Advances in Additive Manufacturing of Materials: Current status and emerging opportunities

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Lecture 21

Introduction to Cellular structure and Topology Optimisation

Let us continue our discussion on the lattice structures. In this lecture I will start with that lattice structure basic definition, classification of lattice structure and topology optimization. Why lattice structures is important? Lattice structures not only play an important role in additive manufacturing of components for biomedical applications and so on, but it has essential, it offers specific advantages which other homogeneous structures cannot offer. For example, weight reduction, sandwich cores or strain isolation or vibration control, these are like structural related advantages. Then thermal applications related heat exchangers, flame arresters, heat shields and acoustic liners, they are also lattice structures play an important role. In the fluid related applications, catalytic carriers, packaging and buoyancy, in biomedical bone integration, cell growth and biomimetic and why this biomedical is playing an important role I have explained sufficiently while I have described about the bone structures and so on.

there are two key requirements of the lattice structures. One is the unit cell that is the combination of material and pore and this particular unit cell that has been shown here, it has a lattice dimension is 4 millimeter and you can clearly see these are like specific nodes. and these are like specific struts here. struts and nodes these are the two important elements and that can be used in combination to describe any unit cell of a lattice structure that I am going to explain mathematically very soon.

Then another requirement is repetition that is the unit cell is repeated in 3-dimensional space and the resulting pattern need not be regular and can include more than 1 unit cell. these are generic classification of the lattice cellular structures. The first one is the structural shape. structural shape wise, lattice structure can be classified as regular and then pseudo-random. Then pseudo-random can be either warped or conformal or random.

Random can be natural, foam or Voronoi type of structure. Cell geometry like cubic, octet truss, triple periodic minimum surface, TPMS structures or others. Cell topology wise, it can be either open, hybrid or closed. And cell element dimensions wise, homogeneous, gradient and heterogeneous. these are like generic classification of lab cellular structures.

It is not possible for me to get into very, very finer details into the description of each of these classes of lattice structures that is beyond the scope of this NPTEL course. However, what I would like to show you very fundamental concepts which is mathematical based explanationsto describe this kind of a lattice structures. Before that, I will give you some examples. This is the classic honeycomb type of structures which is used for biomedical and non-biomedical kind of applications, 3D truss and this is the unit lattice cell these are like, this is the one strut, this is the second strut, this is the third one, this is the fourth one, this is the fifth one, this is sixth one, this is seventh one. natural versus design lattice structure, let us look at some of the natural structure like scoria and pumice rocks.

You can see this is the classic examples of the natural structure. These are like a closed cell structures. This is like an octet truss lattice structure. This you can see it is a more clearly porosity gradient. This is like a more bone like structure.

You remember the bone structure before if I go back a few slides back that when I have explained that how the bone structure is. Now if you look at this particular cancellous bone structures which is present at the inner side of the bone structures or spongy structures or porous structures. For example, this kind of porous structures, this if you recall that particular image then you will be able to appreciate that this kind of a porosity gradient structure. can in principle mimic the bone structure because if you see that look at the outer surface of this particular design, the outer surface has very dense small pores. But the inner material you can say it is a large porous structure.

Now, in the classification I have mentioned, this is the Voronoi kind of a structure which is part of the random structure. let us first explain that what is Voronoi diagram. This is a basic mathematical logic with features of nearest adjacency. I repeat these particular four words, features of nearest adjacency. And it skillfully reflect many expression features in nature like patterns in dragonfly wings or in the giraffe fur or cracked earth and can be expressed both in two-dimensional and three-dimensional space.

These are like two-dimensional points. and the Voronoi cells has been shown and this is like a 3 dimensional points in the 3 dimensional points and the Voronoi cells are being distributed. And why Voronoi diagram is important if you look at the some of the natural structure let us say butterfly wings right you can see very closely this is also Voronoi kind of a structure. Now what a Voronoi diagram? I will explain to you mathematical description now. So let us say this is the random arrangement of points These two are the sheets within the design domain.

Now you connect this kind of sheets point in a meaningful manner that you can essentially get some of the equilateral triangles or isosceles triangles. Now, these triangles, you consider these midpoints of each side and then draw a perpendicular bisector. I repeat, you connect the seeds points that which are randomly distributed in the 2-dimensional To form a specific number of triangles and these triangles you identify the meet points and then draw a perpendicular bisector. that means it will bisect these particular sides of each triangle into half and in a perpendicular manner. And you will notice that each of the perpendicular bisector will meet perpendicular bisector for the nearest or adjacent triangle and that will form in another pattern.

the way I am drawing, these are actually meeting points or these are like centroids of that different triangle. and these centroids will form again another kind of a pattern. And now these initial seed points will be somehow will be essentially located at the center of this kind of a hexagonal array of things structures. this is called Voronoi tessellation. I hope you are able to follow me while when I describe this Voronoi diagram.

mathematical description for any set of points on the m dimensional Euclidean space. let us say P1 to Pn these are like set of points right. It is a one dimensional matrix that belongs to a m dimensional Euclidean space R to the m where 2 is less than equal to n less than infinity. n value should lie between 2 and infinity. and P_i is certainly not equal to P_j.

so that each point is a distinct point, they are not repeated points. The Delauney triangulation is the line between P i and P j divides the space into two parts called Delauney triangles, H. This is a Delauney triangle, H. Then identification of the bisector lines, as I said that you put these bisector lines Then you connect them, then you get the Voronoi tessellation where V of pi is essentially described by this kind of a mathematical equations and then this particular array of gamma of $\{V(p_i),,,, V(p_n)\}$ that is called the Voronoi diagram. this is a little bit in depth in the mathematical description, but I am sure that you are able to appreciate that how uniquely this voronoi tessellation is being constructed in this particular case.

Now, let us describe the lattice structures based on the strut based lattices. this is called struts and this is called nodes lattice structure is a type of cellular structure with non-stochastic geometry. Stochasticity means randomness. Stochasticity is the inherent attribute of any biological system. But what I am talking about this lattice structure, these are cellular structures with non-stochastic geometry that means non-random geometry and as an architecture which is formed by regular geometric arrangements of periodic unit cells over a space.

And then, strut based structures refers to the open-celled arrangement of strut elements with defined connectivity at specified nodes. And these lattice arrangements are readily tessellated to fill space and allow highly efficient paths for the stability of external and internal loads. Now, a few slides back, you remember that I have taught you about the body centered cubic structure. This is the BCC structures. this is the unique arrangement of atoms on the crystalline materials Another one is the face centered cubic structure, this BCC and FCC have been very widely used also to construct this kind of a cellular structures.

what you see here, these are the examples of the BCC based structure. This is a BCC based structure. Now, when you construct the BCC based strut structures, if you add another strut in the Z direction, then essentially you form BCCZ structures. The C is your FCC based structures and this is your FCC Z based structures. FCC is face centered cubic structures.

You remember face centered cubic unit cell like you have motif or basis in each of the cube corners plus motif and basis in each of the center of the faces. in a cube, you have 8 cube corners. 8 cube corners will be occupied by motif or basis. In a cube, you have 6 faces and in the center of the 6 faces, you have 6 motif or basis. And there is other kind of things, these are simple cubic structure or octet truss structure and you have a diamond structure.

And these diamond structures also I will come to that in few minutes actually when I will go into more depth into the lattice structure. Now mechanical response of a strut based lattice structures. Now there are two type of mechanical responses that we are talking about. One is a bending and one is a stretching. Now bending and stretching these are the two fundamental mechanical response that one can expect that this material, this kind of strut structure can behave in the most fragile manner.

So if you try to bend it, this structure may not be very stable, it will break. And the same is true for the stretch dominated and that is why more the bending and stretch dominated structures essentially considered while understanding the design stability of this lattice structures. Maxwell is the person who proposed a set of mathematical condition that must be satisfied for the loads to be stabilized in a mechanically robust manner. if you consider any random lattice structure let us say for this one. You have 4 struts and you have a 4 nodes.

You agree with me? These are like 4 nodes. what Maxwell said that capital M is equal to S -2j + 3, for planar 2D systems. This is certainly planar 2D systems. here this one M is equal to S - 2j + 3.

What is the S value? S value is 4. What is the J value? J value is 4. 4 +3,= 7 - 8. M = -1.

that means M is less than 0, this understiff bending dominated response. What is these particular cases? S = 5, strut = 5, J, the nodes, number of nodes = 4.

essentially what you have is that 5 - 8 + 3, so M = 0, so this is just stiff structure. And here in this case how many s is equal to 8 and how many are nodes , so that j is equal to 5. 8-10 + 3, so is equal to 1, m = 1. if m = 1, that means m greater than 0 is over stiff this is the stretched dominated response. This is a bending dominated response, this is stretched dominated response.

And this actually you can see more details in this particular paper which I have referred here. Where s and j are number of struts and nodes respectively, m is less than 0 that is bending stresses developed in struts, m is greater than or equal to 0 that is the axial tension and compression in struts. this kind of strut geometry is very important. Now, you understand what is the mathematical foundation to analyze whether the structure sensitive or will experience more bending stresses. because of the unique arrangement of struts and nodes or this kind of lattice structure will experience more axial tension and compression in struts because of the unique combination or distribution of struts and nodes.

I repeat if M is less than 0 bending dominated response in M is greater or equal to 0 that is a stretching dominated response. these are like different cell types, you have body centered cubic structure, I mentioned body centered cubic structure you have strat is 8, you have nodes is 9, your Maxwell number is -13. struts aligned to load direction is no and strut inclination angle is acute angle 35.3 degree. You have a body centered cubic Z-struts as I said you add perpendicular along the Z direction more struts.

it is derived from BCC structure but you know it is not essentially BCC lattice structure BCCZ and here you have 12 struts it is minus 9 Maxwell number is less than 0. it is bending dominated structure according to this particular definition. M is less than 0, bending stresses developed in struts, bending dominated response. Then, face centred cubic, number of struts is 16 and this is also, nose is 12 and then angle is 45 degree, M value is -14. essentially, strut aligned to load direction, no.

this is also bending stresses developed. Then comes 20, then 12 and then 90 degree and 45 degree FCCZ means this FCC based structure means along the Z directions and then Maxwell number is -10 and strut aligned to load direction is yes. triply periodic minimal surfaces are characterized by having the smallest possible surface area. that means there is certain periodicity and when it is called triply periodic means that periodicity value is to be defined by another constant k value and k value the way this has been defined K_i is equal to 2 pi n_i/l_i where i is = x,y,z and t is the constant to alter the relative density. there are 3 different ways that one can describe this TPMS structure. Before that let me complete the

TPMS structures definition.

TPMS are characterized by having the smallest possible surface area. Then surface area is extremely small while filling a given volume in 3 dimensions without any self-intersections. if you look at the nature or you know daily life let us soap film. soap film is a classic example is a minimal surface area. Driven by this some of the examples in regular life or nature, researchers have designed three different type of structures under this category of TPMS, triply periodic minimal surface.

And if you remember this classification of the cellular structure I mentioned, TPMS essentially falls under the cell geometry definition. based cellular structures and there you can see that Schwarz diamond there that Ud that isosurface can be described as sin (K_xx), product of sin (K_xx), sin (K_γy), sin (Kzz) plus combination of sin cosine and sin cosine and cosine sine minus t. t is the constant to relative to alter relative density. Schoen gyroid, this also t is that essentially constant to alter relative density and neovius it is that un, this is the cosine function cos (k_xx) + cos (k_γy) + cos (kzz), 3 times of this sum of this cos functions plus product of these three different cos functions. sum of principal curvature one has to also note there at every point is 0.

Now, this kind of structures that it looks very attractive, it is all very complex, it somehow represents some of the most complex structures. But as I said, when you use this additive manufacturing or 3D printing, one has to also make sure that printability or manufacturability of this structure is also established. Now, topology optimization, this is another concept, it is a mathematical method that spatially optimizes the distribution of material within a defined domain in terms of size, shape and weight by fulfilling given constraints previously established and minimizing a predefined cost function you start with the CAD model. Then you run the topology optimization for this specific additive manufacturing.

One can do finite element methods design verification. Then one can do additive manufacturing mechanical and material verification. this is the typical rational flow which is used in the topology optimization process. Now, if you look at the solid isotropic material, as I said isotropic means it is a homogeneous material without any directional dependence or spatial directional dependence variation of the properties. Now, in the solid isotropic material, one of the concept that is very important, is the solid isotropic material with penalization approach and that in combination with finite element analysis is used for numerical computation of the structural optimization objective and constraints. Now, if you see some of the mathematical expression, so whenever you see that this is a bold notation, bold notation means it is either in the matrix form like for example, governing equation is KU is equal to F.

K is the global stiffness matrix, U essentially the nodal degrees of freedom and F essentially load vectors respectively. Now you have to minimize that C function. And what is the C function? C essentially is the objective function as a function of P and then C function if you essentially minimise, then C function can be described as F transpose. if the transpose of the F function, F is your load vectors. F^TU, and that is equal to U^TK , U transpose K and U transpose, U is your nodal degrees of freedom, K is your global stiffness matrix and that is summation of E is equal to 1 to N.

rho e to the power p ue transpose Ke and ue. So these are the kind of different terminology that has been mentioned here. u_e and k_e are element displacement vector and stiffness matrix respectively. And N is your total number of elements in these particular structures. This is some of the example, this is a free cantilever beam but if this beam has this kind of structures then how it will respond. So, in the finite element problem the equilibrium equation is discretized by using that equation 2 that is the governing equation and SIMP penalization is imposed as a power law on the density values given by the penalty factor that is shown in the equation 1.

Just to continue with this SIMP approach, I essentially bring your attention on this particular equation that is the constraint what is the design constant G rho which is nothing but V which is as a function of rho / V0 - F. What is V? V is the volume of solid region and V0 is the volume of design domain and f is the prescribed maximum volume fraction of material allowed in the design domain. And this rho minimum is essentially minimum physical density non-zero to avoid singularity of K that is 0 less than rho minimum less than rho E. So this design derivative in equation 3 can be easily found as it depends on the design variables rho E here and derivatives of the nodal degrees of freedom with respect to design variables are required with direct differentiation being applied. this is the last two slides on the SIMP, essentially you go back to this governing equation KU = F, then you can multiply this with the Lagrange multiplication that is C is equal F transpose U plus lambda transpose F minus KU Now if you equation 5 if you differentiate then you get that equation 6 and equation 6 can be very simplified to get this particular form.

Then lambda satisfy the adjoint equation F transport minus lambda transpose K is equal to 0 and equation 8 resembles that essentially that same equilibrium function that is number 2 and then if you the choice if lambda is equal to U the displacement derivatives vanish from equation 6 then you get the sensitivity of the objective function with respect to the design variables. And this final solution can be given as the A is your design domain and B is your solid isotropic material with SIMP solution. This is some of the case study examples. You can see the torsion leading case scenario of the arm. These are topology optimization results of the excavator arm or these are 3D printed and installed arm.

This is some of the examples. This is more examples of the topology optimized lattice structures. Now you can see when the topology optimization is there, this rho c by rho, this is a different value. What is the rho c? That is you remember that is your rho is a vector for all the design variables and then you can find out that what is the force line distribution in the finite element analysis and then what is that isostatic line regeneration that provides all the alignment shape and size. And then finally you have to do 3D print this kind of a structures and also test it under the performance simulating environment just to show what is the stability of this kind of a design lattice structures. So to conclude this particular important lecture on the lattice structures, I must mention that it is important to have the fundamentals right, this kind of mathematical foundation of that.

What is the Maxwell number m and how that bending dominated or stretched dominated structures can be built and they can be analyzed. How different TPMA structures can be designed and what is again the mathematical foundation to design those kind of structures and more importantly I have mentioned this designing is one thing but one has to also establish the manufacturability of these designs so that these designs can be essentially used in real life applications. Thank you.