

# MINERAL ECONOMICS AND BUSINESS

Prof. Shantanu Kumar Patel

Department of Mining Engineering

IIT Kharagpur

## Lecture 14: Cutoff Grade - 3

Hello everyone, and welcome again to this course on Mineral Economics and Business. This is our lecture number 14. This is the third and last part of our lecture on cutoff grade. So, what we were seeing for this cutoff grade calculation is that Lane's algorithm is a three-step process.

**Cutoff Grade for Maximum Profit: Lane's Algorithm**

- Step-I:** Determination of the economic cutoff grade - one operation constraining the total capacity
- Step-II:** Determination of the economic cutoff grade by balancing the operations
- Step-III:** Determining the overall optimum cutoff grades

Step one is to determine the economic cutoff grade, considering one operation constraining the total capacity. Shantanu Kumar Patel  
Department of Mining Engineering, IIT Kharagpur

Step one is to determine the economic cutoff grade, considering one operation constraining the total capacity. That we saw in our last class, and the second step is to determine the economic cutoff grade by balancing the operations. The third step is to determine the overall optimum cutoff grade. So, step number 2 and step number 3 we will see today.

## Step-I: Determination of the economic cutoff grade - one operation constraining the total capacity

$$P = (s - r) \times \frac{1-g^2}{2} Q_m - c \times (1 - g) Q_m - \left(m + \frac{f}{M}\right) \times Q_m$$

$$P = (s - r) \times \frac{1-g^2}{2} Q_m - \left(c + \frac{f}{C}\right) \times (1 - g) Q_m - m Q_m$$

$$P = \left(s - r - \frac{f}{R}\right) \times \frac{1-g^2}{2} Q_m - c \times (1 - g) Q_m - m \times Q_m$$

So before that, for the first step, what we derived was three equations for the maximum profit, considering mining as a constraint. Kumar Patel  
Department of Mining Engineering, IIT Kharagpur

So before that, for the first step, what we derived was three equations for the maximum profit, considering mining as a constraint. The first equation we derived, considering the concentrator as a constraint. And the third one is the refinery as a constraint.

So, we derived these three equations here and then plotted them for the profit versus our cutoff grade. And then we saw that all these curves, for example, the blue line here shows the total profit curve for considering mining as a constraint. So, it is going to a peak around 0.1 cutoff grade. And then before that, the curve is going down, and after that, after this point 0.1, the curve is going down. So, the maximum profit is occurring at 0.1.

## Step-I: Determination of the economic cutoff grade - one operation constraining the total capacity

	Case-1	Case-2	Case-3
G (lb/ton)	0.1	0.4	0.16
Total profit (\$)	4100	2600	3400
Life (years)	10.0	12.0	12.18

So, this has been tabulated here in this here in this slide and you know we have three things here the the cut off grade G let us say and the total profit and the life of the

So, similarly for you know the concentrator and the refinery case. So, this has been tabulated here in this here in this slide and you know we have three things here the the cut off grade G let us say and the total profit and the life of the mine. So, in the first case where mining is a constraint or cut off grade is we found out is 0.1 and the total profit is 4100 dollars. for the second case it is 0.4 and the third case this is 0.16 and we have the respective you know the total profit. So, and what we say this 0.1 is our g m we term we give a terminology here and this case to this this is called g c and the third one is called g r.

And also, we can see the life of the mine changes if the cut-off grade changes for the three cases. In this case, because the total quantity to be mined is 1000 and the per year production maximum is 100, so this becomes 100. Similarly, if the cut off grade is 0.4, we are going to send 600 tons of ore to the processing plant and the capacity of this plant is 50. So, the life of the mine is 12 years here and in the third case, So, if g is the cut off grade we send 1 or we produce 1 minus g square by 2 into Qm. So, where Qm is 1000 and g is 0.16 here.

So, if you put this value and divide with the capacity of the refinery which is 40. So, this becomes 12.18. So, this is ah from our previous lecture that ah we we found it out.

## Step 2. Determination of the Economic Cutoff Grade by Balancing the Operations

- In the first step, it was assumed that only one of the operations was the limiting factor to production capacity.
- A second type of cutoff is based simply on material balance.

So, now, ah you know the the second step is ah or maybe in the first step it was assumed that ah only one operation was the limiting factor to ah production capacity ah

you know and and then we ah find out a second type of cutoff grade ah based simply on the material balance.

Shantanu Kumar Patel

Department of Mining Engineering, IIT Kharagpur

So, now, ah you know the the second step is ah or maybe in the first step it was assumed that ah only one operation was the limiting factor to ah production capacity ah you know and and then we ah find out a second type of cutoff grade ah based simply on the material balance. ah.

So, for that, we have a table here which shows the concentrator feed as a function of concentrator cutoff with the mine operating at capacity. What this means is, in the first column here, we have  $Q_n$ , which is the mine amount, constant at 100 tons per year, as we have assumed. So, in this case, in the second column, we are producing 100 tons, but we are changing the cutoff grade from 0 to, let us say, 0.9. So, in this case, if the cutoff grade is, let us say, 0.1, that means we are sending 90 to the concentrator

or maybe 90 to the concentrator per year and sending 10 tons to the waste pile. So, that is why in the third column we have 90. Similarly, if we set a cutoff rate of 0.8, 20 tons will go to the concentrator out of this 100 tons, and 80 tons will go to the waste pile. So, in this case, to balance both operations, we know that the capacity of this concentrator is 50 tons per year. So, which is shown in the row that is being highlighted here.

## Step 2. Determination of the Economic Cutoff Grade by Balancing the Operations

Concentrator feed as a function of concentrator cutoff with mine operating at capacity

Mined amount ( $Q_m$ ) (tons)	Concentrator cutoff grade ( $g_c$ ) (lbs/ton)	Feed going to the concentrator( $Q_c$ ) (tons)
100	0	100
100	0.1	90
100	0.2	80
100	0.3	70
100	0.4	60
100	0.5	50
100	0.6	40
100	0.7	30
100	0.8	20
100	0.9	10

So, we are sending 50 tons to the concentrator.

Jibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel  
Department of Mining Engineering, IIT Kharagpur

So, where we are producing 100 tons and the cutoff grade is 0.5. So, we are sending 50 tons to the concentrator. So, both operations are being optimized. So, this has been plotted in this figure here, with the total profit on the y-axis versus the cutoff grade. So, and the total profit curve for two cases.

So, and you can see that from the previous case this is being balanced at 0.5 cut off ah similarly for refinery product is a function of concentrator ah cut off with concentrator operating at capacity. So, this means you know the capacity of the concentrator is 50. So, if we keep 50 as constant in in the first column which is the amount to be sent to the concentrator. and what we we can do is change the cutoff grade here from 0 all the way to 0.9.

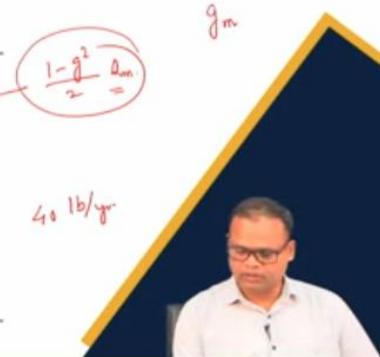
So, and then we can find it out that you know the average grade that we are going to send the concentrator is a function of  $g$  ah which is you know  $1 + g$  by  $2$  ah if we with this  $g$  equal ah sorry the  $g$  or the cutoff grade is 0.1. So,  $1.1$  divided by  $2$  is  $0.55$  here. ah and if this is ah there the refinery product that we know that ah you know this is  $1 - g$  square by  $2$  into  $q_m$ . So, ah so this this is you know if it is  $100$  ah we we can calculate the corresponding ah amount of the product that will be produced from the refinery is  $27$  Similarly, we can change the cut off grade ah you know increase the cut off grade and we can find out the total quantity of the product that we can get it from the ah you know

refinery. But in this case ah you know the refinery has ah ah you know capacity of 40 ah pound per year.

## Step 2. Determination of the Economic Cutoff Grade by Balancing the Operations

Refinery product as a function of concentrator cutoff with concentrator operating at capacity.

Amount to be concentrated ( $Q_c$ ) (tons)	Concentrator cutoff grade ( $g_c$ ) (lbs/ton)	Avg. conc. feed grade ( $g_c$ ) (lb/ton)	Refinery product ( $Q_r$ ) (lbs)
50	0.1	0.5	25
50	0.2	0.55	27.5
50	0.3	0.5	30
50	0.4	0.65	32.5
50	0.5	0.7	35
50	0.5	0.75	37.5
50	0.6	0.8	40
50	0.7	0.85	42.5
50	0.8	0.9	45
50	0.9	0.95	47.5



So, this cut off grade you know is called  $g_c$  into  $c_r$ . So, and if you see the previous one that we saw in our previous slide that is our  $g_m$  and  $c_m$ .

Department of Mining Engineering, IIT Kharagpur

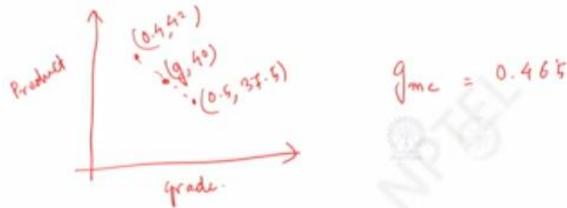
So, which is here in this row. So, to balance both these ah you know concentrator and the refinery we have to make a or put a this you know cut off grade as 0.6. So, this cut off grade you know is called  $g_c$  into  $c_r$ . So, and if you see the previous one that we saw in our previous slide that is our  $g_m$  and  $c_m$ . So this has been plotted again in this profit versus cutoff grade here, where you can see that if it is 0.6, both the operations are being balanced.

And this is our  $G_{cr}$ , which is 0.6. The third case is refinery feed as a function of mine cut-off with the mine operating at capacity. Again, we are keeping this 100 constant, meaning the mine is producing 100 tons per year every year, and what we are doing is changing the cut-off grade here from 0.02 all the way up to 0.9. So, in one of these cases, let us say if the cut-off grade is 0.2, the refinery will produce some amount, which is 48.0, which again we have seen is  $1 - g^2$  by 2 into  $Q_m$ . So, here  $Q_m$  equals 100.

So, you put the value of 100,  $g$  equal to 0.2. So, this will give us a value of 48. So, you know, if you keep the mining, you know, mined-out amount constant every year and keep changing this cut-off grade, this is how, you know, the product obtained from the refinery will change.

In this case, the refinery has a capacity of 40 pounds per year, which falls between, let us say, these two ranges. We do not have an exact value for 40 pounds per year, but we can see this is happening between 0.4 and 0.5.

## Step 2. Determination of the Economic Cutoff Grade by Balancing the Operations



So, this can be plotted in a plot where we can put the product on the y-axis and the grade on the x-axis, where for 0.4, we have 42 tons produced, and for 0.5, this is 37.5 tons. So, if we want a cut-off grade  $g$  for which it will match our refinery capacity, which is 40 tons, if we do linear interpolation, we can find this  $g$ , which we call it  $g_{mc}$  into  $m_r$ , as 0.465 which is shown here for the similar plot. So, this  $g_{mc}$  is 0.465



© 2019 IIT KGP

Prof. Bibhuti Bhusan Mandal & Prof. Shantanu Kumar Patel  
Department of Mining Engineering, IIT Kharagpur

### Step 3. Determining the Overall Optimum of the Six Cutoff Grades

- There are six possible cutoff grades.
- The other three ( $g_m, g_c, g_r$ ) are based upon capacities, costs and the price.
- Three ( $g_{mc}, g_{cr}, g_{mr}$ ) are based simply upon the grade distribution of the mined material and capacities.
- The objective is to find the cutoff grade which produces the overall maximum profit considering the mining, concentrating and refining constraints.



Now, there are 6 possible cutoff grades that we saw that you know from from the 2 previous 2 steps that the 3s are  $g_m, g_c$  and  $g_r$  and then another another 3 are  $g_{mc}, g_{cr}$  and  $g_{mr}$ . Now, the objective is to find out a cutoff rate which produces the overall maximum profit considering mining concentrating and the refinery constant. So, this is our step number 3 in Lane's algorithm.

### Step 3. Determining the Overall Optimum of the Six Cutoff Grades

The local optimums for each pair of operations are first considered. The corresponding optimum grades for each pair ( $G_{mc}, G_{cr}, G_{mr}$ ) are selected using the following rules:

$$G_{mc} = \begin{cases} g_m & \text{if } g_{mc} \leq g_m \\ g_c & \text{if } g_{mc} \geq g_c \\ g_{mc} & \text{Otherwise} \end{cases} \quad G_{rc} = \begin{cases} g_r & \text{if } g_{rc} \leq g_r \\ g_c & \text{if } g_{rc} \geq g_c \\ g_{rc} & \text{Otherwise} \end{cases} \quad G_{mr} = \begin{cases} g_m & \text{if } g_{mr} \leq g_m \\ g_r & \text{if } g_{mr} \geq g_r \\ g_{mr} & \text{Otherwise} \end{cases}$$



and it says you know if  $g_{mc}$  is less than  $g_m$ . So,  $g_{mc}$  will be small  $g_m$  if it is more than  $g_m$  is more than  $g_c$  in that case you know  $g_{mc}$  will be  $g_c$

So, for that what says the local optimum for each pair of ah operations are first considered So, the corresponding optimum grades for each pair G capital Gmc capital Gcr and capital Gmr selected using the following rules. So, we have three conditions here where we have mining m and c. So, we have you know three cut off grades here gm, gc, gmc .

$$g_{mc} = \begin{cases} g_m & \text{if } g_{mc} \leq g_m \\ g_c & \text{if } g_{mc} \geq g_c \\ g_{mc} & \text{Otherwise} \end{cases}$$

So, similarly we can calculate for grc the conditions are given here and for G capital Gmr.

$$g_{rc} = \begin{cases} g_r & \text{if } g_{rc} \leq g_r \\ g_c & \text{if } g_{rc} \geq g_c \\ g_{rc} & \text{Otherwise} \end{cases}$$

$$g_{mr} = \begin{cases} g_m & \text{if } g_{mr} \leq g_m \\ g_r & \text{if } g_{mr} \geq g_r \\ g_{mr} & \text{Otherwise} \end{cases}$$

So, for our case you know in the first case where we have mining and concentrator. So, we have gm to 0.1, gc equal to 0.4 and gmc equal to 0.6, 0.5. So, in this case you know you can see gmc is greater than gc. So, g m c is not less the less than g m ah, but it it satisfies our second condition.

So, in this case, our G m c equals g c, which is 0.4. For the second case, where we have a refinery and concentrator, we have g concentrator equal to 0.4, gr equal to 0.16, and grc equal to 0.6. In this case again, if we check g r c less than g r, which is not the case because g r c is 0.6 and g r is 0.16. So, but it satisfies the second condition where g r c is greater than g c. So, capital G c r becomes g c. which is 0.4.

For the third case, similarly, where we have mining and refinery, so GM we know is 0.1, and GR equal to 0.16, GMR equal to 0.465. So, in this case again, it satisfies the second equation here, where GMR is greater than GR. So, capital GMR becomes GR, which is 0.16, which is the second condition here. Now, the cutoff grade is numerically the middle value, as per this algorithm, that the cutoff grade is numerically the middle value of capital Gmc, capital Gcr, and capital Gmr. So, and we saw from our previous slides that Gmc equals 0.4, Gmr equals 0.16,

g c r equals 0.4, and the numerically middle value in this case becomes 0.4. So, this becomes our final cutoff rate here, which is 0.4, and 0.4 are matching. If it is not, let us say

### Step 3. Determining the Overall Optimum of the Six Cutoff Grades

$$\begin{aligned}
 P &= (s-v) Q_r - m Q_m - c Q_c - f T \\
 &= \$ 2600 \\
 P_y &= \frac{2600}{12} = \$ 216.7 \\
 NPV &= \frac{P_y \{ (1+i)^n - 1 \}}{i (1+i)^n} \quad 15\% \\
 &= \$ 1174.6
 \end{aligned}$$

So, you know, this ends our lecture on the cutoff rate, where we took a simple example of ore distribution for a deposit, and then we found out the three different steps of

how we can determine the cutoff rate to get the maximum profit for the mine. Mandal & Prof. Shantanu Kumar Patel  
Department of Mining Engineering, IIT Kharagpur

in another case, we will find out that this is 0.35 and this is 0.16. and 0.4. In this case, the cutoff grade will be 0.35 for the example that Len has considered. For this, the final cutoff grade becomes 0.4. So, in this case, the life of the mine, considering the mining is constant, becomes 1000, the total quantity to be mined divided by 100, which is 10 years.

And for the, and if it is, and the cut-off grade is 0.4, in that case, we are going to send 600 tons to the concentrator and 400 to the waste pile. So, this is for the concentrator. And, you know, the  $T_c$  becomes, considering the concentrator is 600 by 50, equal to 12 years. And, if you see the third case,  $T_r$ , considering the refinery, it is 1 minus. By 2 into  $Q_m$  is the total amount produced by the refinery, and the capacity of the refinery is 40. In this case, if you put the value of  $G$  equal to 0.4 and  $Q_m$  equal to 1000 here. So, if you find out that this is 420 divided by 40, equal to 10.5 years. So, in this case, we can see that, you know,

it will take the maximum time to process the processing part, or the concentrator will take the maximum time, and the life of the mine in this case becomes 12 years. So, if you see the total profit equation,  $P$  equal to  $s$  minus  $r$  into, you know,  $Q_r$ , minus  $m$  into  $Q_m$ , and, you know,  $c$  into  $Q_c$  minus  $f$  into  $T$ . So, in this case, we have  $s$  equal to 25,  $r$  equal to 5,  $Q_r$  equal to, you know, the total amount produced is 420.  $m$  is 1 dollar per ton, as we have seen in our previous lecture.

This is, you know, 1000,  $c$  is 2 dollars per ton, and this total quantity is 600 that we are going to tell you.  $F$  we know is 300 dollars, and  $T$  is, as we found out in the previous slide, 12 years. So, if you put all these values, this becomes 2600 dollars. And this profit per year we can calculate as 2600 divided by 12 is 216.7 dollars per year, and if you want to calculate NTD. So, this becomes  $P_y$  into  $1 + i$  whole to the power  $n$  minus 1, whole divided by  $i$ , where, you know, this becomes with 15 percent. 15 percent interest rate this becomes this, so it is  $1 + i$  to the power  $n$  here. So, this becomes 1174.6 dollars. So, you know, this ends our lecture on the cutoff rate, where we took a simple example of ore distribution for a deposit, and then we found out the three different steps of how we can determine the cutoff rate to get the maximum profit for the mine.