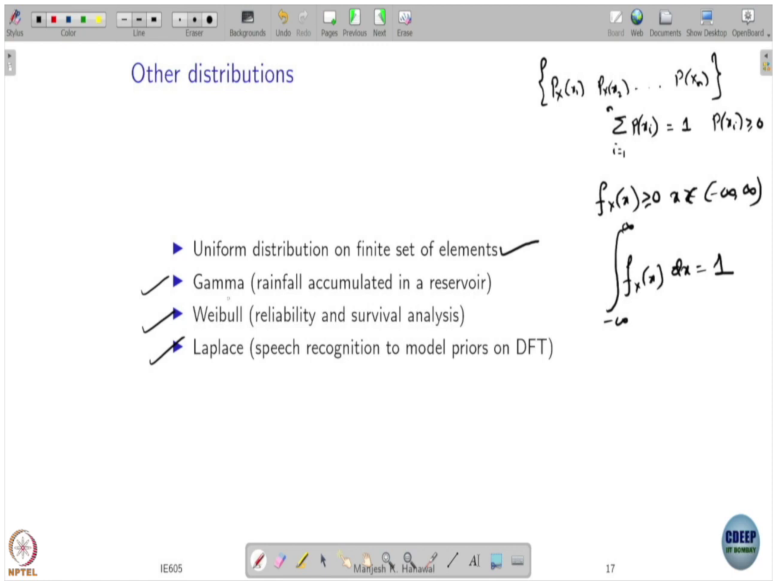

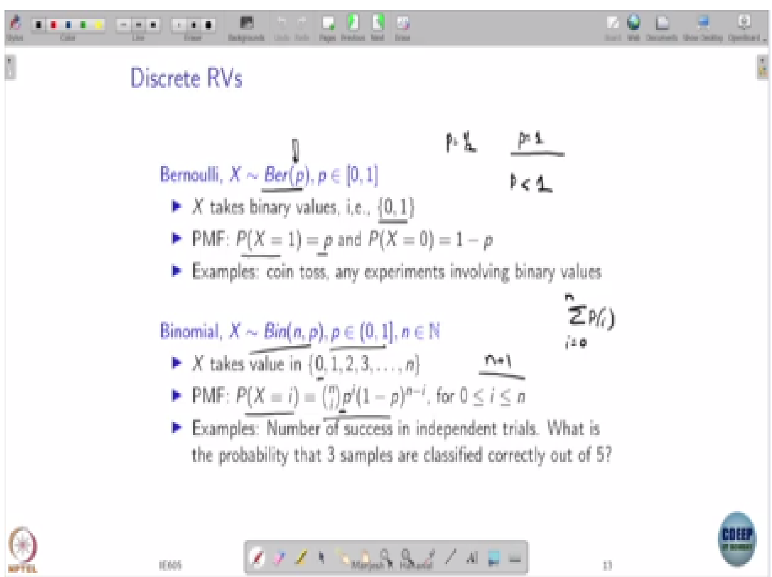

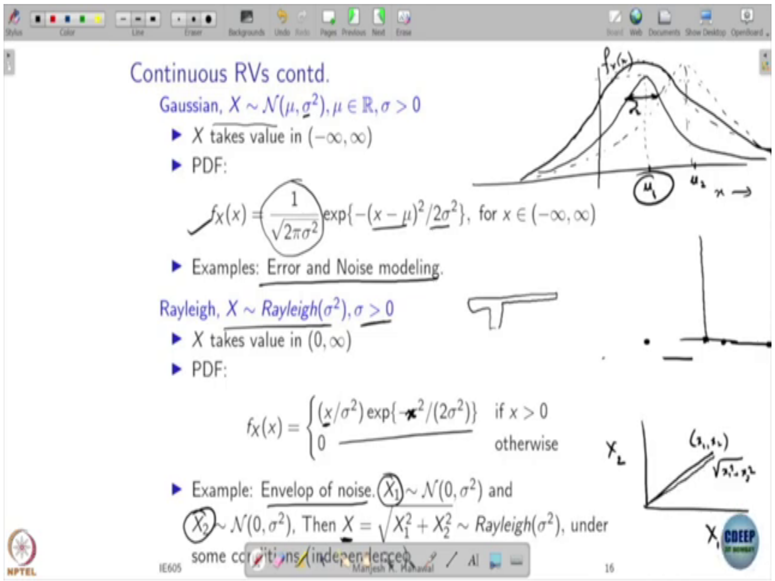

So, in this course mostly we will be focused on the four discrete distribution we talked about and the four continuous distributions we talked about. So, you all of you should be like this discrete random variables for we discussed they should be almost you should memorise on your fingertips and the continuous all these uniform exponential, Gaussian and Rayleigh this should be also, may not be Rayleigh much but Gaussian all this should be on your fingertips.

In addition to this, there are other distributions we already talked about this uniform distribution on finite set of elements, there are again like gamma distribution Weibull distribution and Laplace distribution, we will discuss them as they, as we require them. Otherwise, one can come up with different possible distributions. All we need to ensure is if in a case of a discrete random variables, like if it is like you have this, let us say this is like some probability mass function of some random variable, this is a valid probability mass function as long as i equals to 1 to n is sum to 1 and all these P(xis) are positive.

Any kind of probability vector, you can come which add up to 1 and positive this is a valid probability mass function. And similarly, you can come up with some function for all x. And this is positive and as long as this is 1, this is another valid probability density function for some random variable, you can come up with whatever you want. Like that people have come up with different but those have some special applications based on that they are called Gamma, Weibull and Laplace, I will, we will use them whenever we incur them.

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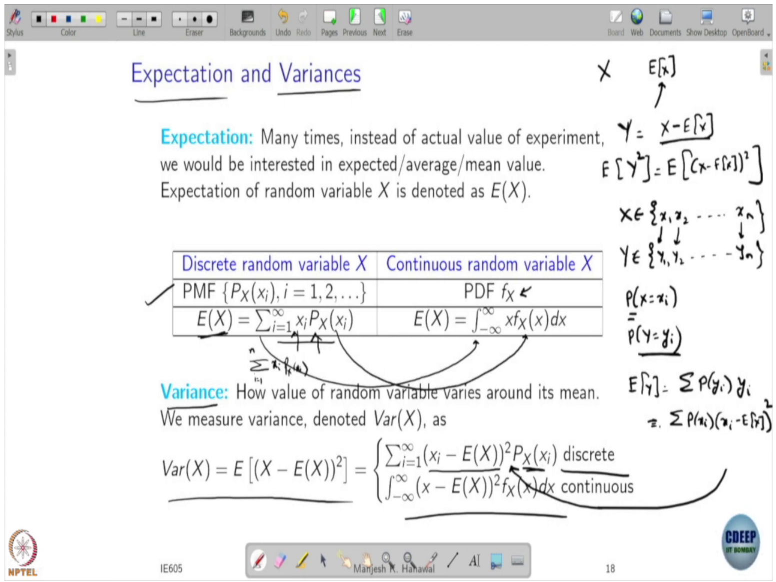

Now, often when we have this random variables, we would not be much bothered about what is the exact value, but we would be interested in on an average what happens, for example, this entire class is there, maybe some of you will get and it is not necessarily that all of you will get the same grades. But what matters to me is what is the average grade of this class that will define what is the quality of this class. Maybe if some of you got A, fine you that is what some individuals got. And some of you got, let us say C, D, some of you got, but that is not like individual case matters to us like what is the average grade of this class?

Right in that case, like, what each one of you score, it is like a particular realisation like I can treat the score of this class as some random variable. And there are about 60 students and value scored by you as a different realization, I am going to see, for me particular realisation is not so important, what is important is the average score. If the average score is high, then let us say you are the current batch average score is higher than the previous batch average score, I may feel that I will feel that maybe I did a good job or you people did a good job, like overall the class performance is better.

So, like that, it becomes useful to compare the overall behaviour. So, that is where we will look into the expectation and variance, which will help us to understand the aggregated behaviour rather than the individual behaviour. So now, when we have this discrete random variable, we said that, we will have this point muscles, and the expectation of that random variable is simply defined as the weighted sum of the realisation where the weights being their associated probabilities.

So, notice that here my random variable, X is taking values, X1, X2, with a probability P(xi) and what I am doing is xi is being taken with probability P(xi), I am multiplying and summing over all possible values. Even though I have written here i equals to one to infinity, it could be finite, when you have only finitely and like if you have only n terms in there, this is simply going to be i1.

Student: What is DFT?

Professor. Manjesh Hanawal: Just a minute. What is discrete DFT? This is something called a Discrete Fourier Transforms, I mean that that Laplace arises in that context. Like usually in signal processing, when you are processing your speed signals, one has to deal with certain kinds of Fourier transforms, one of them is a special thing called Discrete Fourier Transforms, you do that some special distribution arises, that special distribution is called a Laplace there.

All I am saying is Laplace is one kind of distribution, which has application in this speech modelling. Like that, Weibull is another kind of things distribution, which finds application in reliability and survival analysis? I do not know why some of you who are mechanical engineering, you people study something called reliability course, you study reliability course. Maybe later. So there, you will end up seeing a Weibull kind of distribution.

And when somebody in climate studies, or civil engineering, people may be interested in studying the amount of rainfall accumulated in a reservoir, for example, how much of the water gets accumulated in our Powai lake, like in monsoon season? Maybe that is needs to be studied, we need some models. And maybe that gamma kind of distribution will find applications.

So, depending on application, people have come up with different distributions. And I have just listed them, like instead of going through all of them, this is your expected value in the discrete case. Similarly, like, if I am given a probability density function, what you are going to do is, like when you go from discrete to continuous, the things change from summation to integration, and the PMF change from PMF change to probability density functions, that is what we are doing. And now, you have this expected value of your random variable.

Now, the other quantity we just said what is expectation, variance. Variance is how the value of random variable varies around its mean. So, how the things behave with respect to the mean, we want to capture and one way to capture the quantity is called variance and that is defined like this. Let us say I have a random variable X and I have expectation of X is expectation of X is a random quantity or a constant, is a constant. Let us say I want to define a new random variable X minus expectation of X, can I define a new random variable like this?

So, now, what I want to understand is I want to take square of this and take its expectation and that is exactly my variance, that is what its variance is. Now again, how to compute this, if it is a discrete random variable, you just find out this quantity like this quantity actually, let me ask you, let us say if my x takes values x1, x2 up to xn and when I do this operation, y hypothetically assume that y also takes value y1, y2, yn, that is x1 gets mapped to y1, x2 gets matched to y2 and xn gets matched to yn. Now, I want to understand what is the probabilities of xi? And what is the probability that y equals to yi?

Is there any relation between them? Will they remain same or different? Others? They are going to remain same. Here, I am specifically assumed that there is a one to one mapping. So, whatever the probability I have x1, the same probability with same probability that y1 also like that, now it is now basically finding because of this, if I want to find expectation of y, this is simply expectation of yi times yi, but this is nothing but probability of xi, but yi is now xi minus expectation of x squared, and that is what we have written here.

Maybe I should do boundary pe chala gaya, p(xi) times xi minus expectation of x, good to have it and similarly, in the continuous case, as I said, we are going to replace summation by integration and the discrete probability by the associated density function.

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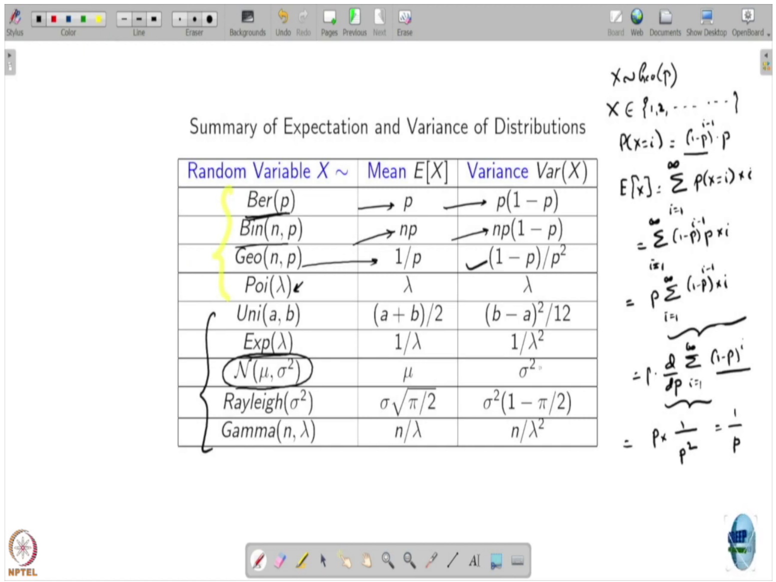

Now, I talked about different distributions right, these are some, like these are some discrete distribution we talked about, and there are some different continuous distributions we talked about. And each of them with comes with a certain distribution. Now Bernoulli random, Bernoulli distribution with probability p. If you go back and compute its mean value it is simply going to be p, whereas its variance is going to be p(1 - p). This is obvious you can calculate.

And binomial np. So, recall binomial has two parameters n and p, and its mean is going to be np and its variance is going to be np into n - 1. So, notice that means and variance depends on their parameters. And for geometric, this is going to be 1/p. So, let us compute for one of them, let us say, let us take X to be geometric. So, what are the possible values X takes in the geometric distribution? So, 1, 2 up to infinity and what is the probability that X equals to i, p, where is q say again,

Student: 1 minus p into 1 minus p power i minus 1 into p

Professor. Manjesh Hanawal: So, there are i - 1 failures and after that there is a success that means, there are success happened in the ith trial. Now, what is the expected value of, I want to compute, what is the expectation of, expectation of X. Now, this is going to be probability that X equals to i into i itself, i equals to 1 to infinity now.

And what is this value this, value is going to be 1 minus pi minus 1 p into i and i equals to 1 to infinity, can somebody, now you can simplify try to simplify this, pi equals to 1 to infinity 1 minus pi minus 1. Now, can you find out what is this value is? What is this in integral summation is? How? So, what is this like if you want to write it as we can write it as d by dp of what is this quantity? p power i.

Student: 1 minus p.

Professor. Manjesh Hanawal: This is not exactly the case. But you can imagine like this, because integration and differentiation, you cannot always interchange like this, some conditions has to be satisfied, you cannot blindly do this. But I will blindly interchange for time being, integration and summation. Now you do this, this quantity, you can find it as $p(1 - p)^2$. Is it correct, this quantity is going to be $(1 - p)^2$, I am blindly assuming you are right and then it is going to be 1/p. So that is what we get it.

And you can also find out variance. So, variance, you need to do a little more computation. Because squared terms are involved. You do it. So, like that you can do. So, some other things to notice is, if I have a Poisson distribution with parameter λ, its mean and variance are also going to be λ only and if I have exponential parameter λ, it is mean is going to be $1/\lambda$, and its variance is going to be $1/\lambda^2$.

And this Gaussian distribution with parameter μ and σ^2, μ is corresponds to its mean. And σ^2 corresponds to its variance. So, like that. So, I want you to go through all of this table even though we have put it here, you should work out this, make sure that you are getting the right things at least once you should do, so that it is like a little bit get drilled in your head.

You do it once at least so that it remains little bit longer term with you. If you do not calculate once even though it could be a little bit manually you have to write some four or five pages of calculation but do it. fine. This is all about the discrete and continuous random variables, and the distributions and their associated functions like expectations at variances