

MINERAL ECONOMICS AND BUSINESS

Prof. Shantanu Kumar Patel

Department of Mining Engineering

IIT Kharagpur

Lecture 08: Ore Reserve Estimation - 1

Concept Covered

1. Traditional Methods ✓
 - Method of triangle ✓
 - Geometrical method ✓
 - Nearest point method ✓
 - Block method ✓
2. Geostatistical Methods ✓
 - Variogram ✓
 - Kriging ✓



JANUARY 2023

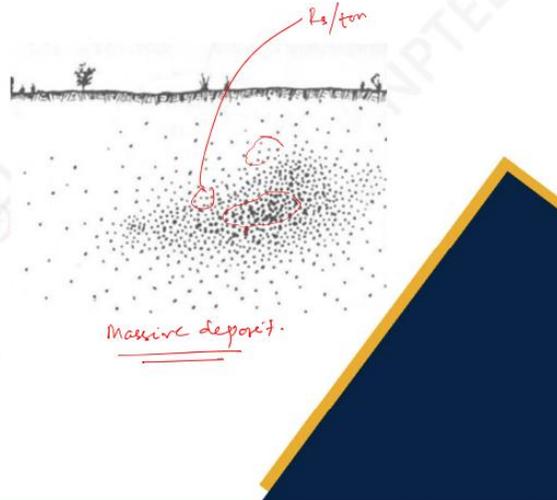
Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur

Hello everyone, and welcome to this course on mineral economics and business. So, this is our lecture number 8, and this is the first part of this ore reserve estimation. These are the concepts to be covered today. We will look into the traditional methods and the geostatistical method because, you know, all those ore reserve estimation methods are broadly divided into two classes. In the traditional method, we are going to look into the method of triangles, geometrical method, the nearest point method, and the block method.

And in the second part of this lecture, which is lecture number 9, we will look into the geostatistical method, and we will see how we do the variogram and the kriging.

Ore Reserve Estimation

- **Ore** is defined as mineral that can be **extracted at a profit**
- Profit depends on:
 - cost of extraction ✓
 - selling price ✓
 - amount of the **valuable mineral present**.
- Grade and size of the deposit is unknown and must be estimated for any mine investment analysis



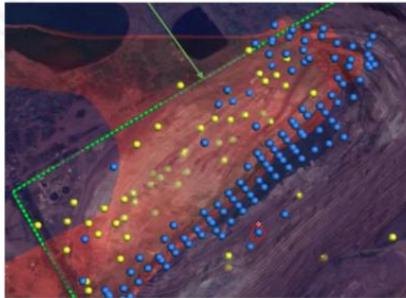
JANUARY 2025

Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur

As we know, ore is defined as a mineral that can be extracted at a profit. So, let us say this is a vertical section of a deposit. The black dots here are showing the concentration of the mineral. If the concentration of the mineral is more at a particular location, like in this location, the grade of the deposit in that location is better. So, you know that, let us say we are starting a mine in this case, and we want to calculate the total profit. This total profit depends on three things: one is the cost of extraction.

So, if you want to take the mineral out, what is the cost per ton. And also the selling price, you know, like if we are, once this mineral is processed, we sell it, and what is the selling price of this mineral? And the total profit that the total amount of valuable mineral present in this case also matters. And this valuable mineral present depends on the grade and size of the deposit, which must be estimated by different methods for any mine investment analysis.

Details of a Borehole



Borehole	Easting	Northing	Grade (%)
1	6165.0	6958.5	1.16
2	6363.1	7121.6	0.38
3	6321.3	7252.1	1.08
4	6208.5	7103.5	0.12
5	5851.6	7216.2	1.06
6	6287.8	6497.7	1.96
7	5809.4	7187.9	0.70
8	5613.3	6868.3	1.94
9	5608.2	7034.3	0.70
10	5323.6	6786.2	0.52



3:02 / 29:15

Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur

So, this is an example of the borehole exploration. Here, the dots are showing the location of the boreholes, and the blue dots and the yellow dots are different colors because they were drilled at two different stages. The right-side table here shows, let us say, an example of 10 different boreholes, and you can see it is showing the easting and northing of the boreholes and also the grade associated with that borehole.

So, or maybe the average grade that we are going to get for that kind of borehole. Now, in this example, this can be, you know, let us say around 1 kilometer. Here, it can be, let us say, 3 kilometers. So, in this kind of case, ah, you know, if you see the deposition, ah, volume and the amount of, ah, drilling that we are going to do, ah, the material we are really getting from, ah, the site is a small fraction of, you know, the total volume. So, from this small fraction, what we need to do is to estimate what the reserve is.

Traditional Method

The traditional methods are based on the following three main principles or rules:

1. **Rule of generalization** ✓

When there's insufficient data at one part of the deposit, the rule of generalization suggests that the characteristics (e.g., grade) in that area can be assumed to be similar to those in other areas where more data exists.

2. **Rule of gradual change**

3. **Rule of nearest point**



Then we have two more rules, first one is called rule of gradual change, second one is called rule of nearest point, we will see these ah rules ah in the next slides.



Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur



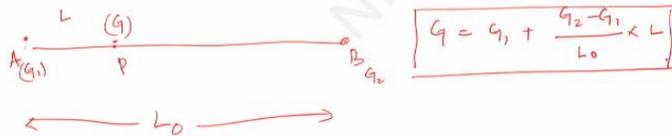
So, this process of reserve estimation is one of, extending the sample data to a larger volume of the material and then usually converting this volume to the weights. So, as we discussed, there are many methods available to perform this task. The method, ah, as we discussed, you know, can be roughly separated into two. The first one is called the traditional method that we are going to see in today's lecture, and the second one is the more recently developed geostatistical method.

So, in the traditional method, are based on three main, principles or rules, the first one is called the rule of generalization. So, what this says is when there is insufficient data at one part of the deposit, the rule of generalization suggests that the characteristic, for example, the grade in that area, can be assumed to be similar to those in the areas where more data exists. What this means is, let us say this is the top view of any kind of deposit and what we did was we drilled more boreholes here, and let us say that boreholes are far apart in here. And what we can assume here, or maybe the rule of generalization says that the characteristic in the part, let us say B, will be similar to that of part A. Then we have two

more rules, first one is called rule of gradual change, second one is called rule of nearest point, we will see these ah rules ah in the next slides.

Rule of Gradual Change

- The rule of gradual change, also termed as the law of linear function, is based on the assumption that all sample elements of a mineral body change gradually and continuously as a linear function along a straight line connecting two adjacent sample points.



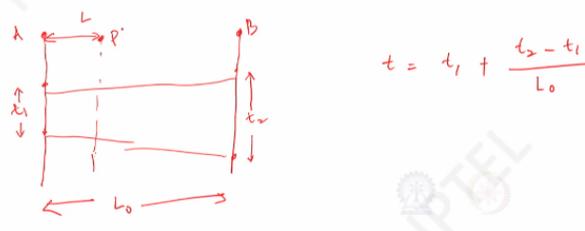
7:21 / 29:15

Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur

So, first is the rule of gradual change. The rule of gradual change also termed as law of linear function is based on the assumption that all sample elements of a of a mineral mineral body change gradually and continuously as a linear function along a straight line connecting two adjacent sample points. What does this mean is let us say we drilled ah ah ah two boreholes ah one at a point a and another at a point b and the distance between this point ah ah two points is let us say and ah what we need to do here ah is at a at a point in between this a b which is at a distance of l, we want to find what is the grade g here ah and and before that you know the grade at a is let us say G_1 and the grade at b is let us say G_2 .

So, this rule of gradual change tells that you know G is the grade at point let us say unknown point P is $G_1 + (G_2 - G_1)/L_0 * L$

Rule of Gradual Change



$$t = t_1 + \frac{t_2 - t_1}{L_0} L$$

- This principle is used in the [method of triangles](#).

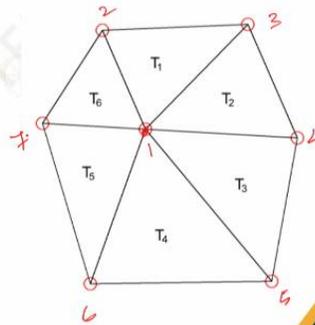
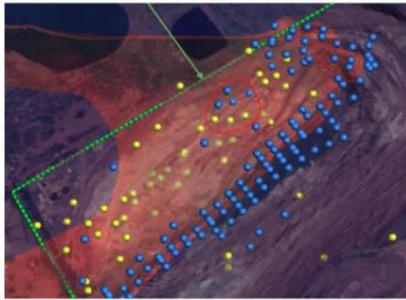
t equal to t 1 plus t 2 minus t 1 divided by L 0 again L 0 is the initial distance between ah point a and point b here and in the distance between point a and point p which is L


 Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
 Department of Mining Engineering, IIT Kharagpur

Also as you know let us say you know let us say take a vertical section at the first point and second point and at first point a the thickness of the deposit is let us say t_1 and at point b the thickness of the deposit is let us say t_2 . So, this distance is t_2 here and this distance t_1 here. Now, if we want to get the thickness of the deposit at a distance L from let us say point A. So, the thickness here will be calculated using $t = t_1 + t_2 - t_1 / L_0$ again L_0 is the initial distance between ah point a and point b here and in the distance between point a and point p which is L . So, this rule of gradual change is you know applied in the method of triangle.

Method of Triangle

In this method, all boreholes are connected to one another by series of straight lines thus dividing the ore body into a series of triangles.



So, in this case you know what we do is we join these boreholes with straight lines



9:21 / 29:15

Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur

what is this method of triangle is let us say let us assume the previous case of the deposit and we are looking at few of the boreholes let us say here. And those boreholes are let us say you know number 1, number 2, number 3, number 4, 5, 6 and 7. So, in this case you know what we do is we join these boreholes with straight lines.

thus dividing the entire borehole in a series of triangles. So, in this case we will you know make the different triangles here and divide the entire borehole into different series of triangles.

Method of Triangle

- Varying altitudes of the vertices represent the varying **thickness of mineralization**.
- Each triangle represents the base area of an imaginary prism of some thickness.
- The average grade of each prism is usually calculated.
- In other words, if t_1 , t_2 & t_3 are the thickness of the mineral prism at three vertices and g_1 , g_2 and g_3 are grades at the correspondent location the average grade assigned to the prism is given as follows:

$$G = \frac{g_1 t_1 + g_2 t_2 + g_3 t_3}{t_1 + t_2 + t_3}$$

$$t = \frac{t_1 + t_2 + t_3}{3}$$

So, let us say let us let us consider 3 ah points here.



Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur

So, once this is done the varying altitude of the vertices represents the varying thickness of the mineralization. Each triangle represents the base area of an imaginary prism of some thickness. So, let us say let us let us consider 3 points here. 1, 2 and 3.

So, the thickness at point 1 will be different than the thickness of point 2 and can be different than thickness of point 3. So, here you know as we said that each triangle represents the base area of an imaginary prism. So, there is an imaginary prism here and the area of this triangle represents the area of that prism. The average grade of each prism is then calculated we will see it.

In other words if t_1 , t_2 are the thickness of the mineral prism at 3 vertices and G_1 , G_2 , G_3 are the grades at the corresponding locations then the average grade can be calculated using G

$$G = \frac{g_1 t_1 + g_2 t_2 + g_3 t_3}{t_1 + t_2 + t_3}$$

So, what this grade here it is G_1 this is G_2 and this is G_3 and the thickness of the ore body is t_1 here t_2 here and t_3 here. and also the average thickness of this you know sum can be calculated as $t = \frac{t_1 + t_2 + t_3}{3}$

So, based on this we have an example here

Method of Triangle

Diagram showing a triangle with vertices P_1 , P_2 , and P_3 . The coordinates are: $P_1(0,0)$, $P_2(40,35)$, and $P_3(60,0)$. The thickness values are $t_1=12$ at P_1 , $t_2=12.5$ at P_2 , and $t_3=10.5$ at P_3 . The grade values are $g_1=0.6$ at P_1 , $g_2=0.8$ at P_2 , and $g_3=0.65$ at P_3 . Handwritten calculations show the average thickness $t_{av} = 11.67$ m and the average grade $g_{av} = 0.684$ %.

Handwritten calculations:

$$t_{av} = \frac{t_1 + t_2 + t_3}{3} = \frac{12 + 12.5 + 10.5}{3} = 11.67 \text{ m}$$

$$g_{av} = \frac{g_1 t_1 + g_2 t_2 + g_3 t_3}{t_1 + t_2 + t_3} = \frac{0.6 \times 12 + 0.8 \times 12.5 + 0.65 \times 10.5}{12 + 12.5 + 10.5} = 0.684 \%$$

4 percent or something

Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur

14:25 / 29:15

So, we have three data points, you know, let us say P_1 , P_2 , and P_3 . What is given here is the x and y coordinate of the point. So, this is x_1 and this is y_1 . Similarly, this is x_2 and this is y_2 . The 60 is x_3 , and this is 0 is y_3 . Here, this 0.06 is the grade in, let us say, percentage, and 12 is the thickness. So, our job is to calculate what will be, you know, the amount of material or mineral present in this volume. So, for that, ah, the first thing we need to do is to calculate the average thickness.

Average Thickness (t_{av}):

$$t_{av} = (t_1 + t_2 + t_3) / 3$$

$$= (12 + 12.5 + 10.5) / 3$$

$$= 11.67 \text{ m}$$

Average Grade (g_{av}):

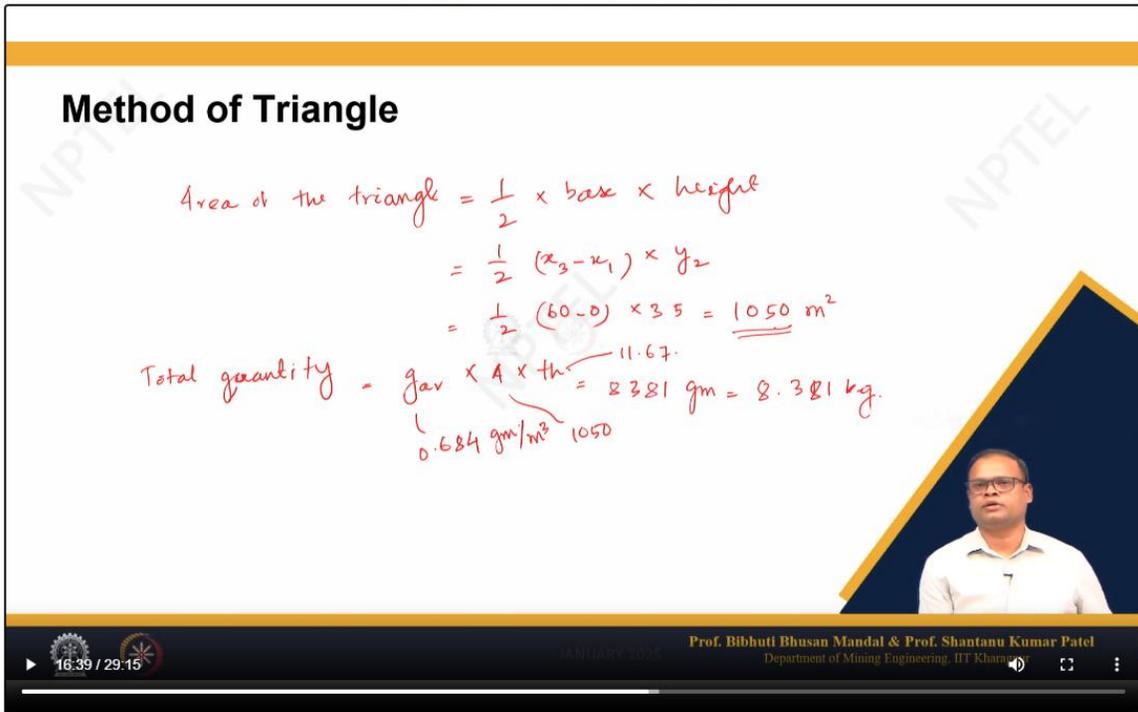
$$g_{av} = (g_1 \cdot t_1 + g_2 \cdot t_2 + g_3 \cdot t_3) / (t_1 + t_2 + t_3)$$

$$= (0.6 \times 12 + 0.8 \times 12.5 + 0.65 \times 10.5) / (12 + 12.5 + 10.5)$$

$$= 0.684 \%$$

Method of Triangle

$$\begin{aligned} \text{Area of the triangle} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} (x_3 - x_1) \times y_2 \\ &= \frac{1}{2} (60 - 0) \times 35 = \underline{\underline{1050 \text{ m}^2}} \end{aligned}$$

$$\begin{aligned} \text{Total quantity} &= g_{av} \times A \times th \\ &= 0.684 \text{ gm/m}^2 \times 1050 \times 11.67 \\ &= 8381 \text{ gm} = 8.381 \text{ kg.} \end{aligned}$$


Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur

So, the next thing is to calculate the area. So, the area of the triangle can be calculated using the equation half into base into height.

Area of the Triangle:

$$\text{Area} = (1/2) \times \text{base} \times \text{height}$$

$$= (1/2) \times (x_3 - x_1) \times y_2$$

$$= (1/2) \times (60 - 0) \times 35$$

$$= 1050 \text{ m}^2$$

Total Quantity:

$$\text{Total Quantity} = g_{av} \times \text{Area} \times th$$

$$= 0.684 \text{ gm/m}^2 \times 1050 \times 11.67$$

$$= 8381 \text{ gm} = 8.381 \text{ kg}$$

So, next is our geometrical method. The geometrical method or the cross section method for estimating mineral inventory is also based on the principle of gradual change. Here each internal block is divided by 2 sections.

Lecture 08 : Ore Reserve Estimation -1

The Geometric Method

- The geometric method or cross-section method for estimating mineral inventory is also based on the principle of gradual change.
- Here, each internal block is defined by two section.



So, ah you know you can you can see here maybe these are the bore holes. So, if you can join this end point of these two boreholes, you can you can generate a cross sectional area or a vertical cross-sectional area for a particular section here. Similarly, you can calculate you know you can generate the area A2 and generate area from the borehole along that plane. So, in this case what we do is the mineral area for each cross section is determined like let us say this is cross section number 1, cross section number 2 and cross section number 3, the area of each cross section is determined.

The Geometric Method

For calculation, the following steps are followed:

- The mineral area of every section is determined (as A_1, A_2, A_3 , etc.)
- Average values for each section is computed.
- Volume for each block is computed.

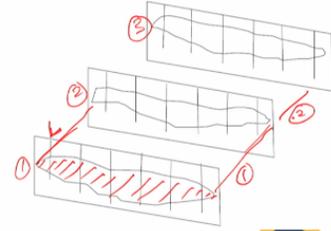
Volume for the block enclosed between section A_1 and A_2 , separated by distance L may be computed as follows:

$$\text{Vol} = \frac{(A_1 + A_2)}{2} \times L$$

$$\text{And tonnage } T = \left(\frac{(A_1 + A_2)}{2} \times L \right) \times F \quad \text{tons.}$$

Where, F is in t/m^3 (F = tonnage factor)

- Values of all the blocks are summed up, and average value is computed



So, the first one is calculated, the second one is calculated, and if there are multiple sections, we can calculate and add them up to get the total amount or total tonnage present.



Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur

The average value of each cross, the average value of the grade in each section, can also be computed, and the volume for each block is then computed. For example, the volume of the block enclosed between the first and second section. It can be calculated with cross-sectional areas A_1 and A_2 separated by the distance L . So, this distance may be computed as follows:

$$\text{Vol} = \frac{A_1 + A_2}{2} \times L$$

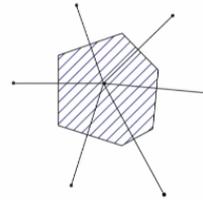
$$\text{And tonnage (T)} = \frac{A_1 + A_2}{2} \times L \times F$$

Where, F is in t/m^3 (F = tonnage factor)

So, in this case, the values of all blocks are summed up, and the average value is computed. So, the first one is calculated, the second one is calculated, and if there are multiple sections, we can calculate and add them up to get the total amount or total tonnage present.

Nearest Point Method

- The method following the rule of nearest points is also known as the area of influence method.
- The procedure is to determine the area of influence of each exploratory point and construction of polygonal blocks with the borehole in the centre of each polygon.



The method following the rule of nearest point is also known as the area of influence method.

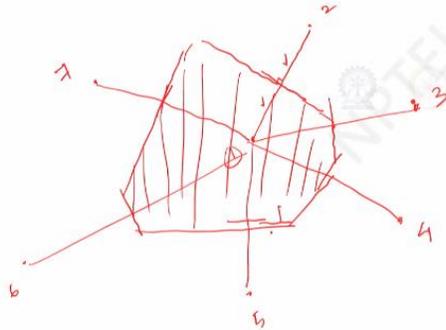
Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur

19:35 / 29:15

The next method is called the method of nearest point. The method following the rule of nearest point is also known as the area of influence method. The procedure is to determine the area of influence of each exploratory borehole point and construct polygonal blocks with the borehole in the center of each polygon.

Nearest Point Method

- The polygons are constructed by drawing perpendicular bisectors to lines connecting all sample points. Each sample point is contained by a polygon.



So, for this, let us say the previous example of the exploration where we have first point here, second point here, third, fourth, fifth, this is sixth, and seventh borehole. So, let us say this point is number 1, number 2, 3, 4, 5, 6, and 7.

So, what this says is the polygon ah, the polygons are constructed by drawing perpendicular bisectors to lines connecting all sample points. Each sample point is ah contained by a polygon. So, you just join all these points. Number 1 and number 2, number 3, number 4, number 5, 6, and 7. So, and then we get, you know, the perpendicular bisector. So, it's just this distance equal to this distance—this is perpendicular.

So, you make it like this. Similarly, let us say this is the middle point, the third middle. Fourth one, fifth one, sixth one so it goes like this, and this is the way we can construct the polygon by joining all those perpendicular bisectors. So, to estimate the mineral inventory, the area of each polygon is measured, and then the area is multiplied by the thickness at the ah at that ah point within the polygon. The grade is assumed to be constant and equal to the mineral ah intercept ah at that ah same point. So, if ah like you know, after getting this perpendicular bisector and getting this area, what we do is ah you know we take the data from for this point. Let us say the grade here is g , and the thickness is $ah\ t$, and the area of this polygon calculated is a . So, this point here, the central point here, is representing

the total area here, and you can calculate the quantity by multiplying your g into a into thickness.

So, this is our volume, and if you multiply with our weight, we can calculate what is the total quantity of mineral present ah.

Lecture 08 : Ore Reserve Estimation -1

The screenshot shows a video lecture slide with a white background and a blue and yellow geometric design on the right side. The slide title is "Block Model". The content includes a bulleted list of points. At the bottom of the slide, there is a small video inset of a man speaking. Below the slide, there is a video player interface with a progress bar, a timestamp of 22:45 / 29:15, and the names of the professors: Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel, along with their affiliation: Department of Mining Engineering, IIT Kharagpur.

Block Model

- The deposit is divided into a three-dimensional array of block
- Grade is assigned by
 - Nearest sample
 - A combination of several samples within a radius of influence
- An inverse distance method is used

This is the last you know method that we are going to ah learn today, called the block method or block model ah.

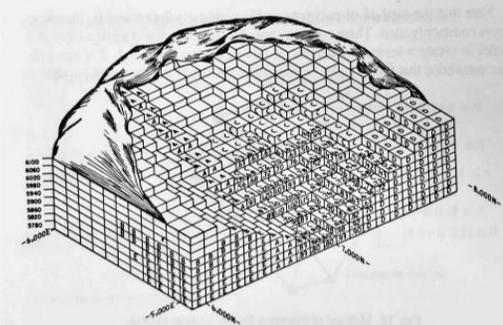
Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur

22:45 / 29:15

This is the last you know method that we are going to ah learn today, called the block method or block model ah. In this case, the deposit is divided into 3-dimensional areas of blocks. ah. And then the grade is assigned by the nearest either nearest point or maybe ah a combination of several ah samples within a radius of influence ah.

And inverse is here in the inverse distance method is used. We will see this thing with an example.

Block Model



BHID	Depth_From	Depth_To	Fe%	SiO ₂ %	Al ₂ O ₃ %
BHNA46	0	11	17.98	38.18	11.47
BHNA46	11	12	49.77	20.56	1.06
BHNA46	12	37	67.31	2.03	1.6
BHNA46	37	78	70.21	2.00	1.45
BHNA46	78	133	66.47	4.14	2.11
BHNA46	133	157	70.33	3.66	1.98
BHNA46	157	158	5.83	80.13	12.44

3-D block model of an ore body (from Crawford and Davey, 1979)

So, like, let us let us—this is a diagram taken from the reference here, and here, you know, the entire thing, let us say, is our ore body. And in this case, what we do, you know, the entire ore body is defined, divided into blocks.

Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur

23:17 / 29:15

So, like, let us let us this is a diagram taken from the reference here, and here, you know, the entire thing, let us say, is our ore body. And in this case, what we do, you know, the entire ore body is defined, divided into blocks. So, each of them looks like, let us say, this. With some width, height, and kind of thickness. So, in this case, let us say once we divide this thing here, let us take one example here the block here. And what we do, let us say our borehole will not be present in each of these blocks, but it will be randomly placed, let us say around these, you know, the assumed block here.

And this borehole data looks like this, you know. This is one of the boreholes, which is borehole number BH and A 46. And this is what is showing like at from depth 0 to 11, the Fe percentage is 17.98. And the SiO₂ percentage is 38.18, Al₂O₃ is 11.47. So, in this case, you know, at different depths, we have different percentages of Fe, which is gradually changing. So, in this case, you know, if you want to assign some grade to this block, what we can do is get some information from the nearest borehole data.

Block Model

The diagram shows a circular block with a center marked with a red 'X'. Nine boreholes are located at various distances from the center:

- G₁ (0.40%) at distance d₁ (200 m)
- G₂ (0.50%) at distance d₂ (200 m)
- G₃ (0.60%) at distance d₃ (150 m)
- G₄ (0.90%) at distance d₄ (100 m)
- G₅ (1.00%) at distance d₅ (150 m)
- G₆ (0.50%) at distance d₆ (200 m)
- G₇ (1.00%) at distance d₇ (250 m)
- G₈ (0.80%) at distance d₈ (100 m)
- G₉ (0.70%) at distance d₉ (150 m)

A radius of influence of 275 m is indicated by a red line from the center to the edge of the block. Handwritten calculations on the right show the formula for the representative grade g :

$$g = \frac{\sum (g_i / d_i^2)}{\sum \frac{1}{d_i^2}}$$

$$= \frac{\frac{0.5}{200^2} + \frac{0.5}{200^2} + \frac{0.7}{150^2} + \frac{1.0}{250^2} + \frac{0.5}{100^2}}{\frac{1}{200^2} + \frac{1}{200^2} + \frac{1}{150^2} + \frac{1}{250^2} + \frac{1}{100^2}}$$

$$= 0.77\%$$

A small diagram below the formula shows a shaded area representing the volume of the block, with the text "volume x grade" written next to it.

So, similarly, you know, you can add up all those blocks present within that volume to calculate what is the total quantity of reserve present in that deposit.

Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel
Department of Mining Engineering, IIT Kharagpur

So, we will see with an example here. So, let us say, you know, this is the block for which we have to calculate the grade, and we do not have a borehole at the middle of the block. So, or inside the block. So, we have to take the information from, you know, the surrounding borehole points. So, in this case, this is the top view of, you know, the block, and we have different boreholes here starting from G1 all the way up to G9.

So, here are a couple of things. So, let us say this is the distance between the center of this block and G2. It is 200 meters, and this is the grade that we get at borehole number G2 for that section. So, in this case, to calculate this grade for that block,

$$G = \frac{\sum G_i}{\sum \frac{1}{d_i^2}}$$

So, here, what we do to get the representative grade at this location is So, we use something called a radius of influence. So, this can be, let us say, 275 meters or something like that, based on what kind of mineral deposit it is. So, within that radius of influence, whatever boreholes are present, we will take the information from them. Other boreholes, like G1, G3, G5, and G8, we are going to neglect here. So, for this, you can calculate this as the first point and the grade.

So, for G2, divided by the distance between them, is 200 squared plus for G6, again 0.5 divided For G9, it is 0.7 divided by 150 squared plus for G7, we have 1 divided by or 1.0 divided squared plus for G4, it is 0.9 divided by 100 squared, all divided by 1 by 200 squared plus 1 by 200 squared plus 1 by 150 squared plus 1 by 250 squared. squared plus 1 by 100 squared.

So, this comes out to be 0.77 percent. So, once you know the grade here, you can calculate the block volume, and you can calculate. So, you can calculate the volume of this. And multiply with your grid. And maybe, if it is, you know, you may need the density to calculate what is the total quantity of mineral present here.

So, similarly, you know, you can add up all those blocks present within that volume to calculate what is the total quantity of reserve present in that deposit. So, this ends our class today, like this lecture number 8.