Engineering Statistics
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Week 11
Lecture 51
Tautology of tests and confidence intervals

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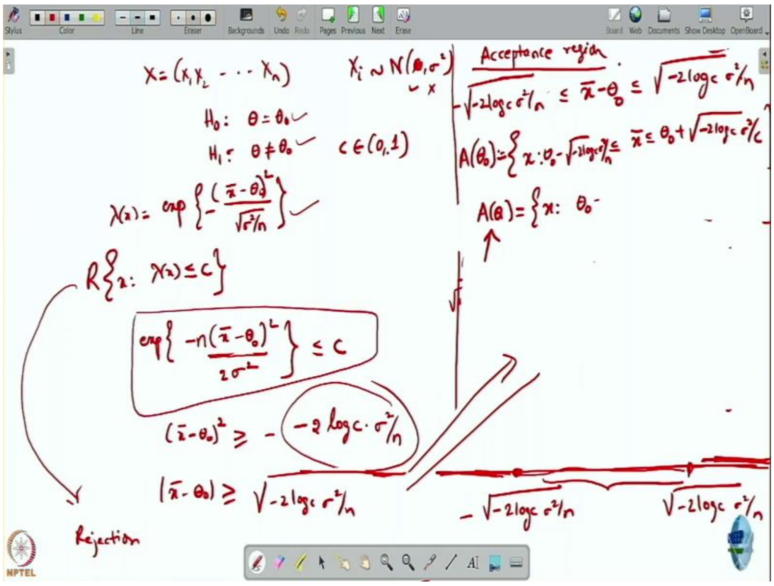

Last time we discussed, started discussing about what are confidence intervals and what is confidence coefficient and how to construct confidence intervals. Now, today we will continue the discussion of how to construct confidence intervals and this time we will see how to use the tests basically the hypothesis tests to come up with a confidence sets.

So, last time we discussed this let us say hypothesis I want to distinguish between two hypotheses H0 or not. And then we know for this when the samples are coming from Gaussian distribution, this is the likelihood ratio test we get. And then if x is given to me, I can test the condition whether this quantity is less than or equals to c and it will give rise to a condition. Now, let us get started with this today, from this point.

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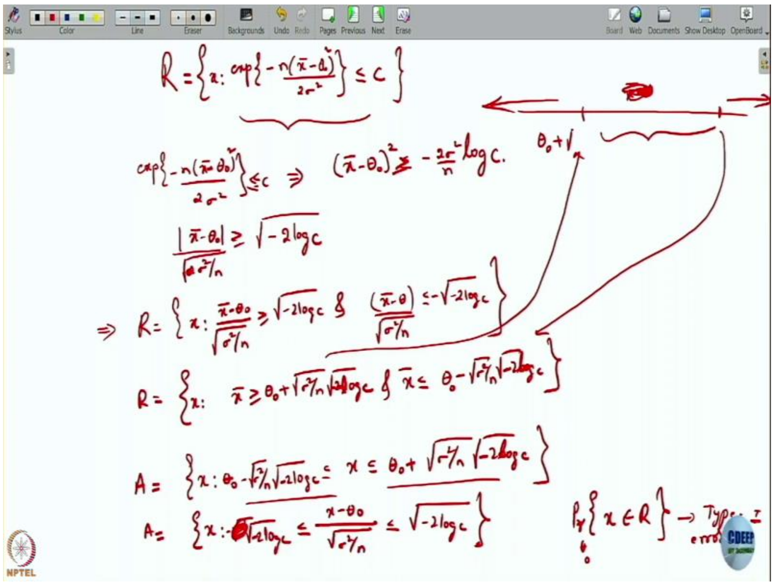

So, we know that exponential minus n x bar minus theta not minus 2 sigma square. This is going to be less than a request to c, this is my rejection region. Now, let us focus on this. Now, we can invert this exponential minus n x bar theta not by 2 sigma square by less than c I can write it as x bar, so there is a squared here I missed, x bar by theta not to be less than or equals to and then this is 2 sigma square by n log c. Now, this is less than or equals to because of the minus sign, there is a minus sign here which I have taken on the other side.

Student: (())(02:46).

Professor Manjesh Hanawal: Okay, sorry, you mean this. That is fine. Now, we also said x bar minus theta not, this should be minus sigma or maybe we said, further I can take it down this two sigma, sorry the sigma square n and the denominator and then square root, do the square root both sides then I am going to get minus 2 log c.

So, this is basically I will get the condition and we say that this is basically the rejection region I can write this is set of all points such that x bar minus theta if this is, yeah, this should be if it is a plus we said that this should be sigma square by n should be greater than or equals to minus 2 log c. And maybe I should have, that is x bar minus theta not by sigma square by n will be minus 2 log c and if and x not minus theta, this is the negative.

We said that this is minus if I take this should be less than or equals to minus 2 log c. And I think there should be a square root here also. If my x bar minus theta divided by this quantity happens to be greater than this, this should happen. And now, I can similarly can come up with the acceptance region. So, last time we said that so, now, I see. So, now, x bar minus theta, let us say or like.

So, x bar let us say plus theta plus, I am just taking this or maybe let me rewrite here itself. If R is equals to such that, if you are x such that x bar is greater than or equal to theta not plus the sigma square by n, n minus 2 log c and x bar is less than, this should be just a minute theta not minus square root n minus 2 log c. So, we said that, this will give me, so this quantity here maybe we can say and maybe this quantity is here.

And whenever my x such that the value falls in this region, I am going to, sorry outside this, either this region or this region I am going to reject and if it falls in between this I am going to accept and that acceptance region is called as x, that acceptance is easier to write. So, this is nothing but theta not plus sigma square n minus 2 log c theta not minus sigma square n.

So, now, I want to do a little manipulation on this side, A is nothing but x such that x minus theta not divided by square root n less than or equals to minus 2 log c and minus 2 log c. So, this is my acceptance region. And now, let us say if I compute probability that my so, maybe instead of this let us say I compute probability that x belongs to R. When the true parameter theta itself, what is the error this one gives you? This will give you a type 1 error.

So, now before I continue to say in a way this is already giving me some kind of hint that I am looking for x to be in this particular interval, when I have to accept x to belonging to this my null hypothesis, it has to belong to this. So, this is if I am going to invert this. Here, I am looking as a function of x.

But what if I fix x here and look for all theta, which satisfies this condition? Maybe that will give me a range of thetas that potentially explains this sample x. In that way we can invert this to get up our confidence set. Before we go into that, let us look, I want to write this in a simplified term.

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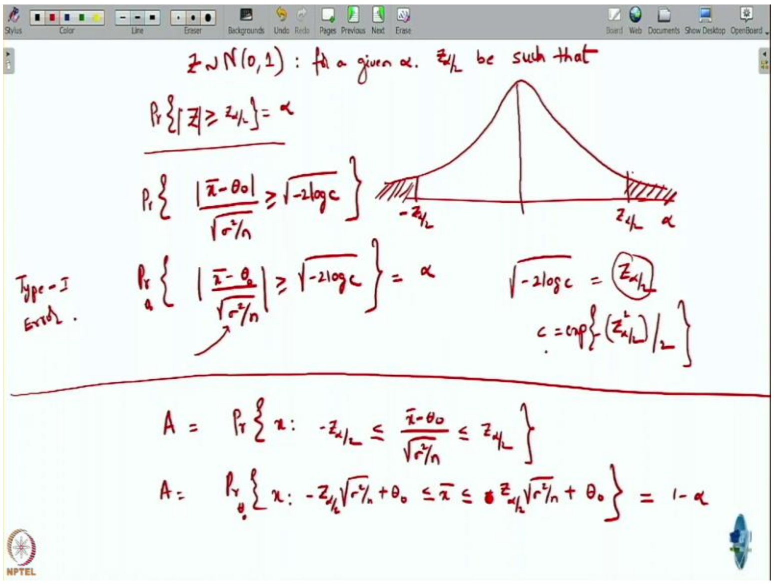

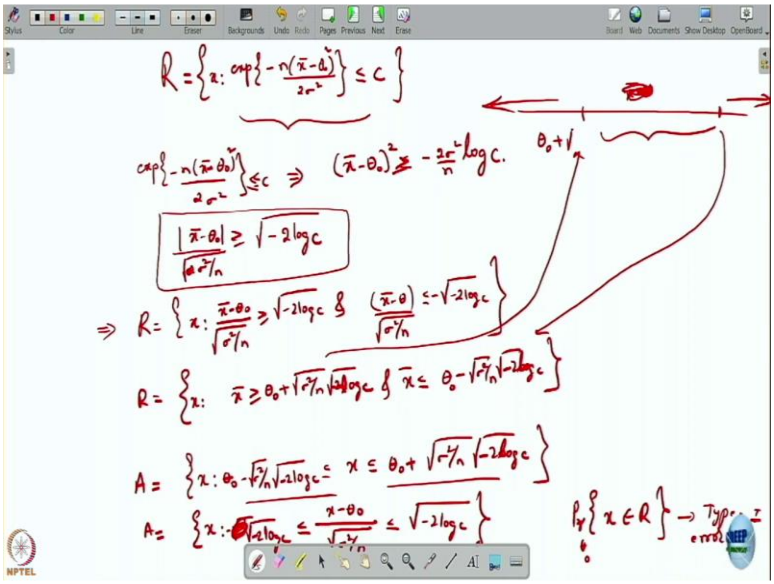

First, let set B standard gaussian normal, sorry standard normal distribution. Now, I am going to say that probability z greater than or equals to some z alpha by 2, this is equals to alpha. This is my definition. Now, I am basically saying that for a given alpha, z alpha by 2 be such that this quantity holds, this property holds. What I am basically saying, suppose this is my gaussian and I am basically looking for probability that z will be taking value larger such that that probability is alpha. Let us say if my z if it takes --

Student: (())(10:31).

Professor Manjesh Hanawal: Haa?

Student: Mod z.

Professor Manjesh Hanawal: Mod z. So, now let us, for time being only put only this quantity, I want to let us right now for the time being, let us use that. If I am going to only look at only what is the probability that beside taking value larger than certain quantities such that this probability. Maybe I want this probability, coverage of this probability to be alpha. And this will happen at some point I am going to call this to be beside alpha by 2.

Now, let us go back to our hypothesis testing problem we had. Probability, so, in the gaussian example, what is the condition we came up? In this previous example, this is our condition which translated to this quantity. So, now let us look into this I want to write this x bar minus theta not, this quantity divided by sigma square n, this is being greater than minus 2 log c.

Now, I know that the quantity here, x bar maybe I will just write it theta not divided by sigma square by n 2 log c. This quantity x bar minus theta not divided by square root sigma square by n this we know is gaussian distributed with mean 0 and variance equals to 1. Now, that is where we may need a mod of z, just a minute, just let me check this.

So, to, I mean I want this definition to be compatible with this, that is what mark I will do is maybe I will just take mod of z here. And in that case, I want alpha the take probability both on the positive side and the negative side, that total to be equals to alpha and where that point happens on the x axis that is what I am going to call it as z alpha by 2.

Student: (())(13:51).

Professor Manjesh Hanawal: Here, it should be minus z alpha by 2. Now, let us now try to say that this is what this is basically the rejection probability? Suppose, let us say I am looking this under the parameter theta naught, under theta not I know this quantity is standard normal, because the mean of x bar is theta not and its variance is anywhere sigma squared. Now, suppose I want this to be set to alpha, alpha is given to me.

Now, can you find out a c that will give me this probability alpha. So, this is what this is type 1 error right now. I want type 1 error to be set c. Now, I am asking give me a test. When I say give me a test here you can say okay, take this LRT test, but the LRT test remember you have to give me the value of c. What should be the value of c? That you should, I should use so that I will get value alpha.

One possibility is what we can do is okay, we can say okay said these two log c simply to z alpha by 2 you can do this. So, if I tell alpha on this gaussian curve can you find me what is the value of z alpha by 2 uniquely. That is unique? Like if I tell alpha, you know that, what is that tail possibility is that will give me the value of z alpha by 2. So, now I do this and from this, I will just do the inversion.

What is this is going to give me? z square alpha by 2 and then I think this whole quantity divided by 2, then exponential of minus of this. So, what I am basically saying, now, I am giving you a method, if for the gaussian sample, if you asked me to set the type 1 error to the alpha, then I am going to choose my c in this fashion exactly. Once you give me alpha, my z alpha by 2 is fixed and then I use that quantity to find out my c, and if I use this c, then I am guaranteeing you that my type 1 error is going to be alpha. So, with this now let us continue.

Now, let us start that my LRT test is such that it is already type 1 error is alpha. In that case, I can simply replace my 2 log c by z alpha by 2 in these expressions. So, what I am going to get, what I am going to get maybe I am starting with my rejection region to be probability that x has that x bar is now upper bounded by theta not plus.

Student: (())(17:18).

Professor Manjesh Hanawal: No, maybe I should, I should, just a minute, maybe I should say x bar minus theta not, let me write this sigma square by n this should be?

Student: (())(17:33).

Professor Manjesh Hanawal: -- z alpha by 2 and on the lower side.

Student: Minus z alpha by 2.

Professor Manjesh Hanawal: Minus z alpha by 2. So, or like I will just start further rewrite x such that minus z alpha by 2 sigma square by n and plus theta upper bounded by this quantity plus z, no, I think known it z by 2 minus sigma square by n plus theta not. So, this is my acceptance region. And this acceptance region when I look under the parameter theta not, when I say that, I look into this under the parameter theta not I know that this probability is equal to how much? How much is this going to be? This is going to be 1 minus alpha y not alpha.

Student: (())(19:06).

Professor Manjesh Hanawal: Yeah, because this alpha was set on the rejection one. Now, this set is the complement of that. So, this is going to be 1 minus alpha.