Engineering Statistics
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Week 11
Lecture 52
Tautology of tests and confidence intervals (Contd.)

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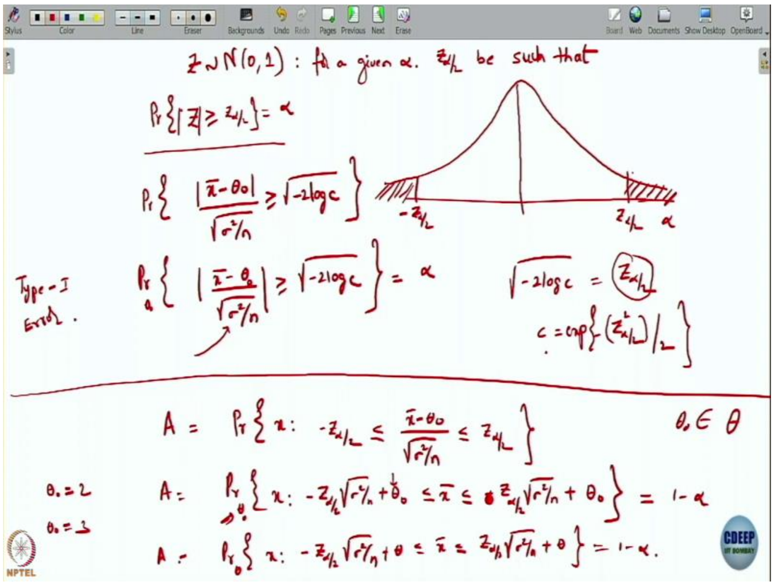

Now, the right-hand side 1 minus alpha this is coming, because of the choice you have made z alpha by 2 and this choice has nothing to do with what is the theta not you are choosing. So, now, is this expression true for any theta not? Or is it true for only some specific theta not?

Student: (())(00:37).

Professor Manjesh Hanawal: Huh?

Student: (())(00:40).

Professor Manjesh Hanawal: Yeah, R in theta not maybe like let us say theta not is some your parameter space, some parameter space. Does it depend on any particular theta not you are choosing or this holds for any theta that is coming from that test base theta?

Student: (())(00:56).

Professor Manjesh Hanawal: Huh?

Student: Only.

Professor Manjesh Hanawal: Only?

Student: (())(01:00).

Professor Manjesh Hanawal: Yeah, I am saying that if instead of let us say this theta not like this theta not I have given you as 2 and this expression holds let us say and tomorrow I will change this to theta equals to 3 does this relation holds or not?

Student: (())(01:17).

Professor Manjesh Hanawal: No, in that case, even the samples are coming with the new theta not. And now I am looking this probability under that true parameter only.

Student: Holds.

Professor Manjesh Hanawal: In that case, it holds, right? It does not matter which theta not you are looking as long as the underlying samples are generated with respect to that parameter, and you are also computing this probability with respect to that parameter this relation holds. So, this is true not necessarily for any particular theta not.

So, I will just write this is my probability under any theta such that z alpha by 2 and sigma square by n. Note that when I say theta not I have to this theta not and this theta are the same. So, now if I change this theta, now, this theta also has to change. Now, what is good thing about this?

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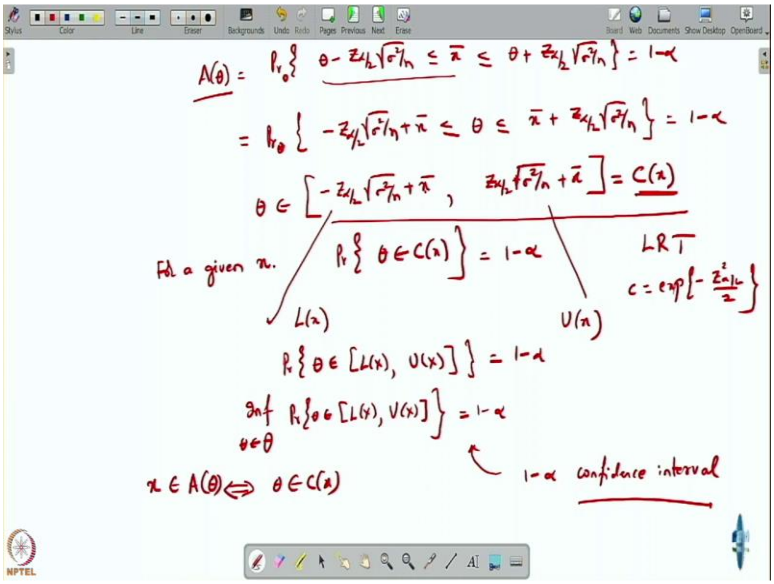

Now, I have defined a set, let us say for a given theta probability that theta that theta minus z alpha by 2 sigma square by n, this guy is less than equals to theta plus z alpha by 2 sigma square by n, this is equals to 1 minus alpha. Now, can I say that, if I just invert it in terms of my parameter, I am just going to invert this.

Now, this is like z alpha by 2 sigma square by n, I want to write now this in terms of theta. So, let us now take one upper bound, I am going to get through this. So, theta is upper bounded by x bar plus z alpha by 2 sigma square by n. And the lower bound for theta I am going to get from this, this is from these two I am going to get I can write it as plus?

Student: Plus x bar.

Professor Manjesh Hanawal: Plus x bar. Now, see what is happening. Now, you have an interval on the theta parameter. That is, you now you are saying that theta belonging to in this range z alpha by 2 sigma square by n plus x bar, this is on end, the other end is z alpha by 2 sigma square by n plus x bar. And what we are saying about this probability that theta belonging to this interval what this probability is?

Student: (())(04:42).

Professor Manjesh Hanawal: Exactly. So, this, I am now going to call this, this is for a given x. Now, I can treat it as for a given x, this thing I can treat it as my confidence set on my parameter. And now, what is the probability that theta belongs to C of x, C of x is this interval this is 1 minus alpha. Now, can you tell me the test we have? What is the test we have? The test we have is an LRT in which we have set c to be equals to that lambda x less than or equals to c.

In that c what is the value has set? We have set exponential minus z square alpha by 2 2. If you set it like this, I know that this is going to happen. And this 1 minus alpha here does not depend what is that theta you are looking into or what is the c we are looking into. So, now, can you tell me we have a confidence interval right now. So, here you can take this to be your L of x and this to be your U of x.

And now, you have guaranteed a probability that theta belongs to L of x and U of x is equals to 1 minus alpha. For this confidence interval sorry for this. let us say you have an interval estimator here. What is the term you used? Confidence coefficient. What is the confidence coefficient of this test or like what is the confidence coefficient of this interval estimator?

Can you recall the definition of confidence coefficient? So, it is simply, we let it be like this, so this is 1 minus alpha. So, what is the definition of the confidence coefficient? It is simply we said infimum over theta belongs to my theta space of this probability, probability that theta belongs to L of x comma U of x. Now, what is this infimum value?

Student: (())(07:38).

Professor Manjesh Hanawal: This is going to be 1 minus alpha because we have 1 minus alpha does not depend on what is your theta. Now, did I come up with a confidence interval which has a confidence coefficient alpha? When this happens, we are going to call this set 1 minus alpha confidence interval. So, did you study this confidence interval before in any other course?

So, I said last time we said we defined confidence estimators and we associated a confidence coefficient with that and we said that if I want confidence coefficient to be let us say 1 minus alpha then we, and we have a confidence interval which gives me that confidence coefficient then we are going to call it as 1 minus alpha confidence interval. Now, obvious, one obvious thing you can note from these two things I have defined A of theta and C of x here. So, what is A of theta? A of theta is set of all those points which I will accept to be coming from?

Student: (())(09:13).

Professor Manjesh Hanawal: Parameter theta. Now, what is C of x here? We will come to that, but we have defined C of x to be this interval. Now, suppose, let us say A of theta is the set of all samples whereas C of x is the set of parameters. So, this is now a set of points and this is now set of parameters. Suppose, some x belongs to C of theta and I have constructed C of x, if x belongs to A of theta can I say that then this theta belongs to C of x?

Student: Yes.

Professor Manjesh Hanawal: Yes, because the way we are constructed automatically guarantees that this is one is that inversion of other. So, it is not only one implication this is also an other implication. If x belongs to A of this, this is true if and only if theta belongs to C of x. So, now, because of this inversion, you will be able to construct nice confident sets from your test.

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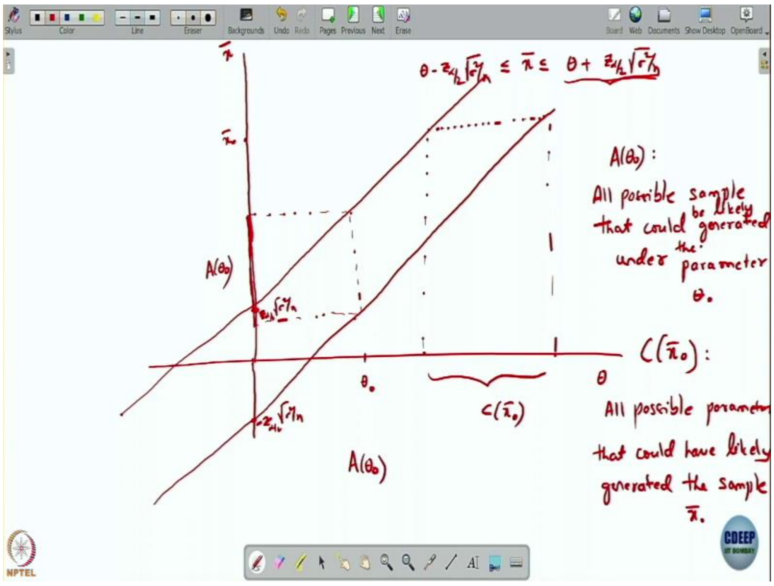

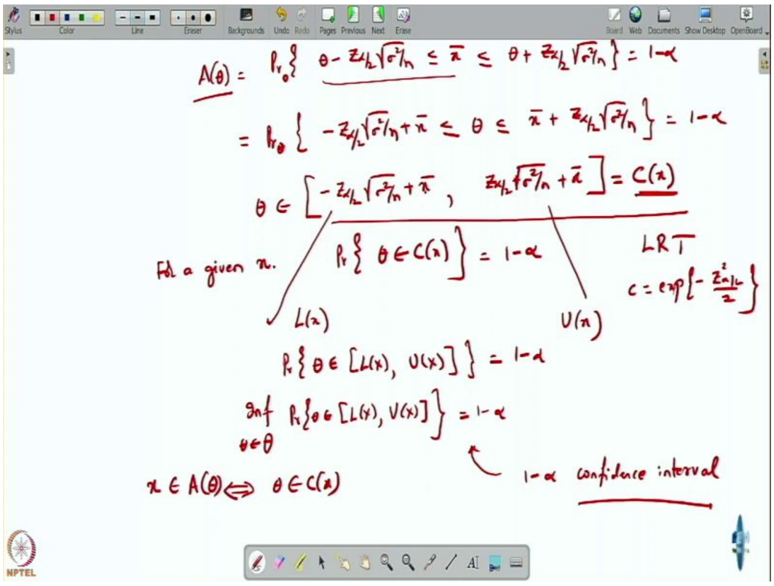

Now, quickly understand pictorially how does this work? Suppose, I want to do a, now I am basically mapping my looking relation between my parameter space and sample space. Now, notice that in the test we have covered always my samples, the random sample is always coming as sample mean. Everywhere, wherever I dealt, mostly I dealt with x bar, that is a sample mean.

I do not need to look into the sample itself, I had to just worry about the sample mean. So, let us say because of that, that is easier the vector has become one scalar for me. That entire random vector is now represent its sample mean. So, let us represent that on the y axis. And on the x axis is my parameter space, for time being assume this theta is one-dimensional, different possible values.

Now, let us plot this. What is the relation we get? We get x bar is less than or equals to theta, for a given theta, let us say z alpha by 2 sigma square n and theta minus z alpha by 2 sigma square n, so x bar is going to lie in this interval. Let us start with let us put theta equals to 0, theta equals to 0 what is the value of x bar, so x bar is going to be either this value or the negative of this value.

So, let us say something this is a positive value, and this is the associated negative value. This is like z alpha by 2 square root of sigma square by n. And this is minus z alpha by 2 sigma square by n. Now, as I increase theta, and let us look into this, this is like one linear function in theta with offset this much. So, let us do this.

And similarly the other one, sorry, I do not know if they are looking parallel a lot, but that is supposed to be parallel. Now, let us see, I have let us say let us take a particular parameter theta not. And I want to construct the set A theta not. From this can you tell me what is going to be A theta not?

Student: (())(14:00).

Professor Manjesh Hanawal: It will be all the points. Let us say if I take this thing. So, this value of x bar is going to be A of theta not for me. And now, let us say I have given a sample and that sample is this something let us say, let us take some x bar 0, some sample is given for which the mean value is x bar 0. Now, how to construct a confidence set for that. So, you do the same thing. You go like this here, and then you do like this.

And then we will get some value here and then some value here and this is going to be what C of x bar x bar 0, C of this entire thing is going to be C of x bar 0. So, with this can you see that it is possible to get from your test to construct a confidence interval in this fashion. And what is happening is basically the test, sorry the test and the confidence interval they are actually looking at a similar things but from a slightly different perspective.

What does this mean? What is the set A theta is looking into? A theta is looking into all possible, so A theta is the acceptance, this is looking into all possible samples that could generated under parameter theta not, that could be likely generated under the parameter theta not.

Now, what is C of x bar 0 is looking into? This is looking into set of all potential parameters that could, under those parameters this sample could have been generated. All possible parameters that could have likely generated the sample x bar under the parameter theta not. So, obviously, this correspondence leads to a theorem which I am going to write now and with that we will conclude this.

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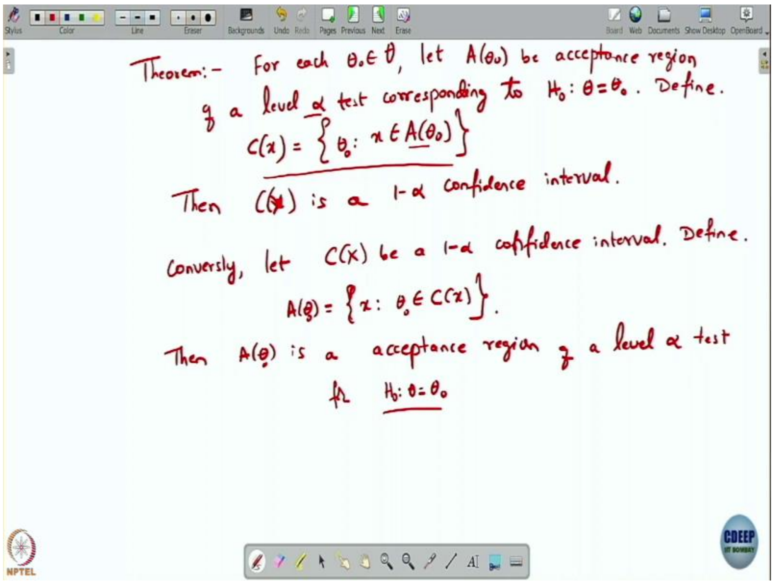

Let us say for each theta not belonging to your theta space. Let A of theta not be acceptance region of a level alpha test corresponding to H not which is going to test the hypothesis that theta is equals to theta not. Now, if you are going to now define C of x in terms of this A theta, you are now going to define C of x to be set of all theta or theta not says that x belongs to A theta not.

Then C of x is a, what can I can claim about C of x, I know that A of theta is a level alpha test and using that I have been able to construct the confidence interval, sorry confidence set or confidence interval. What I can say further about that confidence interval? I can say that this is going to be 1 minus alpha confidence interval. What we just said is if somebody gives you acceptance set of a level alpha test.

So, A theta is given to you and you have been told that it has a level alpha then using the A alpha you can construct a parameter space which is now going to be a confidence set with confidence coefficient of 1 minus alpha. So, that means I readily have 1 minus alpha confidence interval. Now, on the other hand, now, you have given a acceptance region and from this you are able to construct a confidence interval. Now, if somebody has given you a confidence interval, 1 minus alpha confidence interval you should be able to construct that acceptance region with what level?

Student: (())(21:04).

Professor Manjesh Hanawal: With alpha level. So, let us formally write that. Now, conversely let C of x, so I should be writing this let capital X be a 1 minus alpha confidence interval defined A of theta not equals to set of all theta such that x belongs to, sorry I have been given a, so this should be a set of x. So, let x such that your theta belongs to C of x.

Then A of theta is a acceptance region of a level alpha test for H not which is going to test whether theta is equals to theta not. So, now, this is if you now construct A of theta not which is a set given by this then this A of theta not is acceptance region of a level alpha test for this test which is where the null hypothesis is whether to test whether the parameter is theta equals to theta not. So, with this we will stop. I think proof is straightforward. I will leave you to look into the proof.