Engineering Statistics
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Lecture No. 8
Discrete and Continnous Random Variables - II

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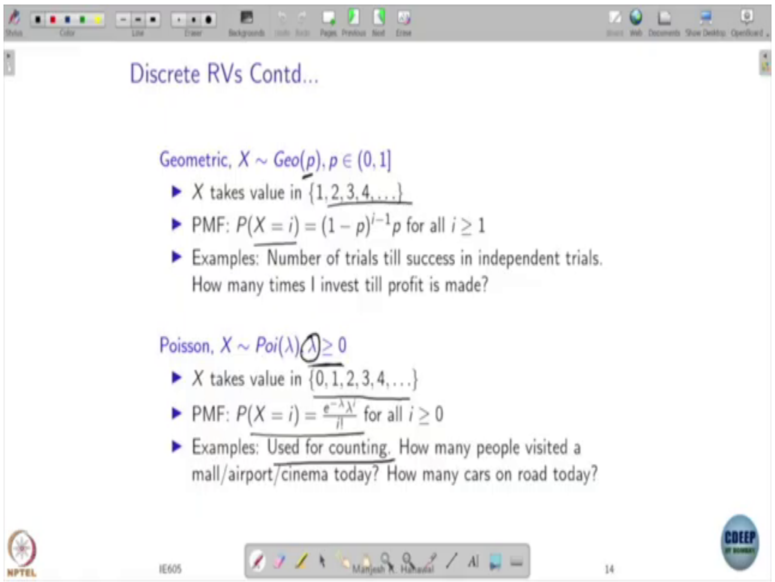

Let us get started. So, in the last lectures, we discussed discrete random variables, and in that we talked about Bernoulli, binomial, geometric and poisson. So, anybody has any questions so far on what we have discussed?

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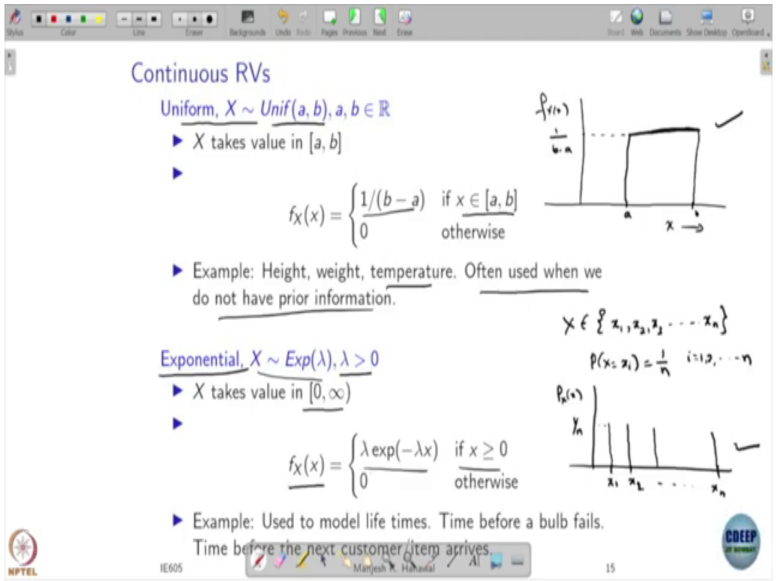

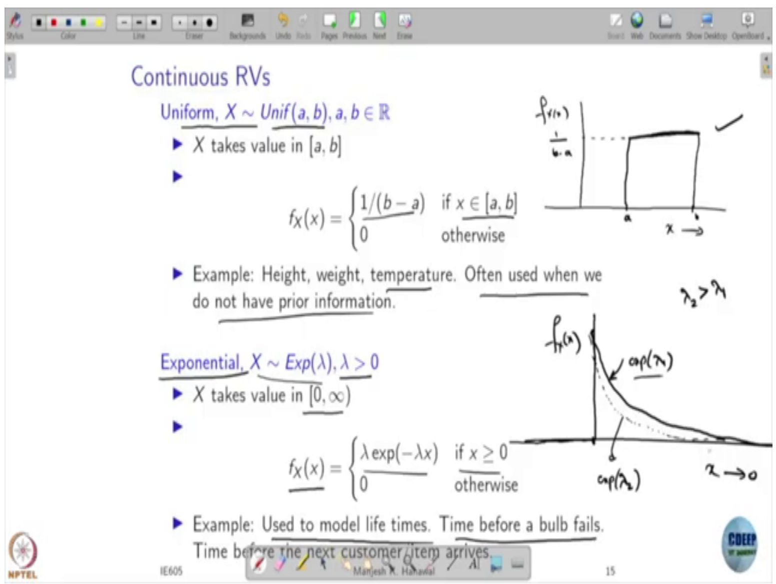

Today we will talk about this continuous random variables. So, as I said, the definition of continuous random variable is the range over which random variable takes value is uncountable. The first example in this case is called uniform. This comes with two parameters, and it is denoted as uniform a, b, where a and b are some real numbers. So, we have a random variable x which is uniformly distributed, a and b are some real numbers.

Now, the parameters a and b says that my random variable X takes in the value in the interval a to b. Now, this is a continuous random variable, there will be an associated probability density function here. And that probability density function is defined for this as in the range a, b it is defined as 1 upon b -a. Notice. that this does not depend on x. So, this function is constant in the interval [a, b], and this is 0 outside.

So, pictorially if I have this, and let us say this is my a, and this is my b. So, this looks like this. And this quantity is 1 upon b minus a. Now, the examples could be many, where you want to use such a random variable. For example, if you want to model somebody‘s height, and weight as some uniformly random variable, in which case you will mention that, my height is between some value a and b. And also temperature you can mention it is between some value a and b all the time.

And this is often used when you do not have any prior information. And you are going to assume that everything is almost equally likely. So, that is why you will see that this function is constant in the range a to b. That means, as we said, the rate of change of probability at any point is going to be the same here.

So, there is an analogue of this in the discrete case also, which actually we did not discuss. So, what could be the analog of this in the discrete? Suppose, let us say now I am slightly instead of continuous now I am talking about discrete. Let us say your random variable is discrete. And it takes value in value, let us call x1, x2, x3 and let us say take up to value xn. Now, I want to say that my x discrete random variable is uniform.

What could that potentially mean?

Student: All xs have same probability.

Professor Manjesh Hanawal: All this xs, they have the same probability that means probability that my x takes any of these values xi is going to be what? 1 by n, this is for i 1, 2, up to n. And now if it is, for discrete we say that it has a probability mass function. And for that, we can draw a probability mass function, this is for x1, x2 all the way up to xn and this is its PMF. And what is this value going to be what?

This is going to be like this, all of them have the same heights. And this value is exactly 1 by n. So, uniform, you can have discrete in that case, all the probability mass functions have the same value. And you can have uniform continuous in this case, this PDF is going to be a constant in the interval [0, 1]. Now, similarly, we can have other random variables, but in those random variables may be like all things or not, need not take the same value.

There could be different like different values can happen with different probabilities. That random variable, One possible thing we are going to study is exponential. And exponential is going to be denoted as we exp(λ). And that comes with a parameter λ, which is strictly positive. Now, this random variable x is a positive valued random variable and it takes value in the entire positive range between 0 to infinity.

And it is a PDF is defined like this $\lambda e^{-\lambda x}$ whenever $x \geq 0$ and it is defined 0 otherwise. So, if this is a PDF, we know that if you integrate it between minus infinity to plus infinity, it has to be 1. Just ensure that if you integrate the area, this function is actually 1, that is pretty straightforward. Now if you want to just look into its figure, what does this function look like?

So, what is the value of this function at less than or close to 0, this is going to remain 0 for all the negative portion of my real value. And it is, at $x = 0$ what is this value is going to be? λ. It is going to start with λ and as x increases, it is going to decay exponentially with the rate my λ here. And that is something look like, like this. Let us say this is some curve this corresponds to some particular lambda, exp $\lambda 1$, some $\lambda 1$.

And now I will take $\lambda 2$ which is greater than $\lambda 1$. Now, if I have to plot that curve, will that be about this curve or it is going to below this curve? Below, it is going to be something like, so this is, this is like maybe going to be like exponential $\lambda 2$. So, it is thus why this factor lambda is called the rate of this exponential. Larger rate, it has a higher rate and that is going to fall faster. And now this is often used to model lifetimes.

For example, how much time my bulb, let us say you have bulbs, how much time my bulb is going to work before it breaks down. So, it is going to be some number maybe it will work for 100, I am going to count time in seconds, it can break down after x seconds or y second whatever, but those seconds are continued values. I am even fractions we are counting in fraction of the seconds also all possible values.

So, it could be like, if the bulb is going to last for longer and longer, that probability becomes smaller and smaller. Because if the bulb is learning, maybe its, its life is now reduced and it may break and there you want to use such an exponential distribution.

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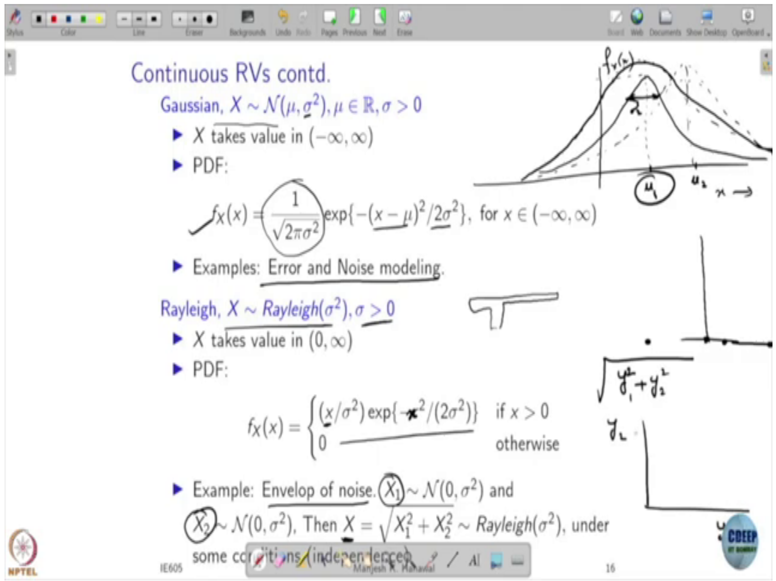

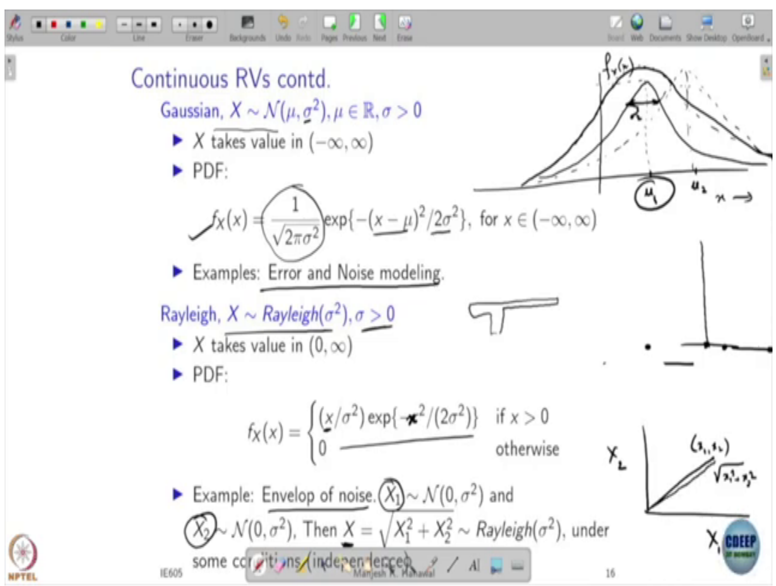

Next is Gaussian. And this is the one of the most thing commonly we're going to use, because of its nice properties. It is denoted by two parameters called μ and σ^2. And it takes value over the entire real line from minus infinity to plus infinity. Notice that x when it was exponential, it was only taking between 0 to infinity. But now we sometimes want both positives and negatives.

So, we are looking for one such possibility, Gaussian, which is going to take value between both minus infinity to plus infinity. And it has this PDF which is given like this. So, $(x - \mu)^2/2\sigma^2$. And now there is also one factor, 1/2 by, $1/\sqrt{2\pi\sigma^2}$. Now, how does it look like? So, μ is one parameter, and if you look into it, this is going to be symmetric around this.

And it is going to take the maximum value at mu and then after that it us going to decay on both left hand side. And if you see that this is going to be symmetric around your main point. And now μ is deciding where the peak happens, but there are two parameters. What does this parameter σ decides? The σ decides the width of this loops. Larger is the, if your μ is larger that means, your loop ships towards the right and right.

Suppose, let us say, this is first a μ1 and you have another quantity μ2. Obviously, μ2 here is larger than μ1. So, in this case your loop has shifted, oh no, like this. And now, again this is symmetric around this point μ2 and σ^2 defines how big is the, how big this loops are. If σ^2 is larger this spread is going to be larger. So, for example, if this is let us say for $1\sigma^2$, let us call 1.

And now if I make it larger than this is going to be more prominent like this, it becomes a bulgy. I do not know if you can look into this, like this upper one is with a larger sigma square but for the same value of μ1. Now, this is often used whenever you have to handle both positive and negative quantities. One of the, that comes commonly in handling errors. Errors could be positive or negative.

And noise, noise could be also positive or negative. So, whenever we want to model such noise you want to use this Gaussian random variable. One example is suppose let us say you, you are trying, you have one target like, one target is here and you want to hit it. And whenever you hit, it may fall short of this target or it may go beyond the target. So, when it goes beyond, it is going to be taken as exceeding, positive when it falls short it could be negative.

Now in modeling that, like that is like an error like you want to be exactly falling here, but sometimes you may exceed, sometimes we may fall short. And, and because every time it will not same things will not happen, sometimes you may fall here, sometimes you may fall here and sometimes you may exceed here.

That is like a random quantity every time you shoot you may be falling depending on so many factors, that will decide maybe wind velocity, humidity, temperature, all these things matter when you are basically firing something, and all those things are random. So, in such case you may want to model them as a Gaussian distribution. Other thing is Rayleigh distribution. So, a Rayleigh is actually now a derivative of Gaussian.

And this is like comes with a parameter σ and it is often denoted as Rayleigh σ^2 and where is σ is a positive quantity. And here this Rayleigh takes a value between 0 to infinity that means it takes positive real numbers. And its PDF is denoted like this. Notice that it is somewhat similar to Gaussian but not exactly the same. It has a x variable coming in, out not only so, there is a typo here, that should have been an x.

So, the x is not only inside the exponential, but it is coming out outside exponential also. And now one property is, one property of Rayleigh distribution is, Rayleigh can be thought of, envelop of two Gaussian random variables. For, for example, let us take you have two Gaussian random variables x1 and another one is x2.

And if you are going to take their squared sum, you are taking x 1 square and x 2 square and take the square root, the new random variable x this is going to follow your Rayleigh distribution. So, this is often thought, can be thought as like envelop of noise. Suppose, let us say whenever, whenever you have some signals, you take their squares and let us say when you want to add this is like finding the amplitude. Like if you have two signals y1 and y2. Let us say this is your, this is let us say this is your y1 axis and this is y2 axis.

And maybe let me call this x. I should have denoted this also. Let us say, let us say this is some x1 axis and this is your x2. And you have some particular point here x1 and x2. And you want to know the distance. And this distance is from original naturally x1 square plus x2 square. If your x1 distance and x2 distance are both are Gaussian distributed, then that distance from the origin is Rayleigh distributed.