Engineering Statistics
Professor Manjesh Hanawal
Industrial Engineering and Operations Research
Indian Institute of Technology, Bombay
Week 3
Lecture no. 15
Conditional PMF and PDF

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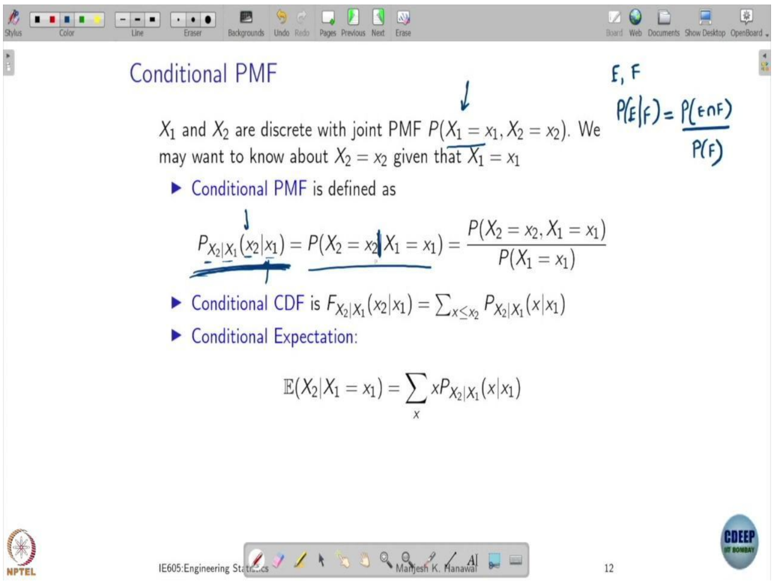

Now conditional probability mass functions. This is nothing but the joint probabilities. But earlier we were interested in joint, but now you want to condition on one happening over the other. We already talked about this conditioning on the events. What was that? How did you define if we have E and F are 2 events? And I am trying to find what is the conditional probability of F given that F has occurred. How did you define it? This is nothing but...

Student: (()) (0:51)

Professor: Divided by F. But now, this notion can be extended to random variables. Ultimately, if I am going to say that X equals to some small some number X1. This is corresponding to some event. So, now asking instead of specific events, I can ask those questions in terms of the random variables, we already have defined joint probability mass functions. Now, we want to extend this to the conditional probability mass functions.

Suppose I want to know what is the probability that the second random variable is going to take value X2 given that the first random variable has taken the value X1. This is a notation for this, but exact definition is this. And this is what it represents that is joint probability of

X1 taking value small x1 and a second and a variable taking X2 given divided by probability that X1 is going to take the value x1.

Now, so notice that now, focus on the notations like every time we may simply write like this. Here it is kind of giving already by this notation, we mean that we are asking for the question that my second random variable, this the one in the numerator, this portion corresponds to this, and X1 corresponds to this random variable. And here, this vertical, this is like this is a notation to say given it is not like a division in this.

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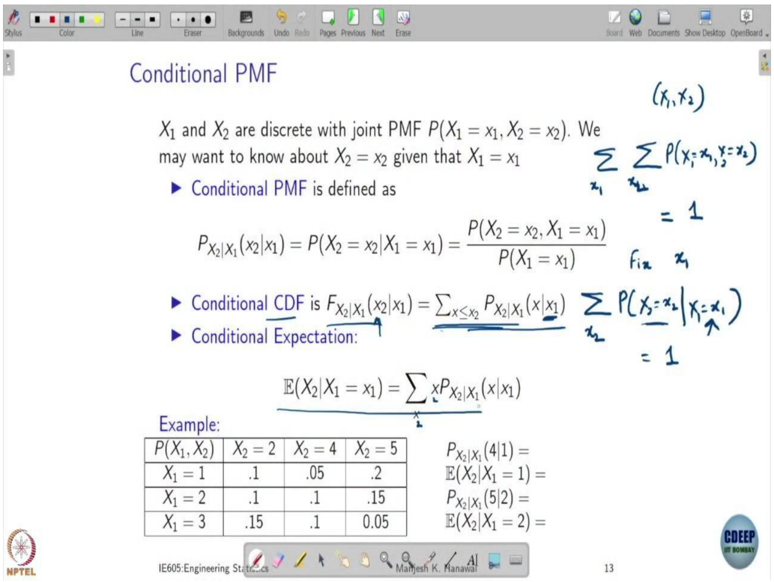

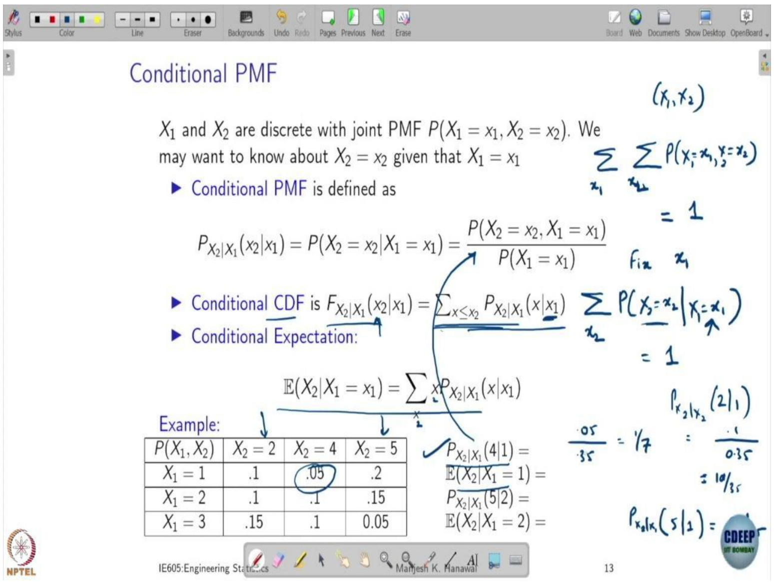

Now, earlier, we argued that if let us say P is a, some discrete random variable, let us say X1 and X2 has some probability mass function. And we said X1, and X2 takes value X2 and being if you add it or all possible values of X 1, let us say this X 1 here, and X 2 here, what this value is going to be?

Student: 1.

Professor: It will be 1. But now suppose I am interested in this conditional probability, like fix some X1 and now I am looking into this probability that X2 is going to take X2 given X1 has already happened X1. Now this is fixed. And now I am looking into the possible values the X2 can take. And now I want to take the summation of this X2, what this value can be? This one, no, I just interpret, what is this? This is I am asking the question. I have already telling that there are 2 random variables. Let us take for simplicity, X 1 is one coin, X 2 is another coin. They will be dependent, right now I am not saying anything dependent or independent.

The first coin I said it has already taken head or let us say it is 1, X 1 is 1 I have taught. Now X 2 I am asking, now I am asking X 2 after my first coin has taken value 1, X 2 can take still head or tail. And now I am asking the sum over those values. It has to add up to 1 like after the first one has shown had, the second one either has to take head or tail, one of these possibilities have to there is no other way. And there is some has to be 1. So, this is again going to be 1 actually. And that is why these conditional probabilities are themselves probability mass functions.

And similarly, you can define conditional ones, conditional probabilities are themselves are probability mass functions, and you just treat it as a simple probability mass function on a given condition and for that you can define your CDF in this fashion. Now here, where X 1 is fixed, and now you are looking for the probability that your X 2 is going to take value less than or equals to X 2. So, I am just adding all the possible values of X 2 till the value x2. And once I know these conditional probabilities, I can talk about conditional expectation now. And this is a conditional expectation of X 2, given X 1 has taken value x 1, all I need to do is now X 1 has already happened x 1. So, I do not need to worry about that X 1 is fixed. I now need to worry about all possible values of X 2.

So, I am taking all possible values of X 2 here, maybe I should written this X 2 here. I multiply it, it is a probability conditional probability. And when you submit over all possible X 2 values, you will get the expected value. It is clear the expectation, those who could not follow like, go back and refer to the slides. Now, let us quickly do an example to see that how much you could understood.

This table is the same table which we have earlier used for the probability mass function. So, there, you try to compute the marginals but now, we are going to compute…

Student: Conditional probabilities.

Professor: Conditional probabilities. So, now let us compute the probability that X 1 has taken the value 1, what is the probability that X 2 will take value 4? So, this one is simple go back and use this formula, let us compute what is the probability that X 2 takes value 4 and X 1 takes value 1, 4 and 1. So, if I had to use this formula, the numerator I got is 0.05. Now, what is the probability that X 1 equals to 1?

Student: 0.35

Professor: This is the sum of this row. And what is that value?

Student: 0.35.

Professor: This is going to be 1 by 7. Now, I want to compute the expectation of X 2 given that X 1 equals to 1. Now, for that what you need to do? You need to compute the probability mass function of X 2 given that X 1 equals to 1. One value already computed here. You have computed what is the value that X 2 takes value 4 given X 1 equals to 1. Now you need to find out what is that it takes value 2 and 5? Can somebody tell me, what is 2 given 1? So, the denominator remains same. What is going to change is this is going to be 0.1. So, this is like 1 by 35, 10 by 35.

And similarly, what is that, p X2 given X1 5 given 1, 20 by 35. So, now you got all the 3 all the possible values of X2.

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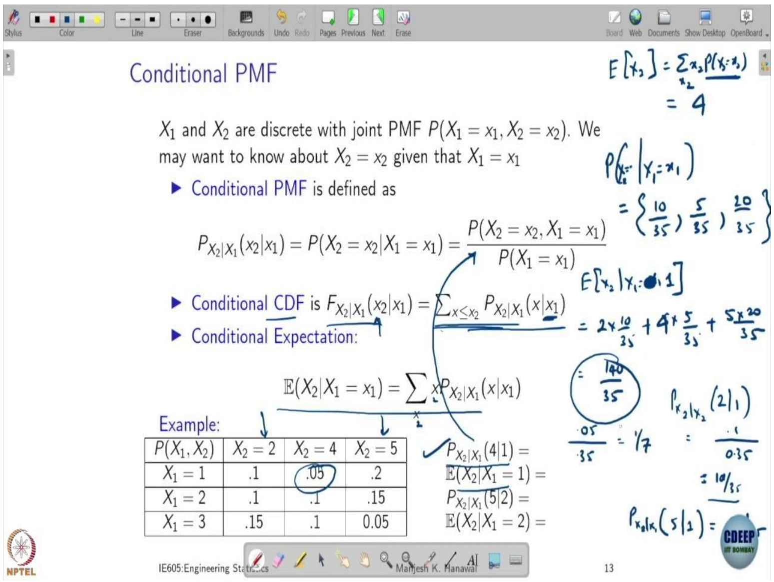

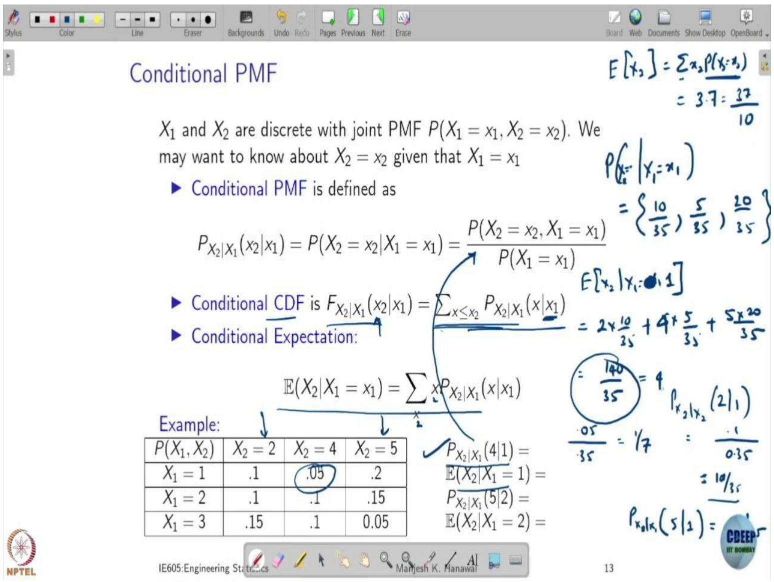

So, first thing you need to now verify is, let me what we get is now probability that I am putting a dot here, X 1 equals to x 1, what are the possible value X 2 is taking? It is taking 3 values 2, 4, 5. And now we just computed its probability mass function? When X 2 maybe write me 2 X 2 when X 2 equals to 2, it was how much?

Student: 10/35.

Professor: It was 10 by 35. And when it was 4, how much it was? 1 by 7 or let me write is a 5 by 35 and when X 2 equals to 5, it was 20 by 35, is this a probability mass function the conditional because it is adding up to 1. So, always if it is a conditional probability is better add up to 1. Now, find the expectation now, it is easy. You have all the things. Now, this is nothing but X 1 equals to x, now this is going to be 1. Now, all you need to do is 2 into 10 by 35 plus 5 into sorry 4 into 5 by 35 plus 5 into 20 by 35 and how much is this? 20, 40, 100, 140 by 35, am I correct? That is it that is expected value.

Now as a quick computation this is expectation of X 2 given that X 1 equals 1. Now, can you compute what is the expectation of X 2 just what is the expectation of X 2. This is unconditional expectation. What is the value of this? This is nothing but x 2 into P of X 2 is taking value X 2 or x 2 and you can compute this marginal from this conditional that joint probabilities you can come compute this marginal. And also how you can compute this. Can somebody quickly compute this and tell me whether you get the same value as this? Are you getting same value, sure?

Student: 4.

Professor: It is just 4. Someone can someone verify this for me?

Student: 4

Professor: This is 4.

Student: (()) (12:38)

Professor: Leave it, why you want to calculate. Just enough know a fraction. I want this value.

Student: 3.7.

Professor: You are getting this as 3.7, you say like 37 by 100, did anybody get 3.7?

Student: (()) (12:59)

Professor: And this is 2. No this is not 2. This is 4. So, conditional expectation is 4 and unconditional expectation is 3.7, so in that way conditional probabilities and unconditional probability need not be the same. Similarly, you can compute it for others, I am just skipping it.

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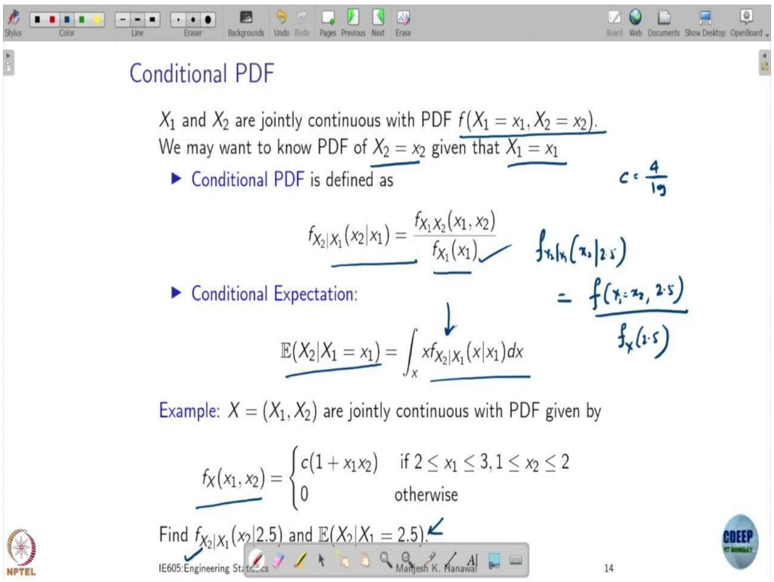

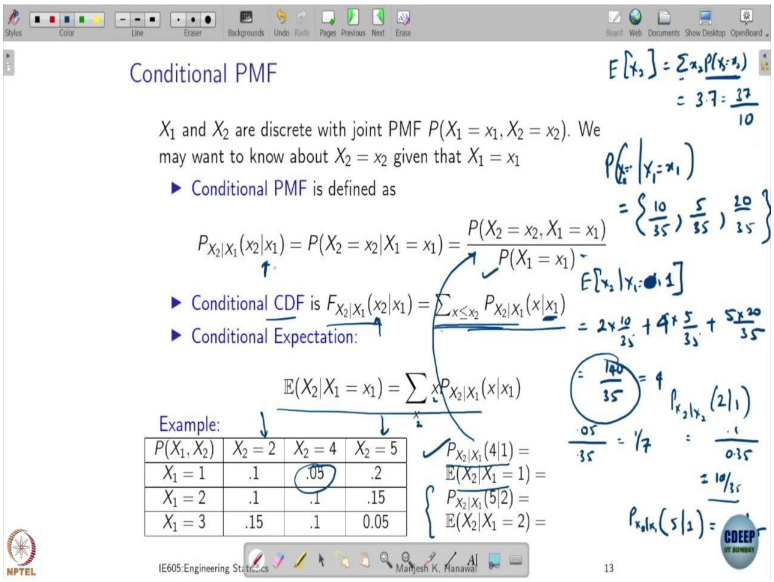

Once we do it for PMF same thing can be done for probability density function also. Let us say you have this joint continuous probability density function then we want to know the PDF of X 2 given that X 1 has taken some value. And by definition, the conditional probability is defined like this.

That is again the joint PDF divided by the marginal PDF on which you want to condition. And similarly, if you want to compute the expectation, you do the same thing. You take the expectation with respect to the conditional PDF. And here again, as an example, I put the same example that I used earlier, where we try to compute marginal PDF from their joint PDF. Now, can somebody quickly compute me? By the way, do somebody remember what was the value of C, earlier?

Student: 4 by 9.

Professor: 4 by 9.

Student: 4 by 19.

Professor: 4 by 19, 4 by 29.

Student: 4 by 19.

Professor: 4 by 19. Let us take for time being c equals to 4 by 19. Now, what is this value of f of x 2 x 1 given that your x 2 and x 1 value is 2.5. How to compute this? For this you need to find f of.

Professor: You want to take X 2 equals to X 2 and X 1 is 0.25 divided by f of x at 0.25. This is the value. And notice that both in this case and this case and this case the value on which you want to condition that this probability need to be positive. Otherwise you will end up with divided by 0 condition. If you are trying to condition on some value x 1 it is better be some value in which there is some positive mass. If there is no positive mass that means value 0 means why you want to condition on something which is not going to occur.

So, whenever you want to do condition make sure that you condition on something which has a nonzero mass and the same thing when you want to condition the continuous case on some point make sure that that point has a positive value the PDF is positive at that point. Like similarly again you can compute the expectation here by finding the like first you need to find the conditional PDF and based on that you can find out this conditional expectation.