Engineering Statistics
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Lecture 44
Type I and II Errors, Power Functions (Contd.)

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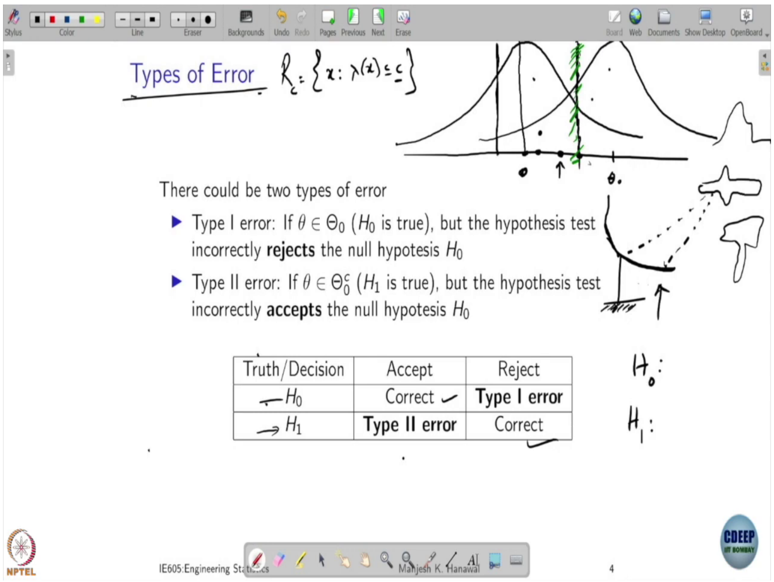

So, have you heard of like false positive, false negative, anyone here? So, what is false positive here? So, what is false positive? Can anyone explain? So, false positive means that is not there, but you are claiming it is there. Is type I error is a false positive? Is type II error is a false positive?

Student: Yes, sir.

Professor: So, it is null hypothesis, but you are accepting is as, it is alternate hypothesis but you are accepting it as null hypothesis, if you are null hypothesis, you are going to consider as the actual thing. Sorry, if in this case, this is going to be false positive, because you are saying it is positive, but the underlying one is actually negative, it is coming from alternate hypothesis.

So, this is like all these false positive, false negative is the mostly used in this machine learning languages. But that is basically statistics, type I and type II error which has been rebranded as false positive and false negatives.

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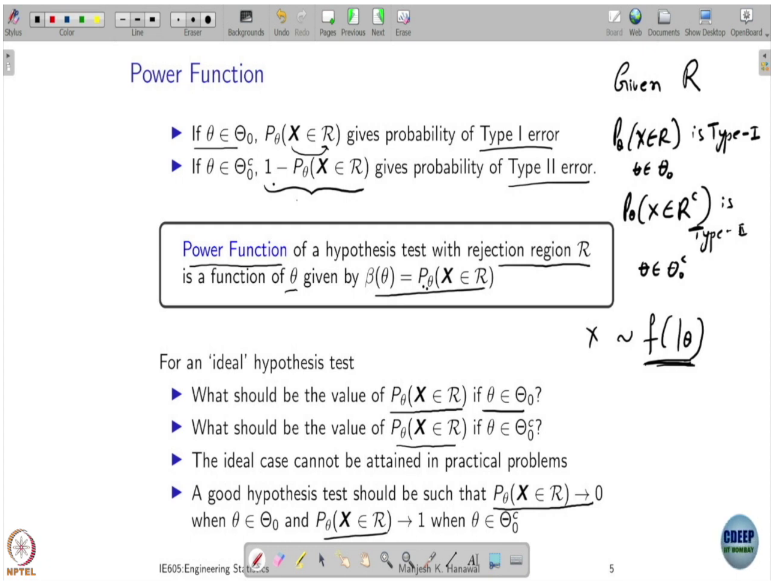

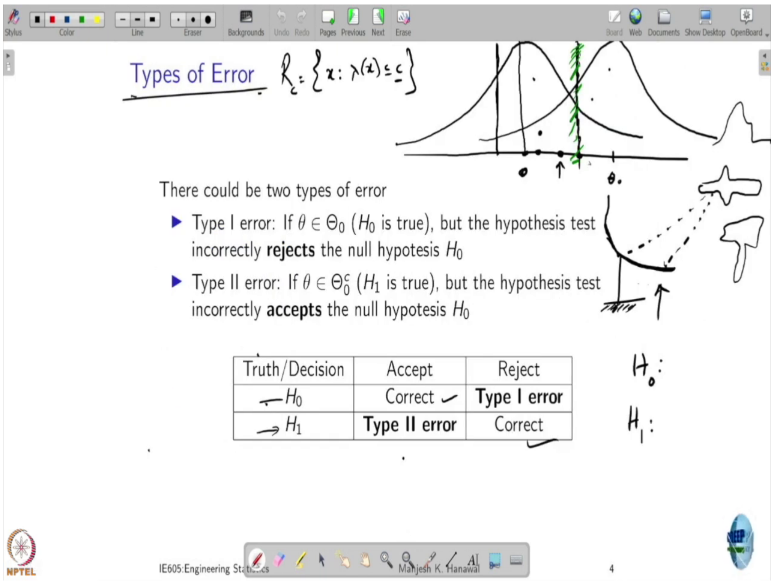

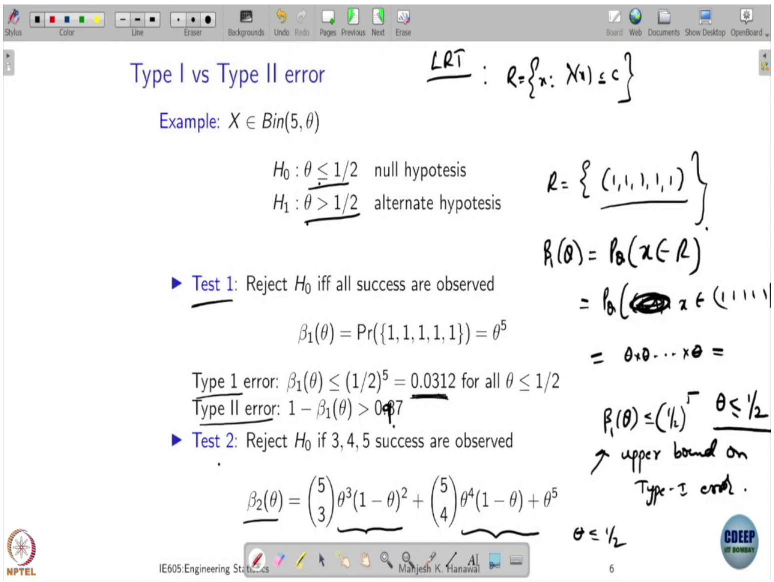

Now, if my theta is coming from my null hypothesis, and I am asking what is the probability that my X is belongs to rejection region. So, what is this is going to give me, the samples under my null hypothesis being treat, being labelled as that belonging to alternate hypothesis. That means, basically this is a type I error. So, the probability of X belongs to the rejection region under my parameter theta, which is coming from my null space is type I error. And similarly, if theta belonging to that theta complement, that means theta is coming from alternate hypothesis, but I am accepting it. So, notice that I am taking the 1 minus of the rejection. So, this is the acceptance probability.

So, it is actually the alternate hypothesis, but I am accepting it that means this probability will give me type II error, this is given your rejection region R. Now, this theta X belongs to R, is type I, when this theta is belonging to theta not. And similarly, this theta, maybe I should say belongs, maybe I will simply say R complement here is type II, when your theta is belongings to theta 0. This leads us to something called power functions. So, power functions are always associated with a given region, rejection region R. Given your rejection region R, power function is a function of theta, which is simply given as type I error.

Sorry, it is simply defined, like this is defined for every theta, it is not necessarily that theta is coming from your null hypothesis or alternate hypothesis. What it is saying that probability that your sample X being rejected under the parameter theta. Sorry, let us interpret correctly. What this P of theta X belonging to R is telling you? It is telling that, so X is here is a they are coming from some PDF. If that underline PDF probability is theta, and you are going to take the probability here with respect to, computing this property with respect to this PDF. And now, that value being rejected, this is going to be giving you the power function.

Now, let us see, how should an ideal power hypothesis test look like? If you have given a rejection function R, what should be the value of P of theta X belongs to R? If theta is coming from your null space. So, if your sample is coming from null space, you want to be rejected or accepted? If that, if your sample is already as if it is coming from the null space, you should be accepted, but here you are talking about the rejection probability. So, this probability should be, it should be low.

On the other hand, if your theta is already coming from your alternate hypothesis, then what should be the probability of rejection, should be higher side. But, as we discussed, it is not possible like to attain both type I and type II error to be large. Sorry, like one is high and another is low, because we said. Like by shifting this boundary. So, this boundary is what going to define your rejection region, you can shift it towards left, to improve your acceptance probability or minimise your rejection probability, but that is going to affect, it to be accepting the sample from the other distribution, as belonging to the null hypothesis. It is not just that the by shifting these boundaries, or changing my rejection region, I will be able to achieve both of it.

So, then but we always want best of both words, what we want is we would be looking for an hypothesis, which is going to make this rejection probabilities very small, or almost tending to 0, when that theta parameter is coming from null hypothesis. And want it to be almost going to 1, when my parameter is going to come to from alternate hypothesis. So, if there is not such an ideal hypothesis, I would be very happy about this. But it may always not be the case. And there has to be some sweet balance, one has to find between these two matrices.

That is I want type I error to go to. So, this is like a type I error, that is the parameter is actual 1, but I am rejecting it. Type I error should goes to 0. Whereas, I am just, this type II error should also be going to 0, but like I am just only focusing on this part without 1 minus, that is going to 1, I want to achieve both of it. Now, let us look into quickly couple of examples about the calculations of, let us take a simple case of binomial distribution with five samples. Sorry, binomial distribution when n equals to five, and theta equals to some number I do not know. And I want to make a hypothesis on my theta, my hypothesis are theta is less than half, and theta is greater than half.

Now, what are my possible test. and how they fare in terms of type I and type II errors? You can always construct your lambda x for some c, and define a rejection region like this. This is going to be your LRT test, but instead of that I am looking into some other test, that are more natural. So, like I am. So, theta is less than half, and theta is greater than half. What could be one natural test? I am saying that if theta is less than half, maybe I will not see all once in my phi samples. Because like if theta is less than half, maybe some of them will be 0. On the other hand, if theta is greater than half, maybe like I seeing, all the ones is more likely, because one observing is more likely.

You are getting this point or not? So based on that, I may come up with one trivial rejection region, say that if all this is binomial that basically this is a Bernoulli, five Bernoulli's. If all these five observations I am going to make, they are all 1, then I am going to take it to be, alternate hypothesis. If not, I am going to take it to be null hypothesis. So, I am kind of being very crude here saying that, if the theta is greater than half, then maybe all of them will be 1, that is why saying, like saying that, only if all the values are 1, take it to be 1, then simply reject it, otherwise, simply accept it to the null hypothesis. So, in this case, my rejection region has only 1 sample, that is 1, 1, 1, 1.

So, you only if I observe all 1s, then I am going to reject, otherwise, I am accepting. So, my rejection region has just one sample, and by definition for beta 1 of theta is what, probability of theta x belongs to R, and here, and x is nothing but here 1, 1, 1. Sorry, here my x what is my this one? This is 1, 1, 1. So, and that means all my xi's are 1, which is nothing but theta to the power 5. Did anybody, everybody follow this? So, I am just saying, and if under parameter theta, observing all 1s is going to be theta to the power 5. Because theta, theta multiplied by 5 times, which is 5 times.

Now, in this case, let us try to see what is the type I error? Now you have this beta 1 of, sorry, this is for test 1, beta 1 of theta is 5, and if this theta is less than or equals to half, then it is if the whenever this theta is less than or equal to half, this is coming from the null hypothesis which I wanted. And now as a function of theta, this is increasing in theta. So, the maximum value of this beta 1 theta, for theta less than request to half, is simply going to be half of 2 to the power 5. So, this is going to give me a upper bound on type I error, everyone agree? So why I restricted these two half? Because I am looking for type I error, in which theta is going to be coming from my null hypothesis, and the largest value of that is like till half, that is why I put half, and I got this value.

And type II error is simply going to be a complement of this. And I just take 1 minus of this, and I will get 0.83, and is this correct? If or something mistake here. So, what is this going to be, 7, 3, 10, plus 1, it is only giving me 0.9, can you check what is 1 by 2 to the power 5?

Student: 1 by 2 to the power 5 is 0.03125.

Professor: So, did I make any mistake? Find out what is 1 minus this quantity.

Student: 0.96875.

Professor: Approximately, I will write 0.97. And type II error is going to be this. So, notice that here type I error is less than this much, this probability. Whereas, type II error is so much, it is saying, it is going to be 0.97, you wanted what? Type II error to go, you also wanted type II error to be small as well. But, if you are going to take such a simple test, you are happy with respect to type I error, type I error is small, but your type II error is very bad. So, type II error is going to be very bad. So, what is type II error here is? Like type II error is it is coming from, so, in this case like when theta is greater than half. And it may happen that, even when theta is greater than half, you may see some 0s in your sequence.

And when you see 0s in yours, you may not see all the time 5, and because of that you may not reject it. So, because of that the type II error could be high. So, there is always trade off, like any test may not be good. Let us look into another case, test 2. So in this case, I want to be slightly smarter. And I know that, out of 5 and majority of them are 1, then I will reject, like majority means at least there are 3 successes, or 4 successes, or 5 successes, then I am reject. Now what for that test, you can compute what is the probability of beta 2? What is your power function, that is this is probability that we are going to see 3 successes, this is probability that you are going to see 4 successes, and probability that you are going to see 5 successes.

So, this one is little more involved. Now, I do not know how to bound it, like if theta is less than or equal to half, what is its value, you can compute, and you can then also compute its type II error bounds, then you can see that it will have a better type II error compared to the first one. So, which are the test one, like obviously by just looking into that test 2 is going to be better than test 1. Simply saying all 1s is not a good reason to reject. So, work out more maybe on this test 2 example, and to see what is the type II error you are going to get? Let us stop here.