MINERAL ECONOMICS AND BUSINESS

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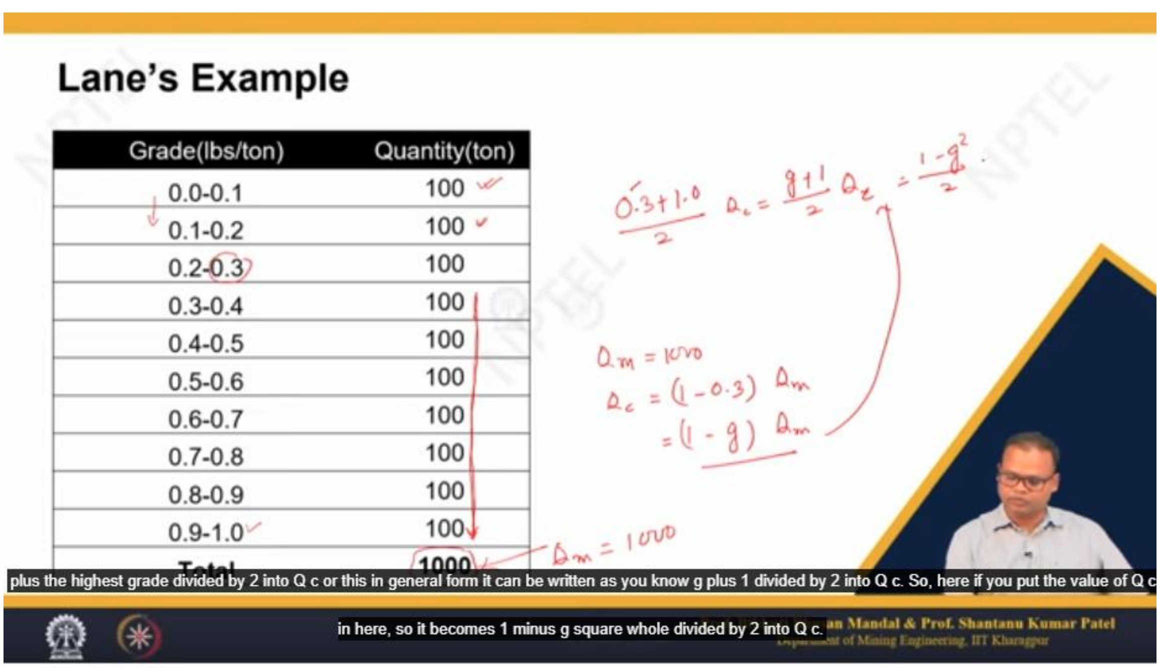
Lecture 13: Cutoff Grade - 2

Hello everyone and welcome again to the course on Mineral Economics and Business. ah This is our lecture number 13 and this is the second part of the lecture on cut off grade. ah In our previous ah class what we were seeing was ah you know this is a typical distribution of ah copper deposit where ah we have in the first and third column ah the ah grade interval For example, you know this one here says that we have from 2.3 to 2.4 percent of copper deposit of 25000 25 into 10 to the power 3. So, 25000 tons of copper.

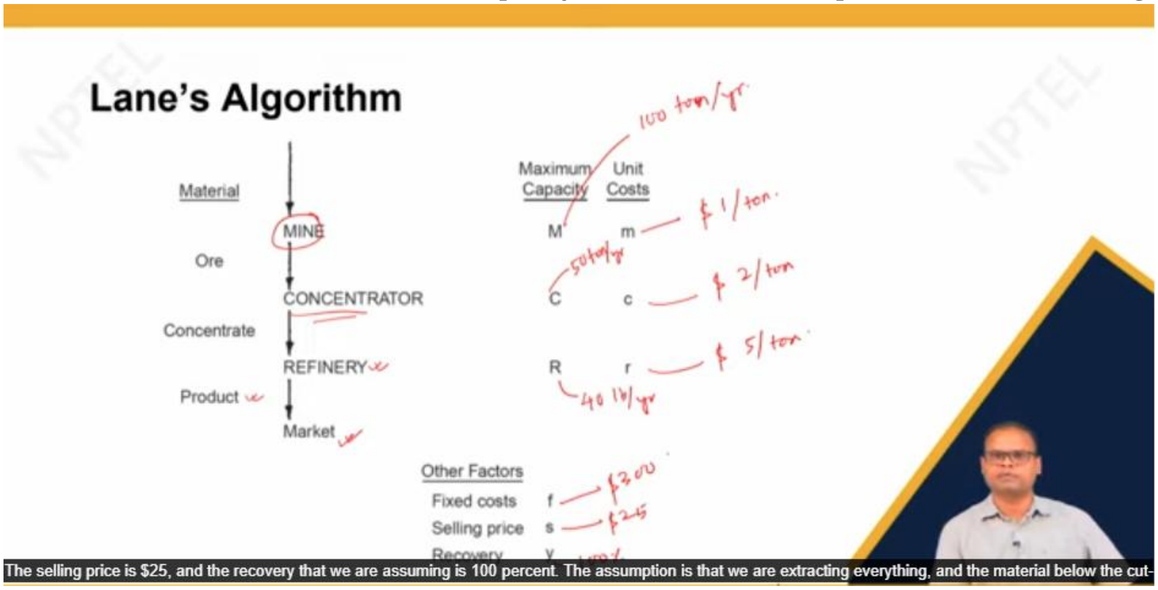
And, similarly you know the the at different class interval the amount of copper present in the deposit is changing. So, and also we talked about let us say if the cutoff grade is 0.8, in this case what we are going to do is to extract whatever above this 0.8. ah to all the way up to the highest ah grade deposit in in in the deposit. So, in this case ah you know the question is ah ah or maybe what we are looking into is how do you decide what should be my ah cutoff grade like it should be 0.8 or 0.6 or or or or what it is. So, for that ah ah you know ah lane consider this ah simple example.



Here you can say that you know in the left side again we have the grade distribution or grade interval ah which is uniformly changing ah from 0 0.0 to 1.0 here and on the right side the quantity of the you know mineral present in the particular interval is ah 100 in all all the cases. So, here we have total amount of mineral that is present or the total quantity of the ore present is 1000. So, it is the our Qm becomes 1000. and let us say the cutoff grade is 0.3.

So, in that case what we are going to extract is anything above 0.3. So, if so, that that quantity can be calculated as you know Qm or Qm we are expecting everything Q m equal to you know 1000, but Qc which we are going to send to the concentrator which is above 0.3. So, that becomes Q c equal to 1 minus 0.3 into Qm. and thus becomes in in general form we can write it you know 1 minus g ah into Qm where g is our cutoff grade.

Similarly, you know if cutoff grade is 0.3 the average grade ah you know we saw in our previous lecture that that we are sending it to the you know the refinery is 0.3 plus 1.0. So, the minimum or the cutoff grade ah plus the highest grade divided by 2 into Q c or this in general form it can be written as you know g plus 1 divided by 2 into Q c. So, here if you put the value of Q c in here, so it becomes 1 minus g square whole divided by 2 into Q c. So, and then also what we saw is we extract ore from the mine and then we send it to the concentrator. The concentrator produces the concentrate which is sent to the refinery and refinery you know produces the final product and it goes to the you know the market. And, what we saw was there is maximum capacity involved for each operation. So, the mining

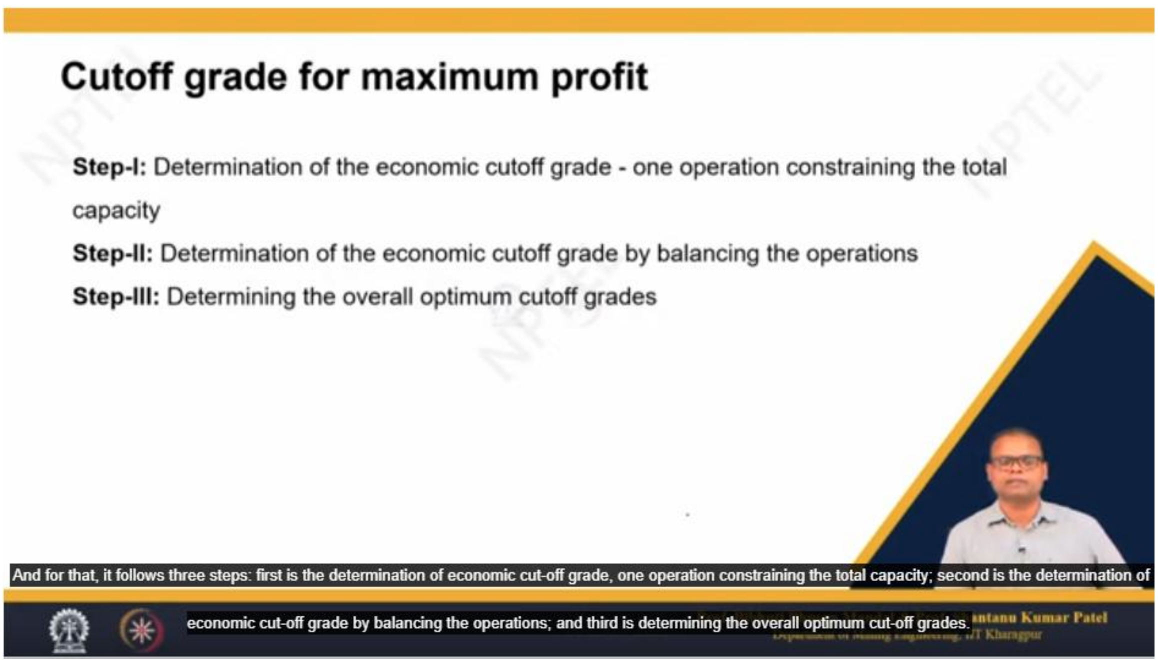


capacity we call it M. So, in the example that we saw is M equal to 100 ton per year. ah C is the concentrator capacity we saw that this is 50 ton per year, R is the refinery capacity which is which you can produce 40 ah pound per year. There is a corresponding cost involved and we saw that you know this is 1 dollar per ah ton of ore. And c is $2 per ton of ore and r is $5 per ton of ore.

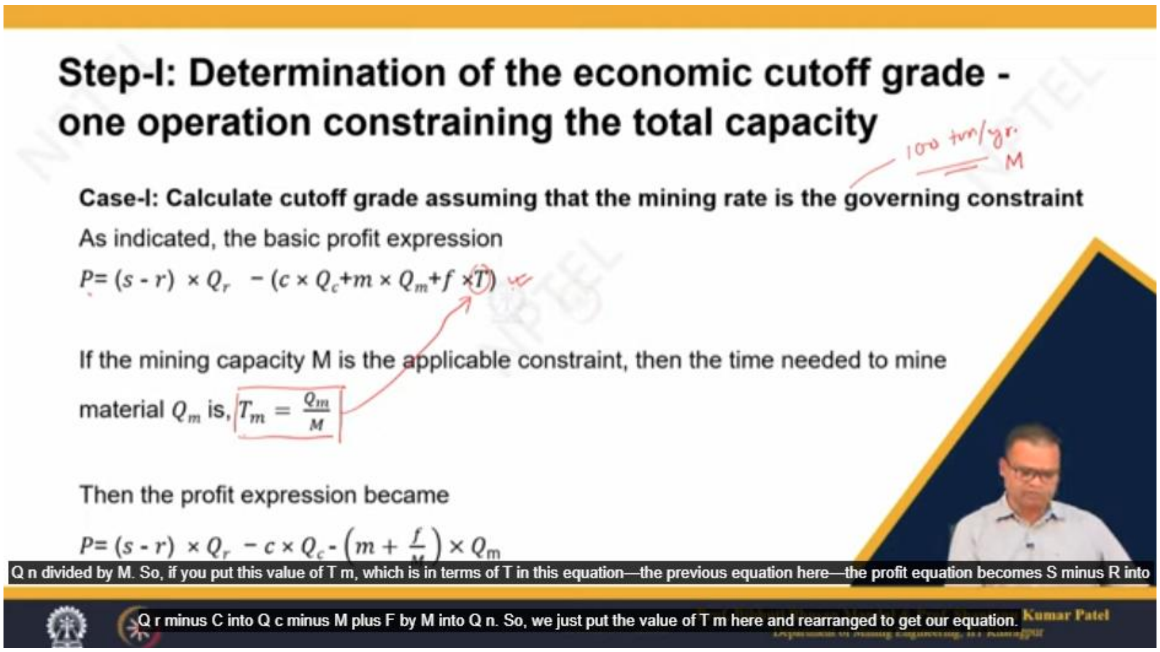
And the fixed cost we have is $300. The selling price is $25, and the recovery that we are assuming is 100 percent. The assumption is that we are extracting everything, and the material below the cut-off grade is going to the waste pile instead of remaining in the ground.

$$Total\ Cost = m\,Q_m + c\,Q_c + r\,Q_r + fT$$

T is the life of the mine. So, and then because the refinery is producing Qr amount. So, s times Qr becomes our revenue, and the total profit becomes revenue minus total cost, which is minus r into Qr, and this term here is there.



So, what Lane's algorithm says is that, in a particular mine, if you want to get the cut-off rate, this total profit should be maximum. And for that, it follows three steps: first is the determination of economic cut-off grade, one operation constraining the total capacity; second is the determination of economic cut-off grade by balancing the operations; and third is determining the overall optimum cut-off grades. So, we will see these three steps one by one.



So, coming to the first step of this determination of the economic cut-off grade, one operation constraining the total capacity. The first case is to calculate the cut-off grade assuming that the mining rate is the governing constraint.

So, what this means is that, in our case, the mining we can do is 100 tons per year. So, this is our constant; we cannot change this or increase or decrease it. So, this is fixed. So, in other terms, we can say that M is fixed. So, in this case,

Ah, you saw that. You know the total profit equation is here, which is P. But if the mining capacity M is an applicable constraint, then the time needed to mine the material or the life of the mine will be

$$T_m = \frac{Q_m}{M}$$

So, if you put this value of T m, which is in terms of T in this equation—the previous equation here—the profit equation becomes

$$Profit(\,P) = ((s - r) \times Q_r\,) - c \times Q_c - \left(m + \frac{f}{M}\right) \times Q_m$$

Assuming g is the cut of grade

$$Q_C = (1 - g)\; Q_m$$

$$Q_r = \frac{g+1}{2}\; Q_C = \frac{1-g^2}{2}\; Q_m$$

Then the profit expression became

$$Profit(\,P) = ((s - r) \times \frac{1-g^2}{2}\; Q_m\,) - c \times (1-g)\; Q_m - \left(m + \frac{f}{M}\right) \times Q_m$$

So, what we did was we put the respective values of Q r and Q c in our previous equation. So, to find the grade which gives maximum profit under this constraint, to get this, what we need to do is to get the derivative of the profit equation and equate it to 0. So, if this is our curve—like we do not know the shape of this curve—but to get the maximum profit, if we differentiate that P equation with g, we can find the cutoff rate g with respect to M.

So, this is our profit equation here. So, this is the or maybe you can write the previous equation as here, as we have written it here.

$$\frac{dP}{dg} = 0$$

We get

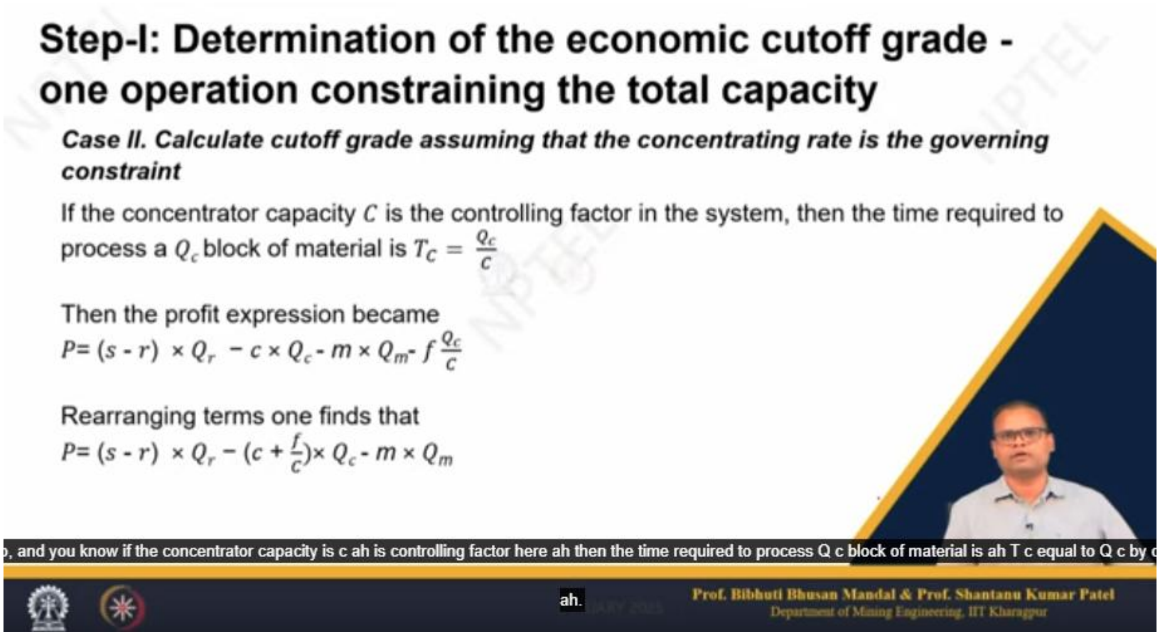
$$Q_m = 0$$

So

$$g_m = \frac{c}{s-r}$$

So, in this case if g m is 0.1 q c is 1 minus g into Qm . So, Qm is 1000 and g equal to 0.1. So, this 0.1 and this is 1000. So, this becomes q c equal to 900. and to to process this 900 tons of mineral the time that is going to take is 900 by 50 which is the capacity of the concentrator.

So, this is 18 years. Similarly, you can calculate you know this time required to the refinery to process is 12.37 years.



So, the case 2 is cut off grade assuming that the concentrating rate is the governing constant. So, and you know if the concentrator capacity is c ah is controlling factor here ah then the time required to process Q c block of material is

$$T_C = \frac{Q_c}{C}$$

So, and if you if you see our previous profit equation and put the value of Tc here.

Then the profit expression became

$$Profit(P) = ((s - r) \times Q_r) - c \times Q_c - mQ_m - f\frac{Q_c}{C}$$

Rearranging terms one finds that

$$Profit(P) = ((s - r) \times Q_r) - (c + \frac{f}{C}) \times Q_c - mQ_m$$

$$Profit(P) = ((s - r) \times Q_r) - (c + \frac{f}{C}) \times Q_c - mQ_m$$

$$Q_C = (1 - g)\ Q_m$$

$Q_r = \frac{g+1}{2} \; Q_C = \frac{1-g^2}{2} \; Q_m$

$Profit(\,P) = ((s - r) \times \frac{1-g^2}{2} \; Q_m\,) - (c + \frac{f}{C}) \times (1-g) \; Q_m$ - mQ_m

So, here we need to remember this is capital C and this is small c. So, again to get the maximum profit, we need to differentiate it with g, and if we differentiate this equation that we saw previously,

$$\frac{dP}{dg} = 0$$

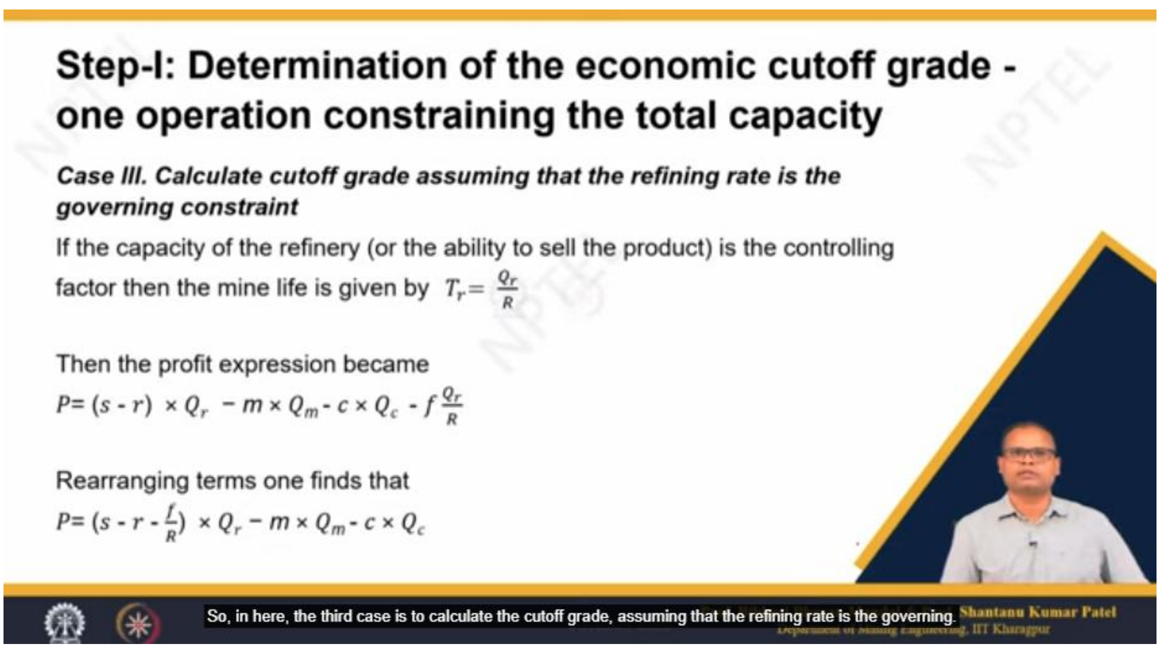
We get

$$\left(\frac{-2g(s-r)}{2} + c + \frac{f}{C}\right) Q_m = 0$$

So

$$g_c = \frac{c+\frac{f}{C}}{s-r}$$

So, if you put all these values, g c becomes c divided by (f c + f)/capital C, whole divided by (S - r). Now, if you put c = 2, f = 300, C = 50, S = 25, r = 5. So, this g c becomes So, if Gc becomes 0.4, 600 tons will be sent to the concentrator and 400 tons will go to the waste



pile. So, this 600 divided by the capacity of the concentrator, which is 50, becomes 12 years. So, to process this total 600 tons of material, we are going to take 12 years, which becomes the T c, the life of the mine.

So, in here, the third case is to calculate the cutoff grade, assuming that the refining rate is the governing. So, in this case, you know the Tr, like previously, the Tr becomes Qr by r. If we put the value of Tr in our last term here, which is f times t, instead of T, we put Qr by r. So, the property expression becomes as shown here. And if we rearrange this property equation, it becomes s minus r minus f by R into Qr minus m times Qm minus c times Qc.

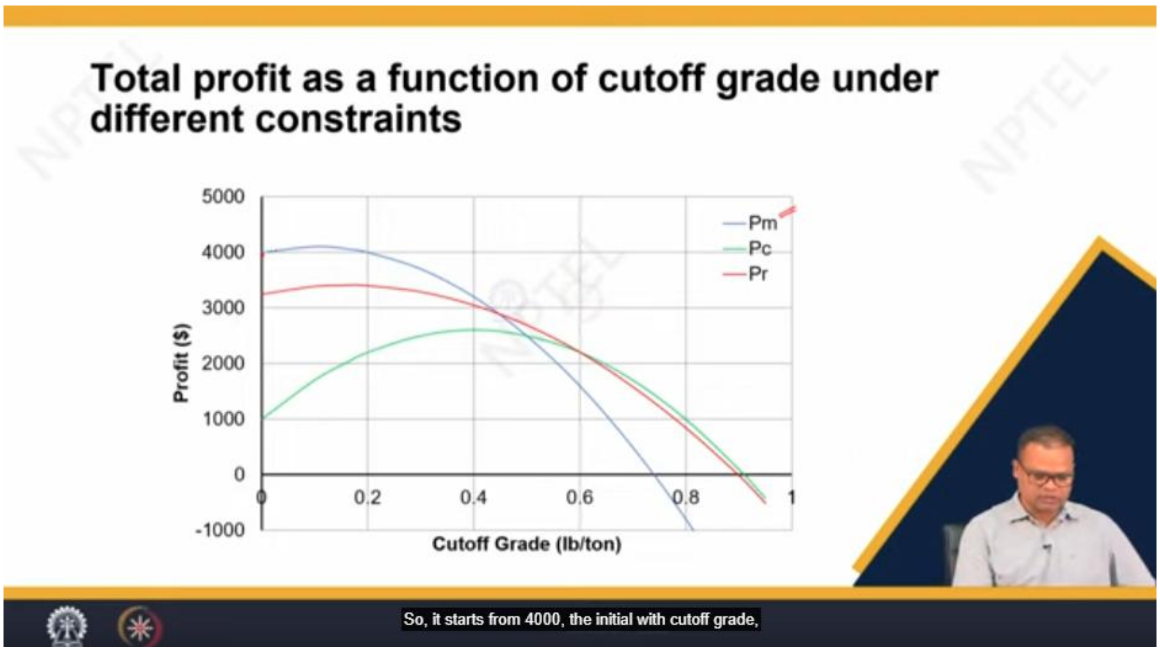
Again, you know, if we put the value of Q c as 1 minus g into Q m and Q r as 1 minus g square by 2 into Q m in the equation here. So, rearrange the profit equation, and it becomes s minus r into f by r 1 minus g square by 2 into Qm minus t into 1 minus g into Qm minus m into Qm, which is written here. So, again, to get the maximum profit for the third case, we need to do d p by d g equal to 0. And if you do the d p by d g equal to 0 for this equation here, this becomes minus 2 g by 2 into s minus r minus f by capital R into Qm minus minus minus plus. c into Qm minus 0, because this third term is 0. Sorry, the third term is constant.

So, this equals to 0. So, if you put the value of the respective value of all these terms here, like c equal to 2, s equal to 25, r equal to 5, f equal to 300, and R equal to 40. So, in this case, the g r becomes 0.16, and the time required to process this amount mineral with cutoff rate 0.16 becomes T r equal to 12.18 years.

So, this is, you know, the total quantity to be processed, which is 1 minus g squared by 2 into Q m. Q m is 1000, and g equals 0.16 here, divided by capacity, which is 40. So, this becomes 12.18 years. So, ah, this is all for the 3 cases. The equations—the profit equations—are written here: equation number 1, equation number 2, and equation number 3, ah, based on their constraints: the mining, concentrator, and refinery constraints. So, if you put the respective values and vary the value of g—so, or maybe, you know, if you put different values of g here, from 0 to 0.1 all the way up to 0.95—the profit from the equations, like from the 3 equations. So, this is equation number 1 that we saw in our previous slide, and this is equation number 2.

In there, if we put ah the corresponding values—and for equation number 3, if we put the values which are related to the refinery constraint, the concentrator constraint, and the mining constraint—in those three equations, for different values of g, we ah put the values in and can find out what the corresponding values of ah or the total profit are. So, here we can see, you know, the values of this total profit are changing at the, you know, the cutoff rate, based on different constraints.

So, the previous table has been plotted here. You know, you can see that, you know, the



first one, with the mining constraint, is the blue line. So, here we saw that, you know, this is where we put the different values. So, it starts from 4000, the initial with cutoff grade,

and then it goes off and has a maximum value at 0.1. So, this is 0.1 that we found out from the derivation and putting the corresponding values, ah, and then, after that, it is going down. So, this is a peak point, and we differentiate at ah this location here to get the maximum. Similarly, for the, you know, the case with the ah concentrator constraint, we found out that the peak is happening at ah the 0.4 ah cutoff grade, and ah, whereas, in this case, the maximum value is happening at 0.5. So, so in for the 3 cases.

So, this is the first step of Lane's algorithm. So, the second and third steps we will see in our next lecture.