MINERAL ECONOMICS AND BUSINESS

Prof. Bibhuti Bhusan Mandal

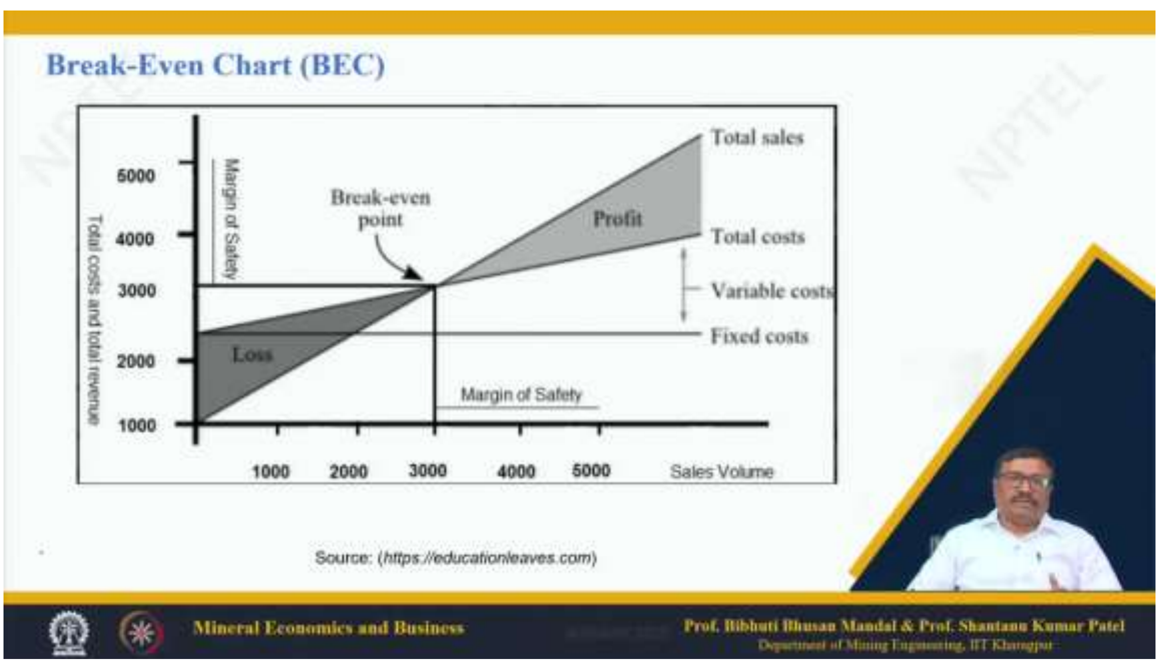
Department of Mining Engineering

IIT Kharagpur

Week 10

Lecture 46 : Problems and Solutions - IV

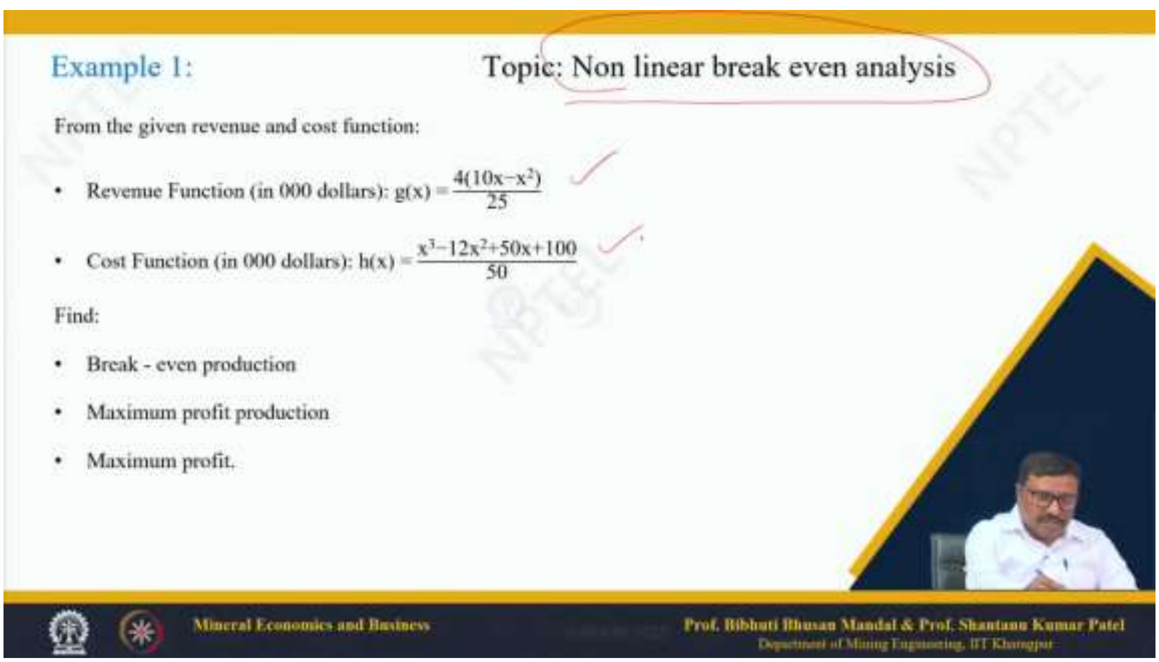
Hello, welcome everybody. Today, we will have one more set of problems and solutions. This is part 4 in the series. We had three previous sets of problems and solutions. We are having a fourth set where we will be dealing with different types of problems, like the non-linear break-even analysis, for example. Next, we will do a set of problems merged together to demonstrate the depreciation of assets and how they behave when you plot them one against the other. As you can see here, just to remind you or let us recall the break-even chart, which is linear. In this particular slide, we are showing the linear one, just to set the mood, and then what complications can arise from the total sales and total revenue if they are not linear.



So, from the linear chart, what you can see here is that this horizontal line is showing you the fixed cost, and then there is a variable cost which, when added to the fixed cost, gives you the total cost. Because this already has these as the fixed cost. So, the whole thing is drawn like this, and this shows you the total cost. So, this part, the difference, is the variable cost wherever you plot. So, this lower part is the fixed cost, and the upper part is the variable cost.

If you add them, then we get the total cost, and this, starting from zero, is the total sales revenue, OK? So, we have plotted the total cost versus total sales and found where these two lines intersect, and we call it BEP or the break-even point. Now, the question of profit and loss is very simple; we can demonstrate it with reference to the break-even point. So, on this side, wherever you are above the total cost, you can see how much profit you are making. At the same time, if you go to the left side, you see this is the difference between the total cost and the total revenue, which is going like this.

So, the revenue is less than the cost. So, you are in a loss that means, this shows simplistically where you are in the profit zone or loss and how much is the margin of safety which we have discussed. Now, in practice the except in very small business, this linear ah charts break even analysis, linear analysis does not hold true. This is much more complicated. because even the fixed cost will not remain fixed cost when the volume of production increases very high.



At the same time the revenue and the the first thing the variable cost that will also change. So, some per unit rupees per unit variable cost will not hold true. When you go for higher production you will see that with the same variable cost you can produce little more, but when you go further high then the variable cost will not remain same then you have to add. So, this linearity will not remain.

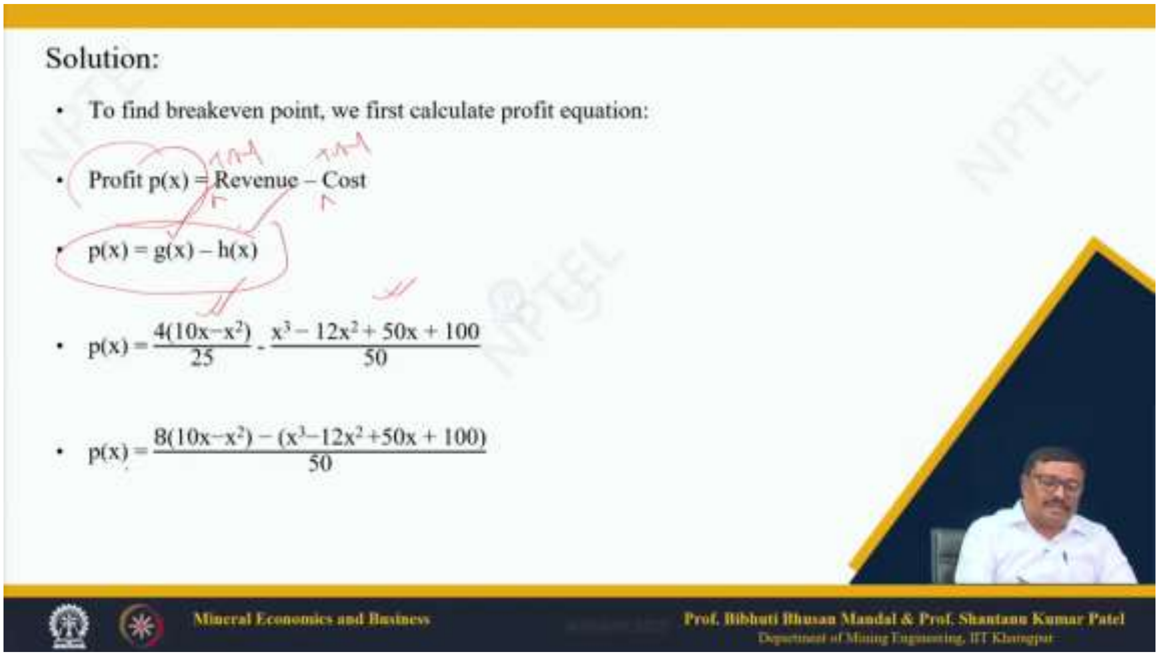
So, what we now ah try to solve is non-linear equations which we have seen earlier in the theory part and we are again demonstrating with one more example here in detail that how we can tackle this situation where the ah neither the revenue nor the cost is linear. In this ah example of non-linear break-even analysis, you can see that the relations both the cases are not ah ah linear at all. So, for example, the revenue function here is given as g(x) which is a function of x that means, the number of units produced is:

- $g(x) = \frac{4(10x-x^2)}{25}$

Whereas the cost function denoted by the h(x) in 1000 dollars again is:

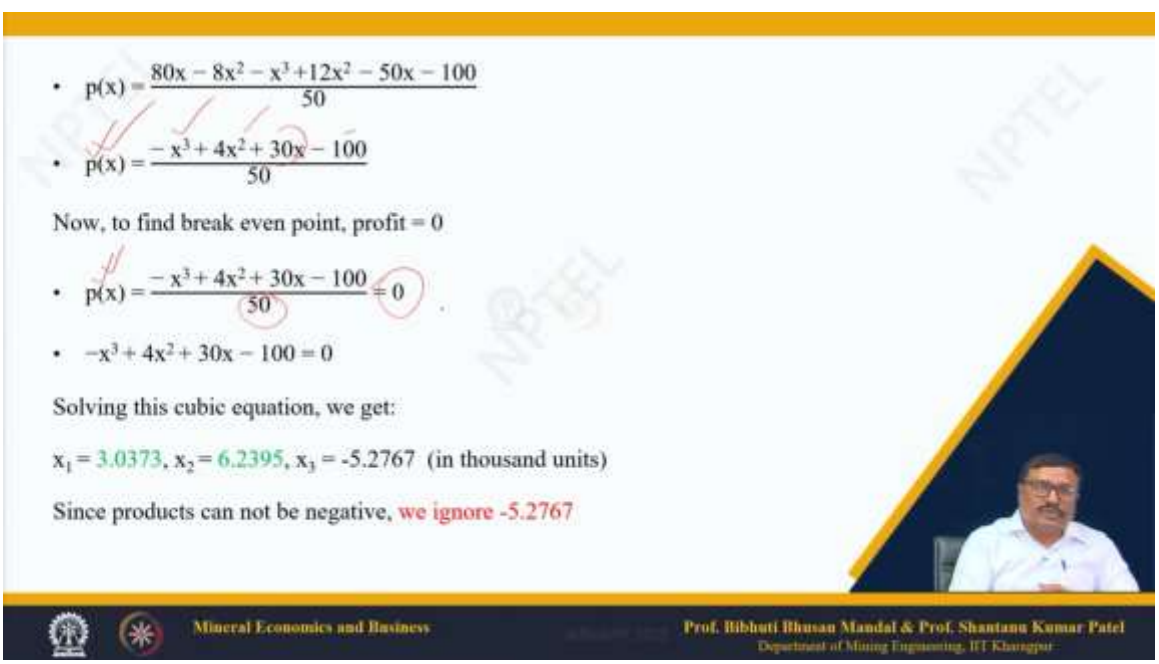
- $h(x) = \frac{x^3-12x^2+50x+100}{50}$

In both cases, these two are not linear at all. What do we have to find out? We have to find out the breakeven. Production number 1: where do we break even? That means the breakeven point. Rather than on the profit side, if you go further, you have to find out the maximum profit production. That means at which number of units will give you the maximum profit and the quantity of maximum profit—the amount of profit, the maximum profit that you can earn when these two equations hold true. Now, to find the breakeven point, we have to calculate the profit.



First, we frame the profit equation. What is the profit equation? Profit, denoted by P, is again dependent on the units of production. So, this is the total revenue. This is total revenue, and this is the total cost.

Total cost. So, this total cost (Tc) minus total revenue (Tr) gives you the profit. So, if you write it in this form, P(x) = g(x) - h(x), where g(x) is the revenue and h(x) is the total cost. As you have seen from the previous slide, g(x) equals this, and h(x), the cost function, is this one. So now, we simplify this and get that P(x) is this.



Now it can be rewritten like this:

- $p(x) = \frac{4(10x-x^2)}{25} - \frac{x^3 - 12x^2 + 50x + 100}{50}$
- $p(x) = \frac{8(10x-x^2) - (x^3-12x^2 +50x + 100)}{50}$

So, now we are further simplifying and finally, we get that the profit function can be expressed as:

- $p(x) = \frac{- x^3 + 4x^2 + 30x - 100}{50}$

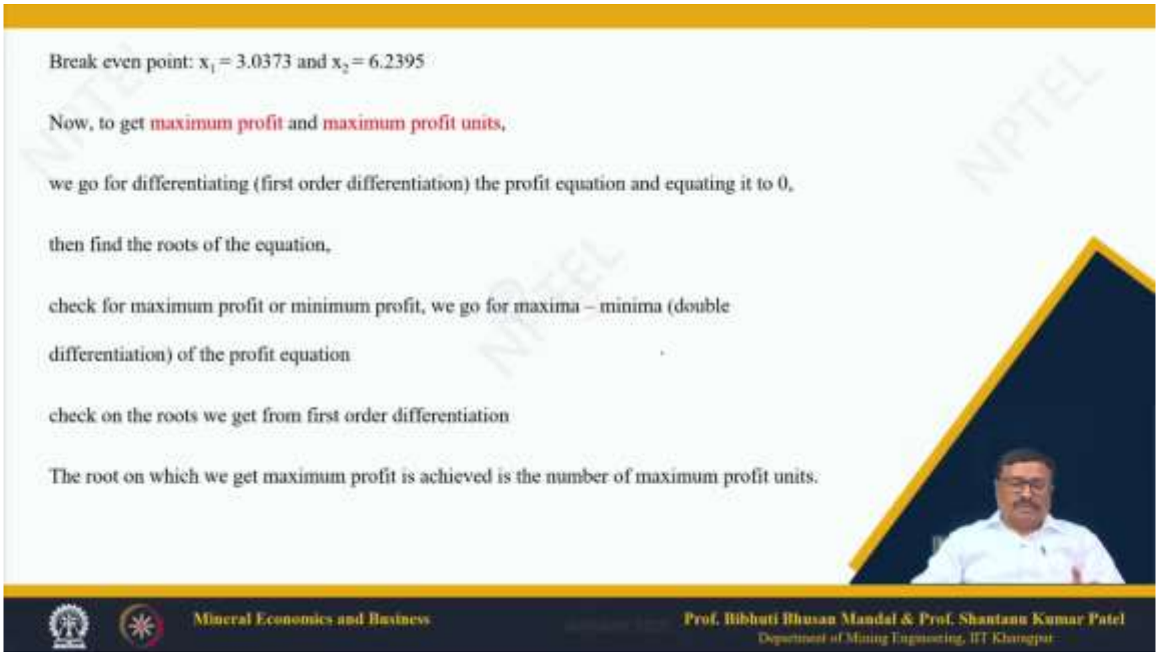
Now, for the purpose of finding the breakeven point as we have done in linear also that the profit will be 0. That means, the total cost will be equal to the total revenue. So, here what we can do that we can make this profit function equals to 0.

- $-x^3 + 4x^2 + 30x - 100 = 0$

This is our profit equated to 0. Solving this cubic equation we get 3 values, one is x1 is 3.0373, this x2 is x 6.2395 and again x3 equals to -5.2767.

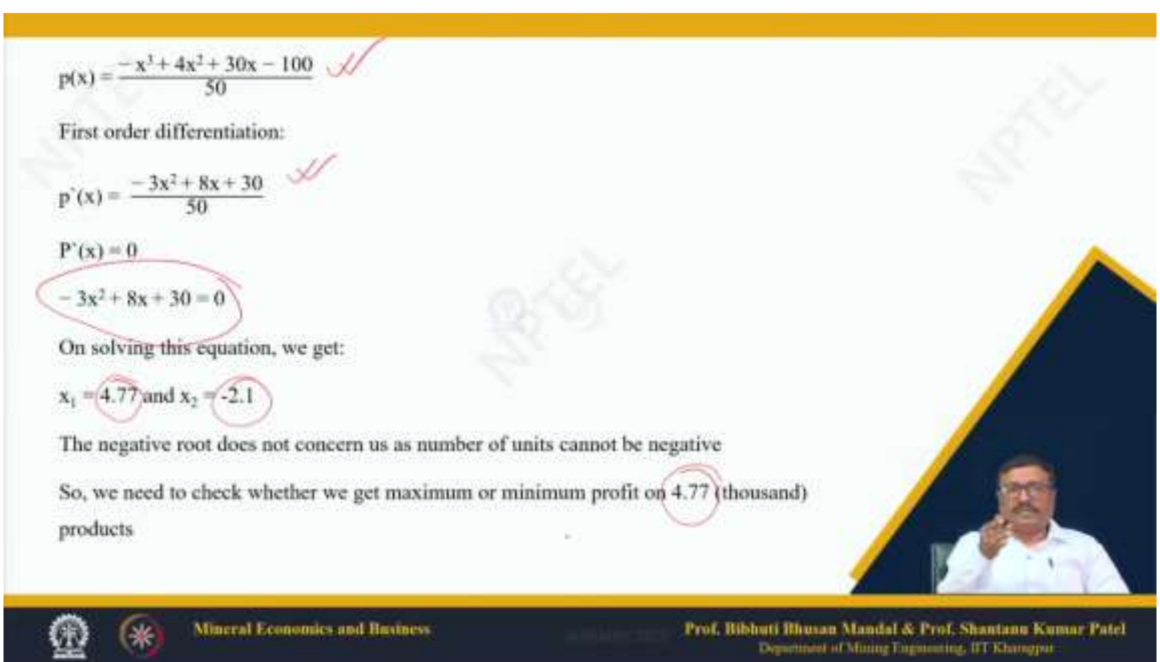
Since all these are in 1000 units, so I have kept it 4 decimal points to take a precaution. So, since products cannot be negative, so we ignore this value, this value we ignore and we consider these 2 values x equal to 3.0373 and x 2 is 6.2395, there are 2 break even points. at a lower that means, if you multiply this with 1000 then say 3037 or 3038 is one this number of units and x 2 will be say 6239 or 6240. This both the at both the cases in both the cases we will reach the we touch the break even points. So, the curve is not linear, it is cutting the cost curve at 2 different points that we will see also in the end of this.

Now, the break-even points are 26.0373 and say 6.2395 or 23, even 240, that is also possible. So, to get the maximum profit and maximum profit units, what we do is we go for differentiating the first-order differentiation of the profit equation, and then we equate it to 0 to find out that. Then what we do is we find the roots of this equation. This is what we have done. Now, to check for the maximum profit or minimum profit, we go for maxima or minima using the double differentiation to see the slopes of the profit equation. So, we have the profit equation, then we go for further differentiation. So, checking the roots we get from the first-order differentiation, the root on which we get the maximum profit is that which will show the number of maximum profit units.

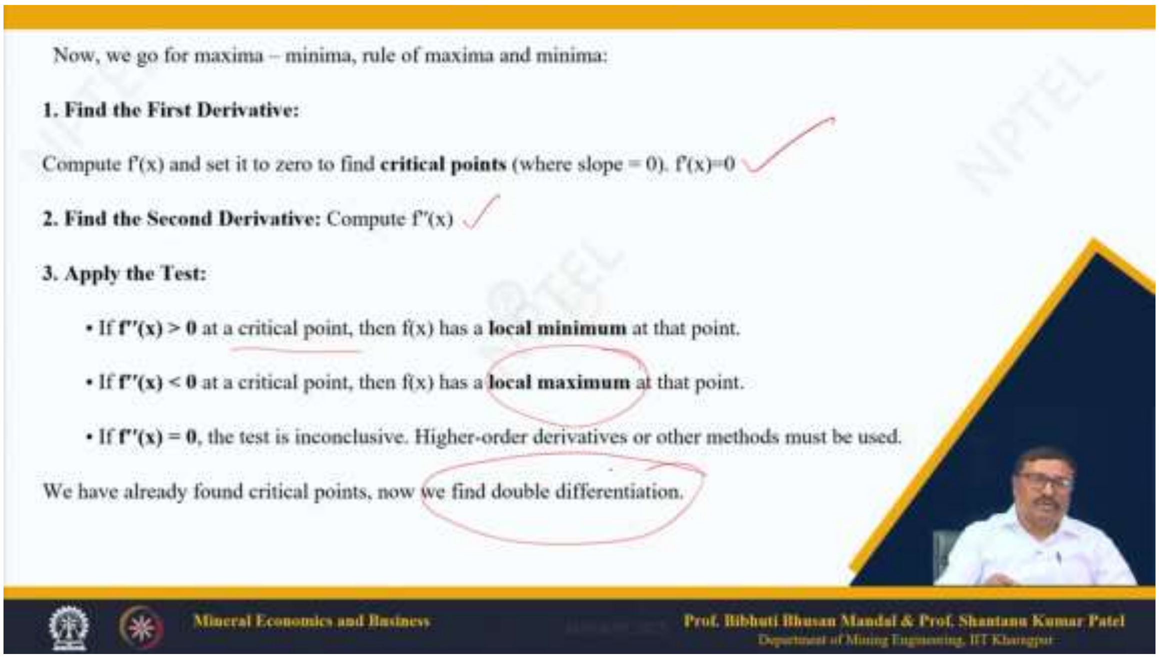


So, we have to differentiate once again. Let us see one by one. So, we had the P(x) profit equation, this we have seen, and then we have done the first-order derivative once here. So, by doing that and equating it to 0, we get $3x^2 + 8x + 30$, and then we equate it to 0. On solving this equation, we get X_1 as 4.77 and X_2 as -2.1. The negative root does not concern us as the number of units cannot be negative.

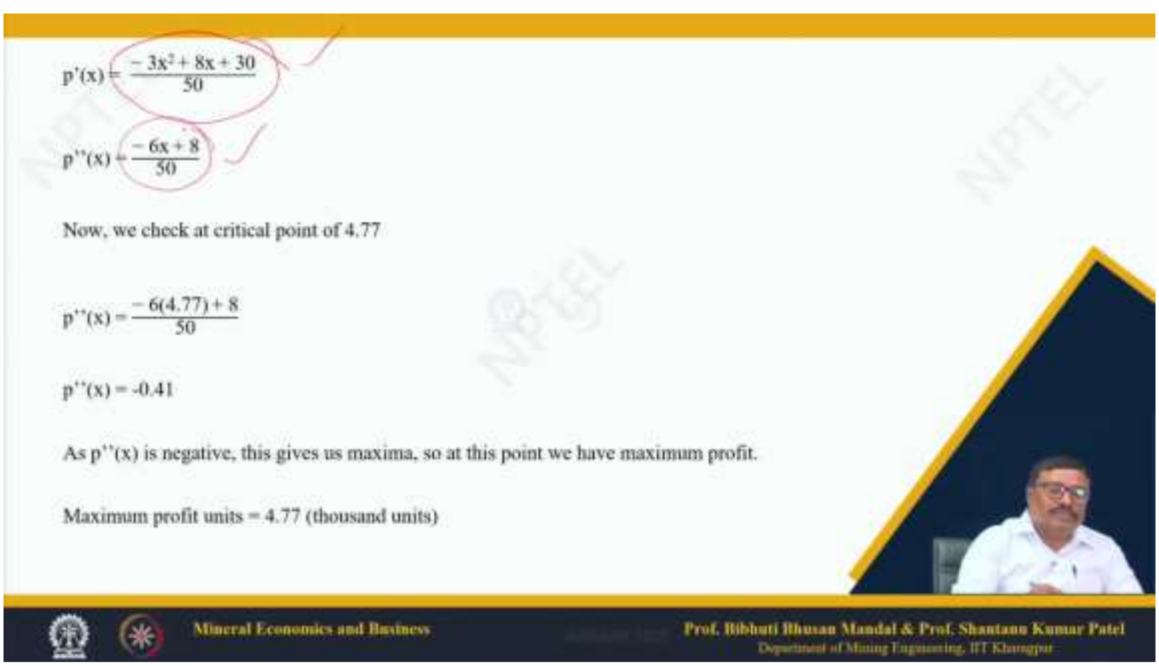
So, we need to check whether we get maximum or minimum profit at 4.77000. Here, this is a question of maximum. We will just check whether we are having the maximum profit or minimum profit because this is where the slope is 0, it is taking a turn, taking a turn. So, find the first derivative, step 1. So, compute f'(x) and set it to 0 to find the critical points where the slope is 0, f'(x) is 0, OK.



Now then we go for the second derivative like the compute f″(x). Then we apply the test like if x is greater than 0 at a critical point then f(x) has a local minimum at that point and f″(x) is less than 0 at a critical point then f′(x) has a local maximum at that point. if it is equal thing equal to 0, then the test is inconclusive and then higher order derivative will be required. So, we have already found critical points, now we want to do the double differentiation to go one step ahead. So, we have this p′(x)already found Then what we do?

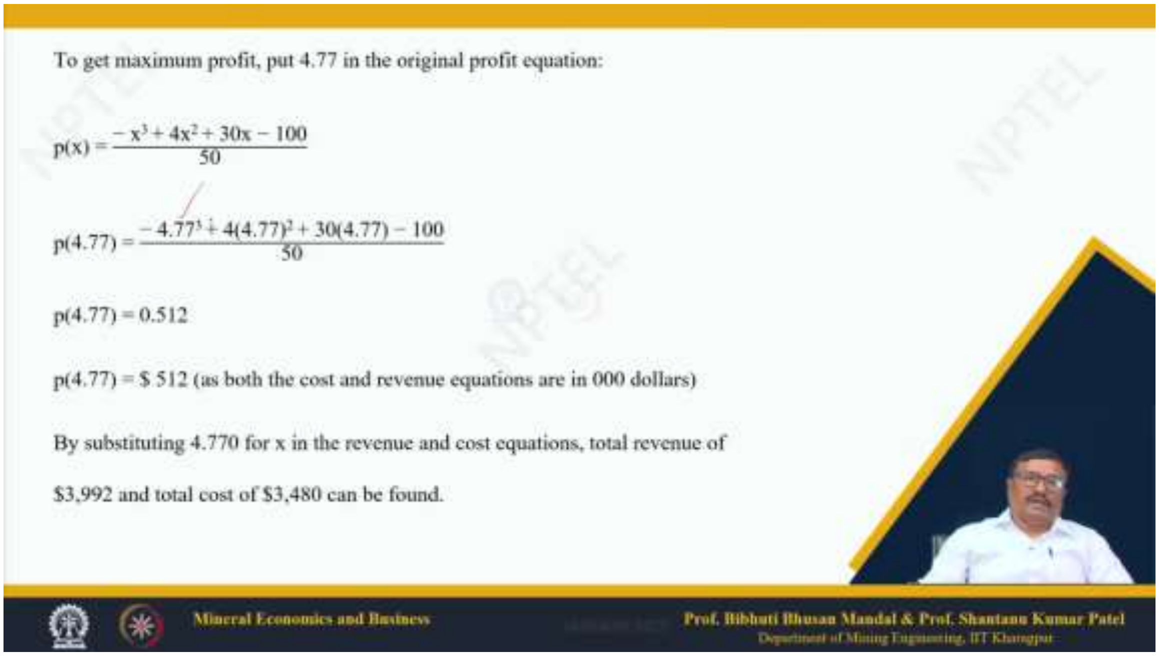


We go for p″(x). That means once again differentiate it and we get -6x + 8/50. So, now we check at critical point 4.77 that means, in 1000. So, this 4.77 we replace in place of x substitute and then we get p″(x)is minus 0.41. As p″(x) is negative this gives up the maxima.



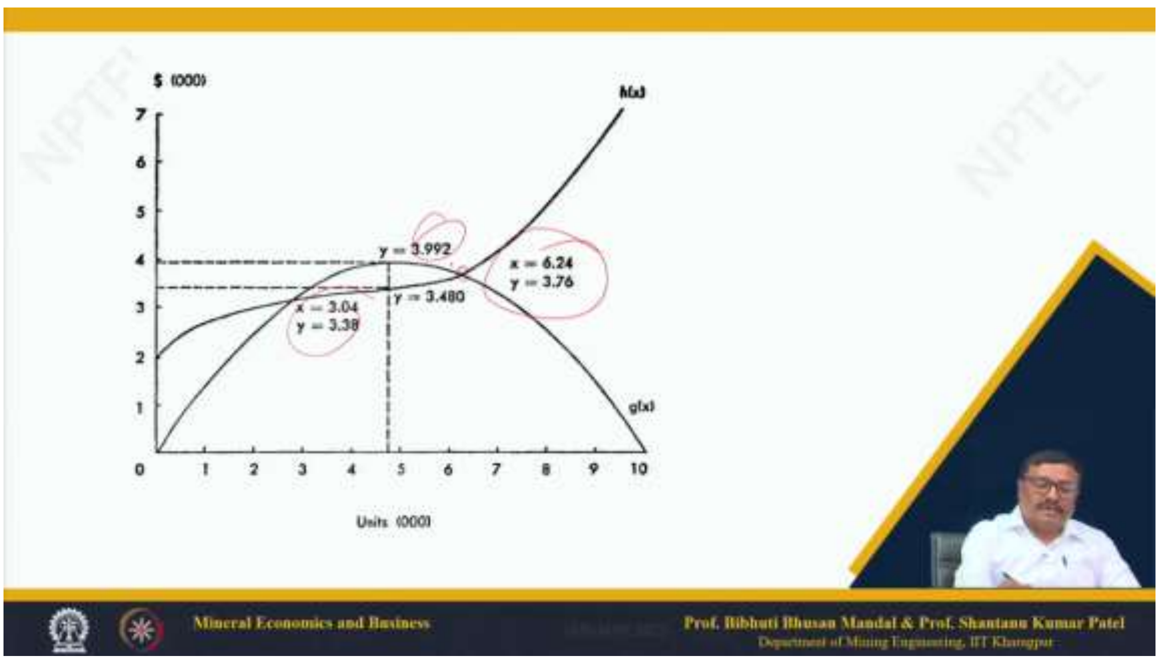
So, at this point we have maximum profit, we do not have to check others. So, by using substituting 4.77, we get the and we get the point where we get the maximum profit. So, the maximum profit unit is 4.77 into 1000. So, it will be 4770 units. Now to get the maximum profit that means, first we we already got where we have the ah maximum profit, how many units will give us the maximum profit.

So, from there now we want to quantify what the profit will be by substituting the value of x in the profit equation. In the profit equation again, we put the value of 4.77 in place of x, then we solve and simplify, and we get 0.512. So, if it is in 1000 units, the amount of profit will be 512 because this is 0.512 multiplied by 1000. So, this will come to 512 dollars because both the cost and revenue equations are in 1000 dollars. Now, by substituting 4.770 thousand for x in the revenue and cost equation, this is where we have used this 4.77 thousand (4770 units) in the profit equation.



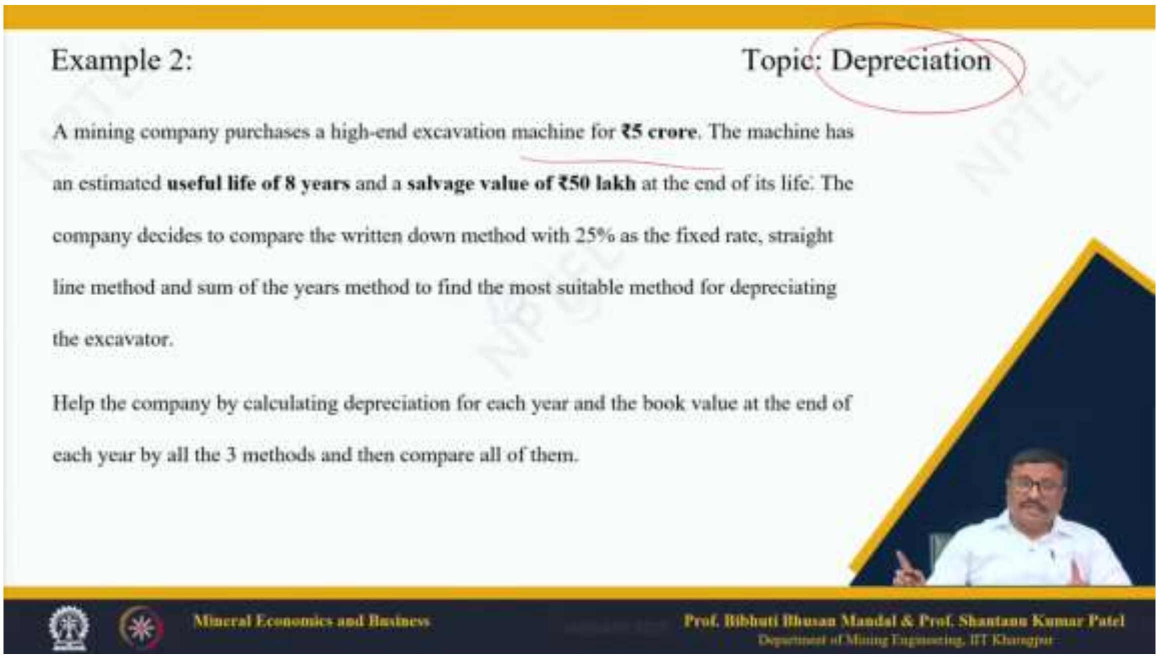
So, we got the maximum profit amount from there. So, if you want to find the total revenue and the total cost involved for 4770 units, you can just substitute them in the equation and find that the total revenue is 3992 dollars, whereas the total cost was 3480 dollars. So, all components have been found: how many units give us the maximum profit, what that amount of profit is, what the total revenue earned is, and what the total cost incurred is at that particular point—all have been found here. And I have shown step by step how to go about solving these problems. Do not go by these numbers here because this is only for illustration. To remind you once again, as we have seen during the theoretical discussion, here the cost and revenue functions intersect at two different points.

That means this is the lower part; naturally, we will consider this as the break-even point, which is the lower one. We should not unnecessarily think of this. But as you can see, when we are transitioning from one break-even point to another, the curve is swelling at a particular point, and the slope is changing and shifting toward the lower (negative) side. So, at that point, at that point, we know that we go for a double differentiation and see that the slope is negative, and then we understand that is the point where we are getting the maximum profit. We consider that particular point as the number of units to produce maximum profit, and from there, we can also calculate the maximum profit.



and related to at at particular point this is the cost and this is the revenue. So, the cost and this is the revenue. So, both all the all the things can be found by solving this problem instead of graphically plotting it. You can use algebraic method to solve and use using the basic calculus you can find out the ah find out the number of points at what points we are intersecting and ah finding the breakeven points then the profit point where we get the maximum profit.

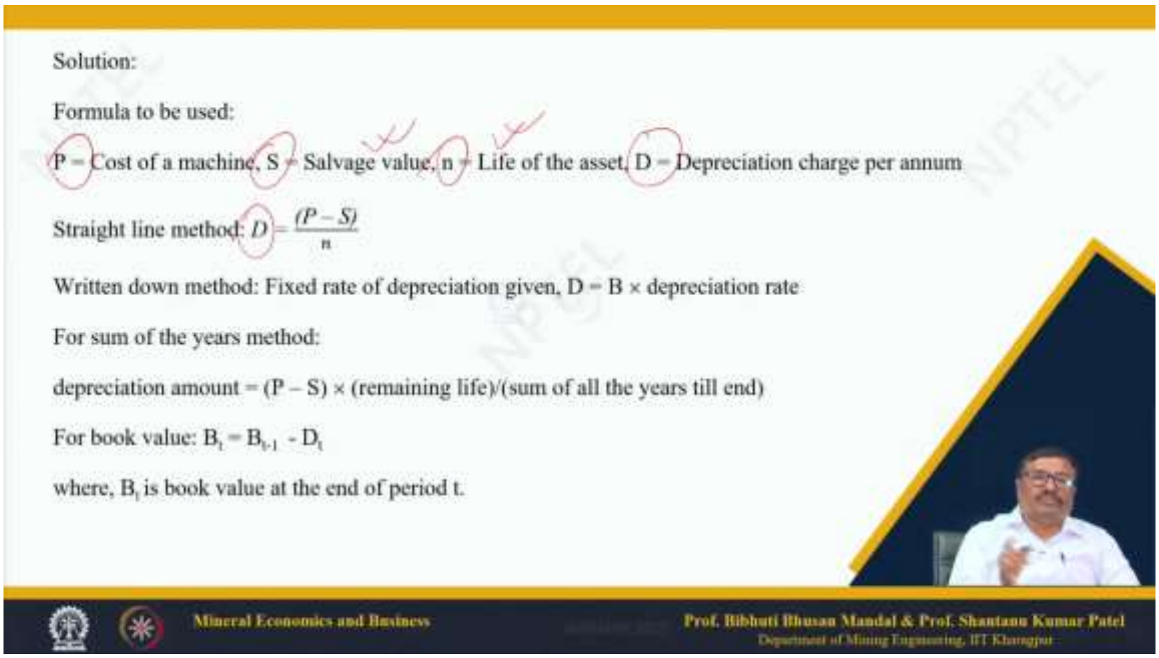
Now, we will be switching over to another set of problems related to depreciation, depreciation which was a big chapter I think you remember. Here what we are saying that the we have formulated a problem a mining company purchases a high end excavation machine for example, for 5 crores. The machine has an estimated useful life of 8 years. the salvage value of 50 lakh at the end of its life. The company decides to compare the written down method with 25 percent as fixed rate.



Now, we are not calculating by 1 – (S/P)^n, no. It is decided that we will be declining the thing using a written down method which is fixed at 25 percent rate. Now and straight line method, sum of the years method and then compare which is the suitable method I mean adopting their principles. So, and we can ah find out the ah depreciation using the techniques that we learnt in the class theoretical class and then from the depreciation then deducting the depreciation from previous years book value we can make schedules of the book values throughout the life of the mind. for all these three methods and then we can compare them physically or at the same time we can graphically plot them to see how do they behave.

Now, for calculating the depreciation, let us remember: P being the cost of the machine, and S, the salvage value, is denoted by S. Then, for the life of the asset, we use n. If D is the depreciation charge per annum, charge per annum. Then, for the straight-line method, we have D = (P – S)/N, where P is the cost minus the salvage value, distributed over N number of years. So, in the straight-line method, the same depreciation occurs every year.

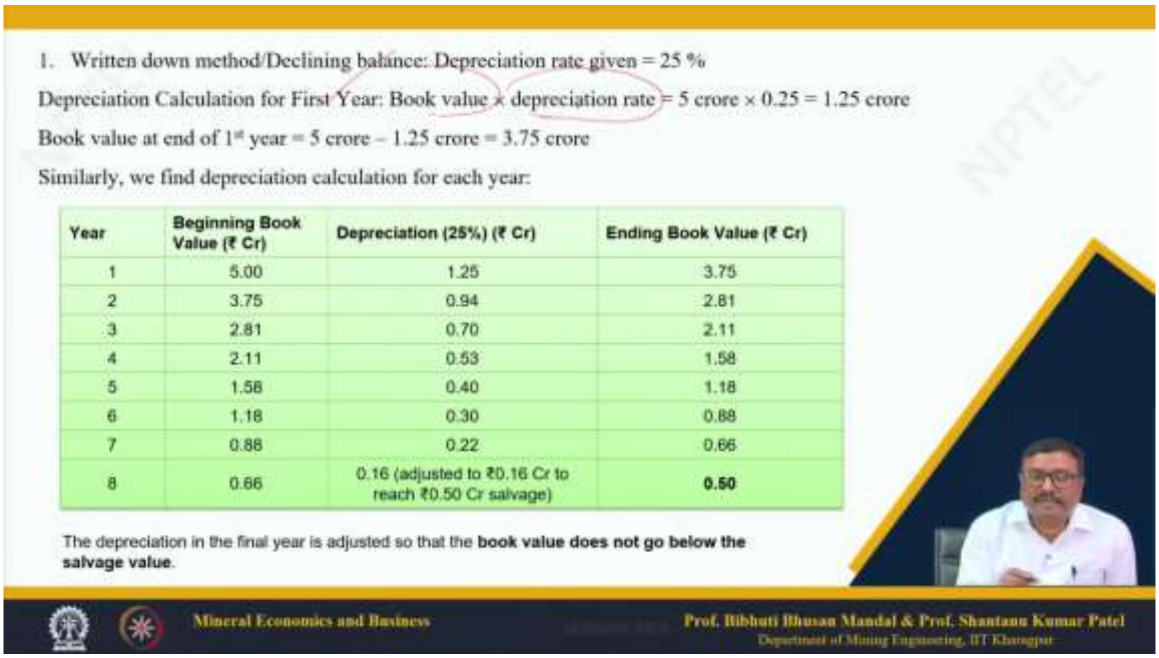
But in the written-down method, or what we call the declining balance method, there is a fixed rate of depreciation, not a fixed depreciation amount - it is the fixed rate of depreciation. So, the depreciation amount is equal to B, the book value, multiplied by the depreciation rate, or in our case, 0.25 or 25 percent.



For the sum-of-the-years method, the depreciation amount is (P – S) × (remaining life)/(sum of all the years till end). For every method, the book value is taken as the previous year's book value - the depreciation amount. So, that will be the book value. So, we have all the depreciation and the corresponding book values. If we find out all these things, then we can make the schedule of depreciation. Now, the written-down method or declining balance depreciation rate is given as 25 percent.

So, the depreciation calculation for the first year is the book value, meaning the previous value, multiplied by the depreciation rate, which is 25 percent. So, 5 crore multiplied by 0.25 or 25 percent is 1.25 crore in the first year itself. So, the book value at the end of the first year will be 5 crore (the initial price) minus the depreciation charged for the first year. So, at the end of the first year, the book value will be 3.75 crore. Similarly, we can go on finding the depreciation calculation for each year.

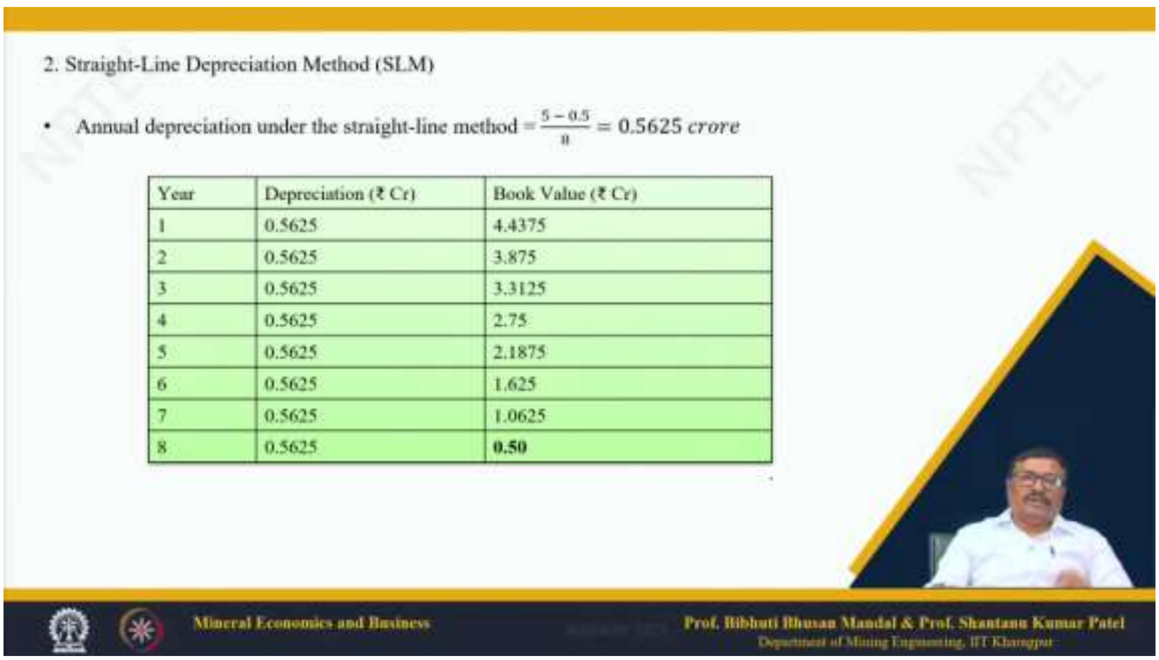
As I have shown, the 5 crore we started with, then the depreciation was charged as 1.25. as 1.25. So, the effective book value will be 3.75 crores, as we can see from here that 5 minus 1.25 is 3.75. Now, in the next year, what will happen? This 25 percent will be loaded on the book value, which means the 3.75 is the book value at the beginning, which will be multiplied by 0.25 to get the depreciation. So, you get the depreciation by multiplying 0.25 or 25 percent with the book value, the book value for that year.



So, now, if we deduct again 3.75 minus 0.94, we get the book value at the end of the second year. Go on repeating this for 7 years, 7 years. So, at last, you see 0.88, then you multiplied by 0.25. So, one-fourth of that, which means 0.22 crore, will be the depreciation. So, finally, it will become 0.66. So, in the last 8 years, what we have done

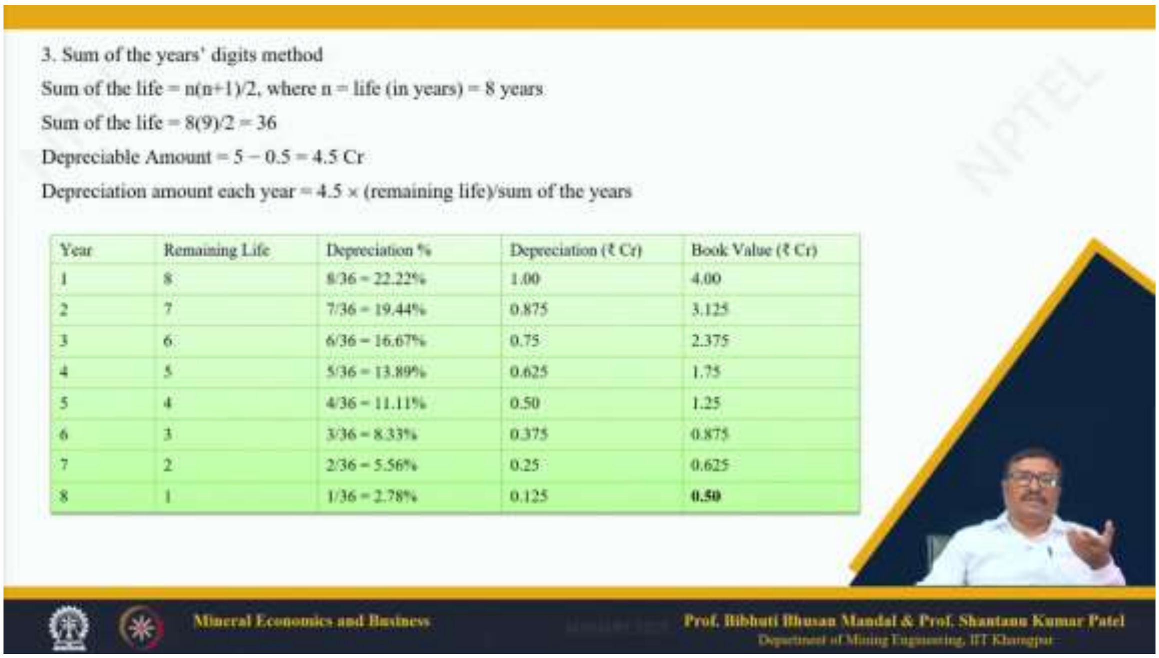
0.66 is the book value, and then we adjusted only 0.16 depreciation instead of depreciating 0.6 again multiplied by 25 percent. What we have done? We have deliberately made it 0.16 so that Finally, we get 0.50 to equate with the book value because we have not calculated with the existing given data, but we have calculated on the basis of a pre-decided rate, which is 25 percent. Naturally, in most cases like depreciation, if you calculate by using the declining balance or return on method, we will see that the salvage value does not match.

Primarily because that in the calculation itself you have never used the salvage value. You have got the price or the cost of the machine when you purchase and go on multiplying and then deducting whatever the depreciation you are finding out from the book value. So, you are repeating the same process and in the whole process you have never used the salvage value. So, at the end when the book value is coming which is different from the salvage value estimated then we deliberately equated to 0.50. So, this is where you can see the beginning of the book value, beginning book value for every year and then depreciation for every year and ending book value at the end of the financial year we get this thing.



book value of the equipment here. Now we are changing the method straight line depreciation method is very simple what we do that we deduct the 50 lakhs salvage value from the 5 crores and we get 0.5625 crores by dividing it by 8. So, now, this 50 lakhs is 0.5 crores. So, (5 - 0.5)/ 8 that will give you 5.5625 crores.

So, in the beginning we had the depreciation first year is 0.5625 and in the see the book value accordingly will be reduced 5 - 0.5625 you get the book value Similarly, you go on every year you have to depreciate same amount and the book value goes down and since it has taken care of the ah the salvage value. So, at the end of the 8th year it is absolutely equal just equal to the 0.50 depreciation, but here the depreciation is flat and it is a straight line method depreciation. Now we are going to another method, sum of the years digit method that that the multiplier the factor that we use for the purpose of calculating the amount of depreciation is done by where the numerator is n life remaining life divided by the sum of the number of years.

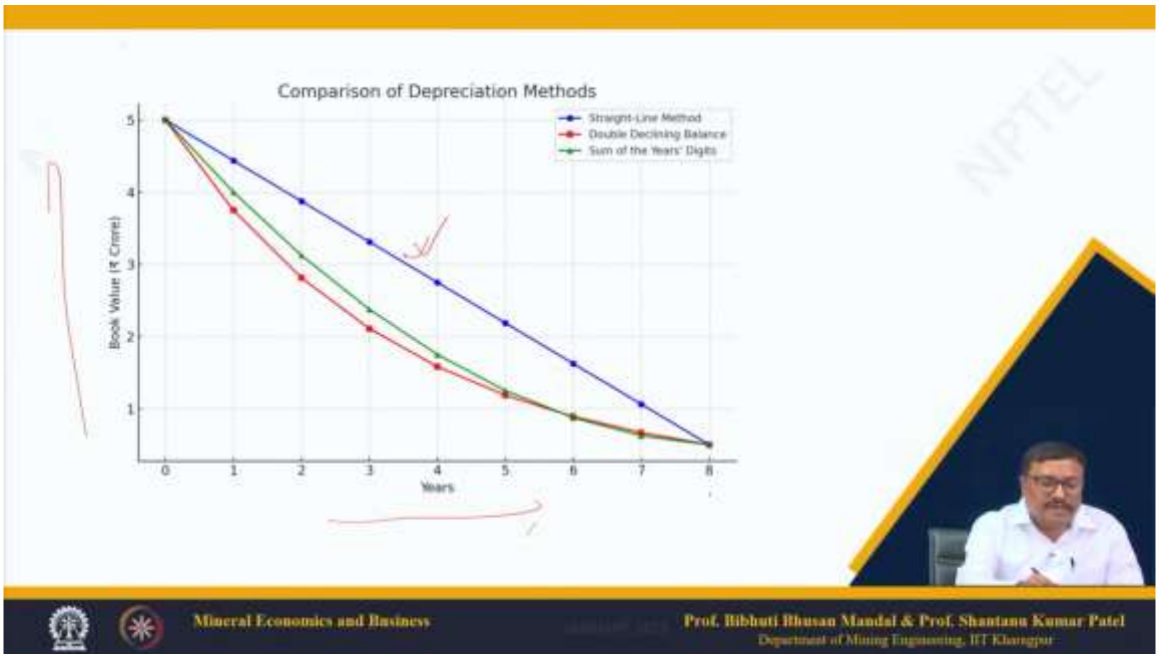


So, that is the factor that we use. For example, sum of the life is n × (n + 1)/2, where n equal to life in years 8 years, sum of the life is 8 × 9 just we have calculated n ×(n + 1)/2. So, sum of the years digit is 36. So, the depreciable amount is 5 minus 0.5 which is 4.5 crore.

So, what we do the depreciation amount each year will be the 4.5 crores for example, into remaining life divided by sum of the year. So, the cost of the machine for example, 5 minus 0.5. So, depreciable amount is 4.5 crores this has to we have to depreciate in the book of accounts. What we do that every year we use a factor which is the n is the remaining life and then the sum of the year digit method we just use this formula and go on getting the depreciation amount.

Now, in the first year say remaining life is 8 years full. So, it will be 8 by 36. So, it is the ratio the factor will be 22.22 percent. So, we get 4, 4.0. Similarly, next year we have 7 by 36 and we get 0.875 and it is 3.125 book value.

That means we deducted from 4; the book value of the previous year is 4 minus 0.875, and we get 3.82125. We repeat the process every year. You can see here in the fifth year we have 5 years remaining, and 5 divided by 36 gives us 13.89. Then we get 1.75 as the book value, and this is the depreciation amount. Continue doing this since you have taken the salvage value into account. So, it is matching here; it is matching here at 0.50. Now, we have a whole lot of depreciation amounts here and corresponding book values in the other column.



So, this rule is ready. Let us plot now. All the values that we got over the years up to 8, and the book value on this side, okay? First one is the straight-line method. As you can see, every year it is falling by the same amount, and that is why the book value, when plotted, is simply linear.

It is depreciating linearly. But if you see the double declining balance, in the first few years, you see it is sharply declining and then it becomes slowly flat here. The first part is very sharply declining. Some of the years are also behaving similarly, even though in the first few years the decline is not as sharp as you would expect from the double declining balance method.

So, what we can see are 3 different methods in the same plots, depending on the policy that the company adopts. And also depending on the item that you are trying to find out the depreciation for, The depreciation does not only depend on the policy of the company, it also depends on certain guidelines given by the government for those particular items. At the same time, what items are you actually depreciating? I mean, is it computers and stationery or chairs and tables? Or buildings, or are you depreciating equipment?

So, depending on that type, we also decide. So, for example, in equipment, in the beginning, we try to use the double declining balance method. Others where it is fully utilized throughout the life, we may prefer these things where the service is meaningless. Here. But here the service is very much meaningful, that in the beginning we fully try to utilize the services and we try to depreciate.

So, we get more output in the beginning and less maintenance. So, we go on depreciating very fast. So, we use the declining balance method for equipment in most cases. I hope I could make these concepts clear by considering all of them together and plotting them together also. For reference purposes, you can use this very book—very good books—one is this one: a practical guide to depreciation under the Companies Act. This is an Indian author, and it is very relevant, very relevant.

And, also in the beginning, we have used this breakeven analysis article, which is old, but the concepts are absolutely valid. In those days, it was new, but the similar ideas we nowadays use are the same; the basic ideas remain unchanged. So, this is a very good article you can easily find on the internet. With this, we come to the end of problems and solutions regarding the breakeven analysis and depreciation. And next, we will take some more examples from other topics. Thank you.