Engineering Statistics
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Lecture 49
Constructiong Confidence Interval from tests

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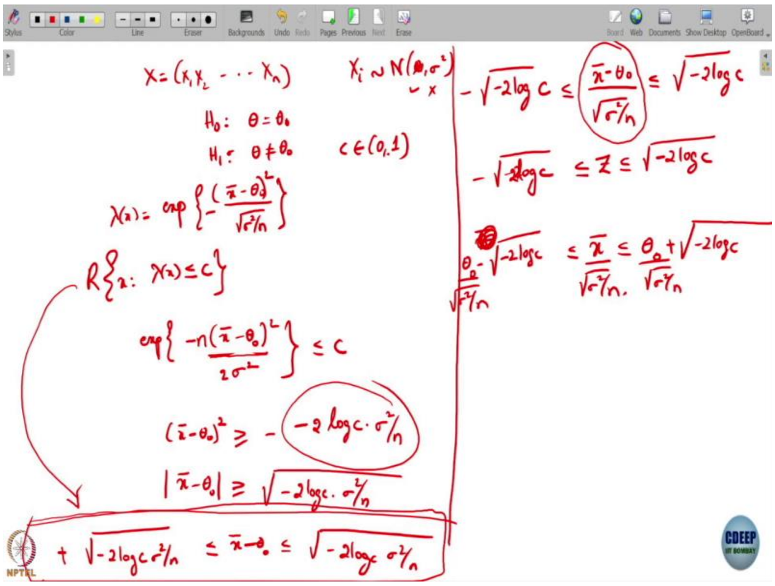

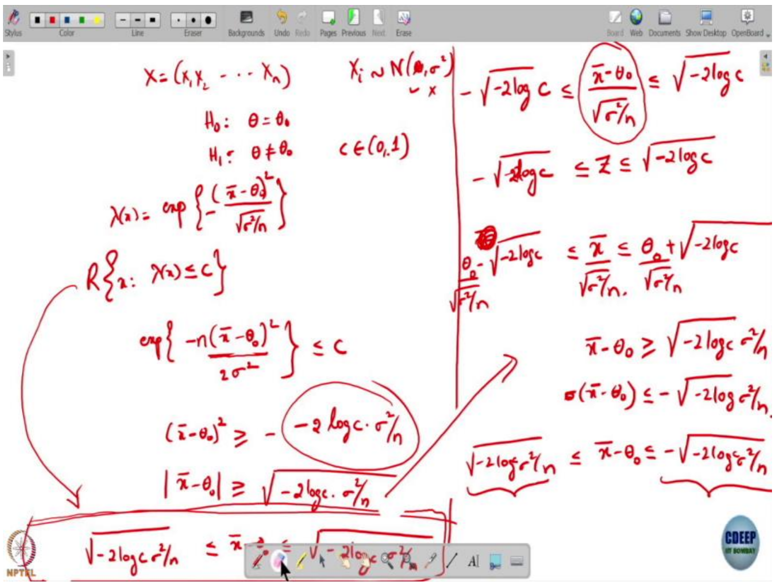

So, now let us go back to our example where we have this X1, X2 up to Xn where each of these Xi's are normal with mu and sigma square, where I assume… let us call it as theta, theta is unknown and sigma square is known my unknown parameter is only theta here. What was our estimator? Sorry what was our hypothesis testing here? Let me now this one I want to see whether my hypothesis 0 is whether theta is equals to theta not and H1 is theta is not equals to theta not, let us consider these two scenarios. What was our rejection LRT what is the value, what is the condition it gave us, what was the value of this LRT?

Student: X bar (())(01:36)

Professor Manjesh Hanwal: X bar minus theta naught divided by sigma square by n and our rejection region was if lambda x is less than or equals to C you reject all those samples. Now, was it this or there was an exponent, it was exponential value of this. I think this was exponential value of this. Can you check? And also, there is a square of course, this gaussian has to play a role here, this structure is coming from the gaussian this is the thing. And now this is my lambda of x and I want this lambda of x to be less than or C and that is my rejection, R is…

Now, what does this translate to? If I had to plug in this, this will translate to x bar minus theta not square greater than or equal to minus log C and sigma square by… logs C minus maybe sigma square I will write like this sigma square by n, that is it. Is there a square root here? Can you check? I think there is no square root there was n here and this was like simply sigma square. So, this part was numerator was simply the exponent in the gaussian term that is simply sigma square and we did some manipulation and we caught an N term here just cross verify this.

Now, this is going to be minus log C and sigma square by n sorry there is also 2 here which I will write 2 here minus, so what is the value of C? What is the range of C here? C 0?

Student: 0

Professor Manjesh Hanwal: 0 no, we know that this has to be between 0 to 1 because lambda x is a ratio which is always between 0 1 and because log C is between 0 1 we know that log of C is going to be a negative quantity and because of this, this whole quantity is a positive quantity, agree?

Now, how does this translate to? This translate to x bar minus theta naught, I can write it as to be greater than or equals to minus 2 log of C into sigma square by n agree? Which I can further say now, this is now x bar, this is like x bar minus theta naught less than or equals to minus 2 log C sigma square by n less than or equals to square root of minus of minus 2 log C sigma square by n, everybody agree with this computation?

Now, what I will do is x bar minus theta not I will get the sigma square by n from both sides now, this is going to be minus of 2 log C and minus of minus 2 log C and as always we know that this is going to be distributed what?

Student: (())(06:47)

Professor Manjesh Hanwal: So, I am going to write this is going to be minus 2 log C, minus of minus 2 log C, agree? Now, see that it is kind of giving me already some range here, what I am doing is? So, what I am going, this is going to give me, if this condition holds if my x bar is such that x bar minus theta is going to lie in this interval I am going to accept that x bar to coming from parameter theta not or no.

This is basically rejection region, I have basically translated that rejection region into this interval here, so if my x bar, so if this x bar right now, I do not know this x bar let me just put it in terms of x bar is now this is going to be theta not plus minus 2 log C sigma square by n this one theta not minus 2 log C, maybe I will just get rid of this sigma square by…in here anyway this is… this quantities are known and this is like so, there is going to be sigma square by n here sigma square by n here and another sigma square by n. What happened?

Student: (())(08:58)

Professor Manjesh Hanwal: This one so, I have taken theta naught this side, that is fine and then theta naught also this side, what is the issue?

Student: (())(09:12)

Professor Manjesh Hanwal: No, this is correct this is correct, when I remove the modulus this is correct, no let us do this when if suppose let say when this is a positive quantity, you are saying this should be there and when it is a negative quantity let me write it. Let me write this this quantity. Whenever this quantity happens to be positive their equality remains like this minus 2 log C sigma square by n.

And when there is a negation of this I have to take negation both sides this is like x bar minus theta not. And so, this is going to be less than or equals to minus of minus 2 log sigma squared by like this and now because of this, let us write this now for how to get this x bar minus theta not is going to be upper bounded by minus 2 log sigma square by n and then this is going to be minus… But see there is a issue here, something is wrong, we know that 2 log C is going to be negative quantity minus 2 log C is going to be a positive quantity. So, this entire thing this entire thing is a positive quantity, but now this entire thing is a negative quantity, how can this happen? Let us do that.

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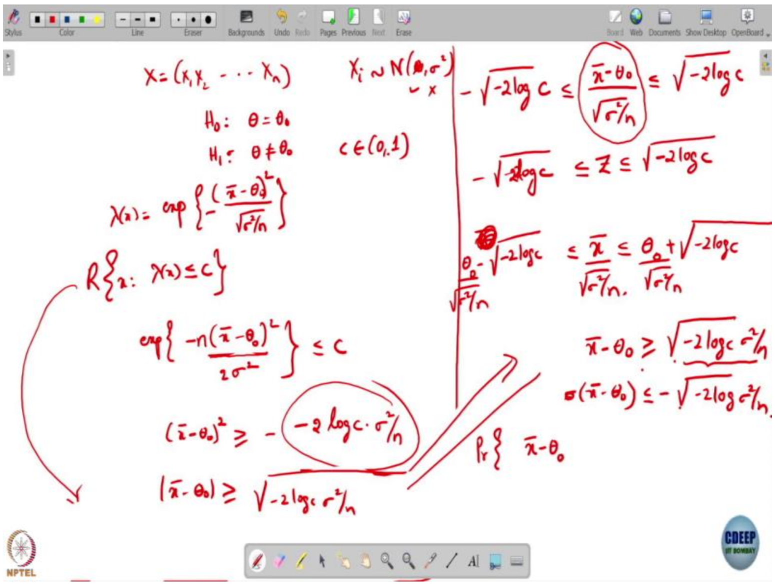

Now, what we have probability that x bar equals to this and this. Now, how we are going to write this? What we have written is actually and write like this and this. So, how can this happen like, so that is what I am saying, this is a inside that is a positive quantity, it has to be greater than positive quantity but less than a negative quantity, how can that happen? We have to start with this maybe let us find up.

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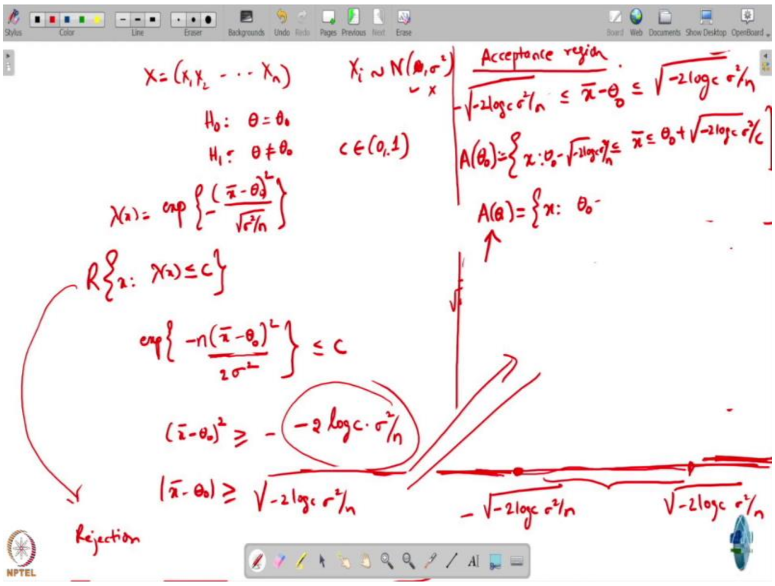

So, this is the condition we have and we know that this x bar minus theta not we know this has to satisfy, how you are going to define in terms of the intervals, now, we have to work out this is true, with this is this true? If this is x bar minus 3 this happens to a positive quantity this either this or this has to happen this is R this is not an and condition.

So, now probability that, now how we will write it x bar minus theta not, now let us say, let us define this I know that my quantity minus 2 log C sigma squared by n has to be here and minus of this and minus 2 log C sigma square by n has to be here. And what I am asking is? This x bar minus theta not should be greater than this and less than this. Whenever my x bar minus theta naught it is so happening whether it is falling in this region or this region what I am going to do?

I am going to reject and I am going to accept whenever it is going to be in this region. So, let us focus on that acceptance region. I think that is where the confusion came. We were not interest… Now, this is going to give me the rejection region and if I have to get the acceptance region then this has to be x bar minus theta naught has to be less than or equals to minus 2 log C sigma square by n and minus of minus 2 log C sigma square by n. So, this is the rejection and this is the acceptance region, this is clear?

Now, I am going to call it as A this region all the x such that x bar minus theta not. So, maybe now I can take that theta not on the other side, whenever this x bar is such that it is theta naught plus minus 2 log C sigma square by n and theta naught minus of minus 2 log C sigma square by n, this is I am going to accept that.

Now, this is for a given theta not, theta not is fixed. And for that parameter I am trying to define this acceptance region and I am going to call it as A of theta not. So, whenever my sample is falling in this region, that x bar minus theta I am going to accept it. Now, the question is, is it possible for us to translate this into the confidence interval.

So, we will discuss that in the next class. But, first thing we need to note here is that does it matter what particular theta is 0 that we are looking here, this could be any theta. So, for any theta, I can look for such conditions and this is what the rest of the things remain the same. And this is what we will use to come up with our confidence interval that should work for any given sample X. So, we will continue that in the next class.