Modeling of Tundish Steelmaking Process in Continuous Casting

Prof. Pradeep K. Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 29
Difference Discretisation Schemes

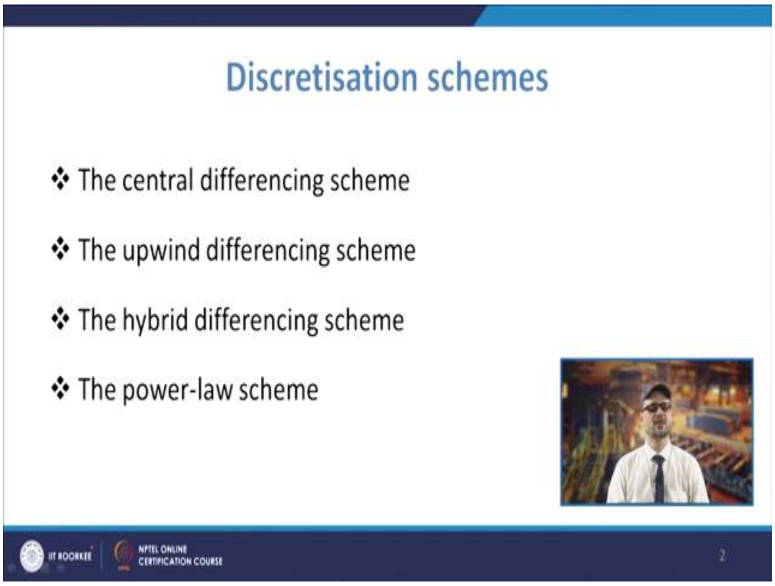
Welcome to the lecture on Different Discretisation Schemes. So, in the last lecture, we talked about the problems related to convection and diffusion, and how to solve them and we have so far used the central differencing scheme because what we have seen is that we need to define the property at the you know control volume cell face.

And there you know how to define that you know normally we take as the you know average of the you know values which is on the east and west side node and that way we are calculating. Now, what we see normally when there will be convection dominated flow? In those cases it will be you know it will not give proper result if we are using those schemes.

So, in fact, what happens that in such cases it will be more dependent upon the upwind node values. So, you know in the case of you know the flow direction whenever there is a flow involved and flow direction is there so, this central differencing scheme does not do well. So, that even we can see we will see later on when we solve a problem based on that.

Now, in this lecture we are going to have the discussion about the different you know differencing schemes or discretized schemes which is available for the prediction of the cell face value you know when we are we have to calculate those values using the nodes which is towards east and west.

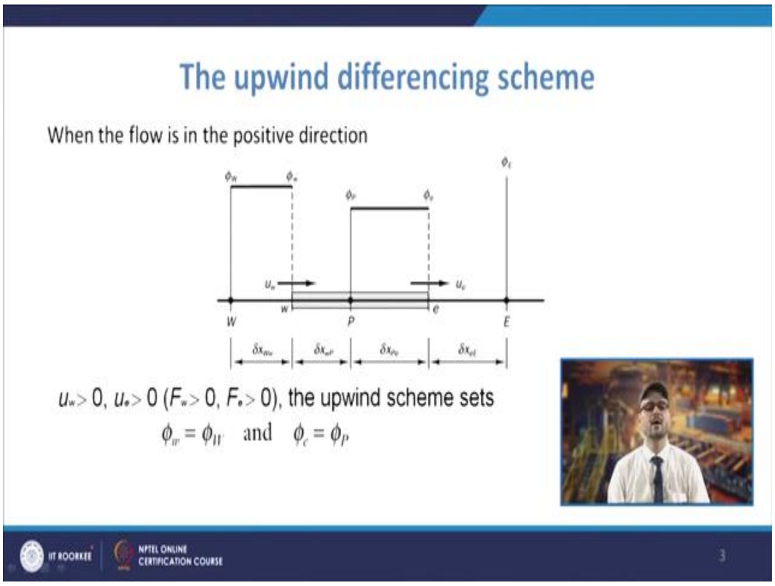
(Refer Slide Time: 02:42)



So, normally the schemes which are used are the central differencing scheme the upwind differencing scheme, hybrid differencing scheme, power-law scheme we have we will also discuss about the higher order schemes like quick scheme. And, there are many more but we will try to have the understanding of these schemes because when we do the modeling of the fluid flow.

You know inside the tundish at that time in most of the modeling tools or maybe softwares if we use, we need to specify these schemes and we must be acquainted with the effect what the different schemes will be generating or which of the schemes are there we should not be used because they will be having the problems while the convergence of the solution or so. So, we have talked about the central differencing scheme earlier where we take the you know values that is based on the both the east and west you know nodal values.

(Refer Slide Time: 04:01)



Then, next comes is the upwind differencing scheme now upwind differencing scheme basically you know it will be normally you know that and when you have the flow direction. So, in case of flow direction the central differencing scheme has the inability now that is being taken care of when you have the chance to use the upwind differencing scheme.

So, basically when if you look at if you have the flow you know which is there and if which is it is flowing in the positive direction. So, you have the you know velocity that is u_w here at the cell face and here you have the u_e. Now, what we do normally in the case of upwind differencing scheme or if it is also known as the donor cell differencing scheme.

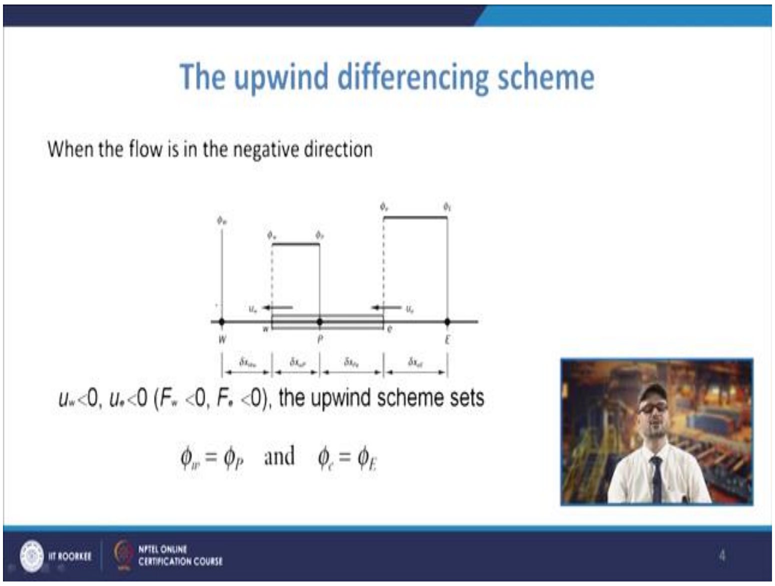
So, it will be taking into account the effect of these flow direction while we try to find the you know value at the cell face. So, the convected value at this cell face that is your west cell face it will be taken as the value equal to the upstream node value. So, normally in the case of upwind differencing scheme you are have when you are calculating the converted flux you know value.

In that case at this face it should be taken as the value at the immediate upstream node that is your w. So, now basically if you try to find the φ_w I mean that is at this face it will be equal to the upstream is this w node. So, you have west node. So, you will have it will be equal to west node value.

Similarly, if we try to have the value of value at e so, for E the upstream node is immediate upstream node is P. So, that is why you get the φ_e as φ_P, because this is the value this is a node where and anyway then you can have the expression in the form of φ_P, φ_W and φ_E.

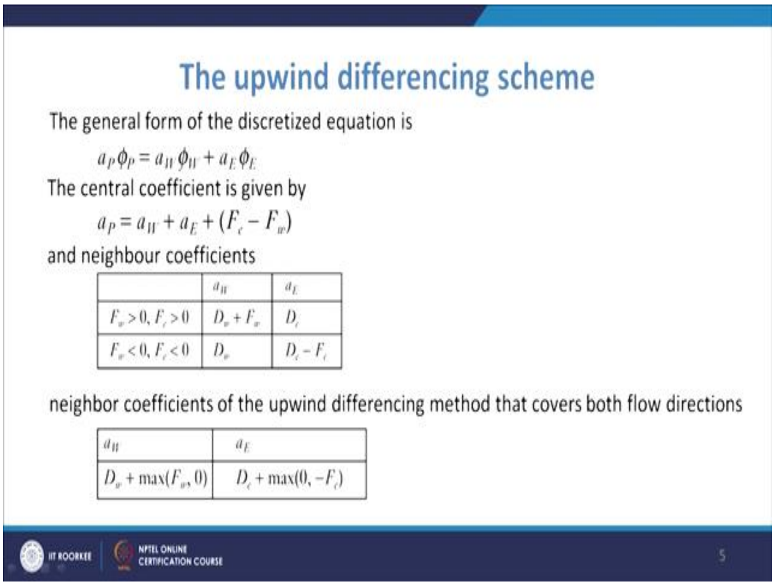
So, accordingly you will have the $a_P\varphi_P$ will be you know $a_E\varphi_E + a_w\varphi_W + s_u$ that is what we get you know you know that equation which needs to be solved. So, this way the you know when is you have a upwind differencing scheme you would get these values as this one and you get this value as this one and then you get one equation.

(Refer Slide Time: 06:57)



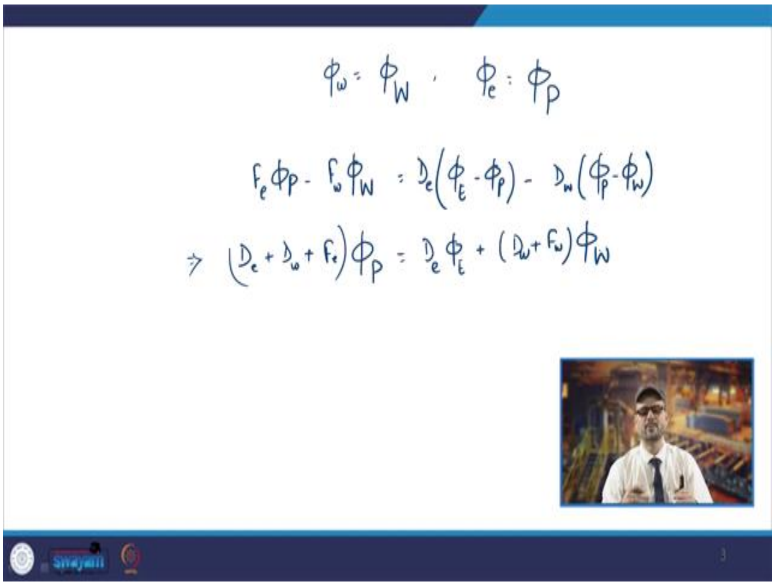
And, if your stream is in this negative direction. So, anyway then you have to take this φ_w. So, it will be if you are having here. So, the upstream you know it is flowing in the directions upstream is P. So, φ_w will be phi at this P node; similarly φ_e will be at φ_E that nodal value. So, this will be used to you know discretise the equation and you get the expression for φ_P in terms of φ_e and φ_w and the source term which needs to be solved.

(Refer Slide Time: 07:42)



And, then you get these values in terms of $a_P \varphi_P = a_E \varphi_E + a_w \varphi_W$. So, your central coefficient will be given by $a_E + a_w + (F_e - F_w)$. Now, what we see that in the case of this upwind differencing scheme you know you get these values you can have the idea about how that you know changes.

(Refer Slide Time: 08:15)



So, what we saw is that you get when you have the flow in the positive x direction. So, your $\varphi_w = \varphi_W$, similarly you get to φ_e. So, you will have the upstream node that is P so, you will have φ_P. So, you will have so, what you can get the values. So, you will have F e phi E and phi E will be phi P.

$F_e\varphi_P - F_w\varphi_W = D_e(\varphi_E - \varphi_P) - D_w(\varphi_P - \varphi_w)$. So, that is what we normally get. And, then if you try to solve so, you can have the φ_P terms on one side. So, if you take φ_P terms on one side. So, you will have this side it will be $-D_e$ and if this side it is $-D_w$ is coming.

So, and this side it is F_P. So, that this will imply that if you take the $(D_e + D_w + F_e)\varphi_P = D_e\varphi_E + (D_w + F_w)\varphi_w$

So, that way you see that there will be changing the equation so that you get it. Now, this will be further you know change. So, that is what you get from here. So, what you see that this will be $(D_w + F_w)$ and you know and similarly you will have theD_e.

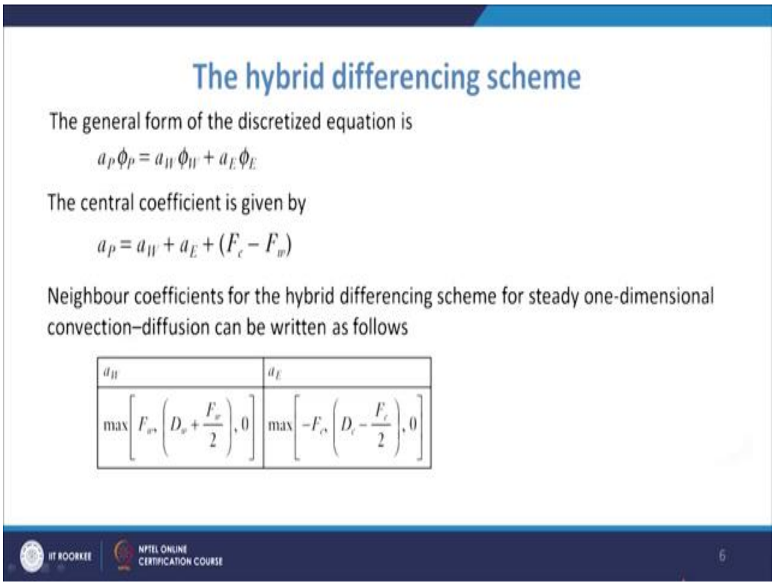
So, that is when $F_w > 0, F_e > 0$, when u is positive in that case F_w and F_e will be that is ρu. So, it will be positive and that is why your a W will be $(D_w + F_w)$ that is what you see the a_w will be the $(D_w + F_w)$ similarly a_e will be D_e and then you will have the expression for a_P . So, $a_P = a_W + a_E + (F_e - F_w)$ that is what you get from here.

So, and when you have the velocity in the opposite direction in that case your upstream nodes having different values and in that case your cell face values will be different because the upstream direction is now in the negative direction. So, for west it will be P and for east it will be E. So, accordingly your expression changes and that becomes again in that case it will be changing.

So, you will have the different you know expression for that case. Now, this is how so, in generalized way your neighboring coefficients you will be having the you know a W as D w plus maximum of F_w and to 0 and this will be D_e plus maximum 0 to minus F_e and that is how you know you these differencing schemes are you know used for predicting.

So, normally whenever you have these you know when the Peclet number is important in those cases especially when the convection flow dominates in those cases the upwind differencing scheme is seen to work better.

(Refer Slide Time: 13:01)



Now, the next scheme which is important is the hybrid differencing scheme and this scheme is used when you know the central differencing scheme will be useful for the diffusional problems when you have the diffusion dominant flow. So, in that case you take the central differencing.

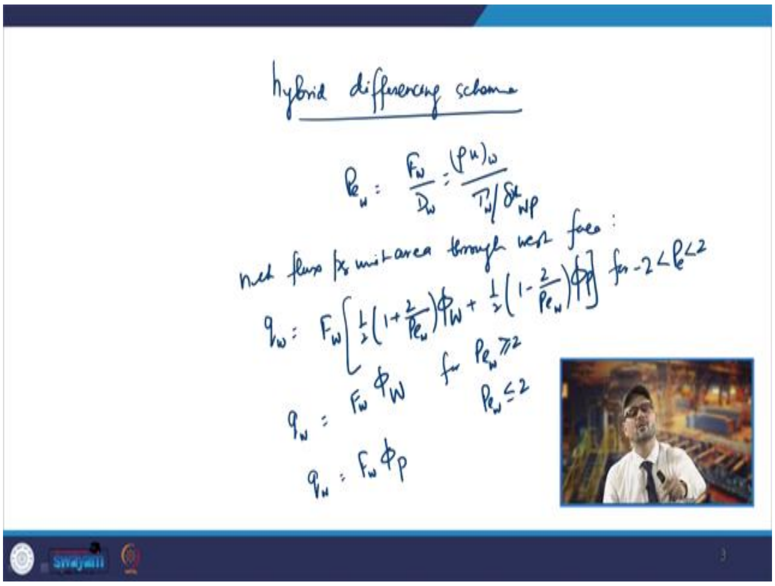
So, you are those central differencing schemes are you know useful whereas, when you have the convection dominated flow. So, you take the upwind you know differencing scheme. So, that is because in the case of convection the upstream node is gone to a actually effects you know more than the downstream ones.

So, in the case of a hybrid differencing what it takes is it will take the effect you know of it will take the advantage you know advantage of both central differencing as well as the upwind differencing and you know normally central differencing will be useful when your Peclet number is less.

So, in those cases and central differencing is basically second order accurate whereas, the upwind differencing is first order accurate. So, also it has the different traits. Now, in those cases what we is done is that this hybrid differencing scheme it will be the using the piecewise formula you know and that will be based on the local Peclet number.

So, local Peclet number will be computed and based on that it will be using that piece wise you know formula you know for finding the net flux through that control volume face. So, we know that the these Peclet number is nothing, but the ratio of F/D. So, you will have you know the.

(Refer Slide Time: 15:13)



So, if you go to the you know hybrid you know hybrid differencing scheme . So, as we know that you have the definition of Peclet number at the west face that will be $\frac{F_w}{D_w}$. So, you find $\frac{(\rho u)_w}{\frac{\Gamma_w}{\delta x_{wp}}}$. So, that is how you calculate these Peclet number at the west face.

Now, what is done is that in the case of the you know the hybrid differencing scheme we find the you know net flux per unit area. So, net flux per unit area through west face. So, that will be computed using a formula and that formula will be like you will have q_w.

So, that will be you $F_w[\frac{1}{2}(1+\frac{2}{Pe_w})\varphi_w + \frac{1}{2}(1-\frac{2}{Pe_w})\varphi_p]$.

So, basically and when this Peclet number will be varying between minus 2 to 2 so, in fact, when the what we have seen earlier that while discussing the boundedness criteria related to the differencing schemes that for ensuring the you know coefficients to be positive for a E especially in that case what we saw is that you know is the Peclet number is more than 2 or $F/D > 2$ in that case is likely to be negative.

So, that is why when it is between minus 2 to plus 2 in that case it is taking you know the see the it is having these effect actually you know of both these φ_w as well as φ_p. So, that is why you know it is known as a hybrid, it is taking the considerations for both these nodes you know in one case and in other cases it will also change.

So, for the value of from Peclet term out from minus 2 to 2 in between you will have this one because minus 2 means even if the flow direction is changing. So, in those cases your you know net flux per unit area or the west face can be calculated using that. Now, if your Peclet number will be more than 2 in that case; so, if you see that convection dominance is more in the positive direction in that case you know it will be switching over to the upwind mode because in those cases the upwind differencing schemes are you know said to be more stable or more useful.

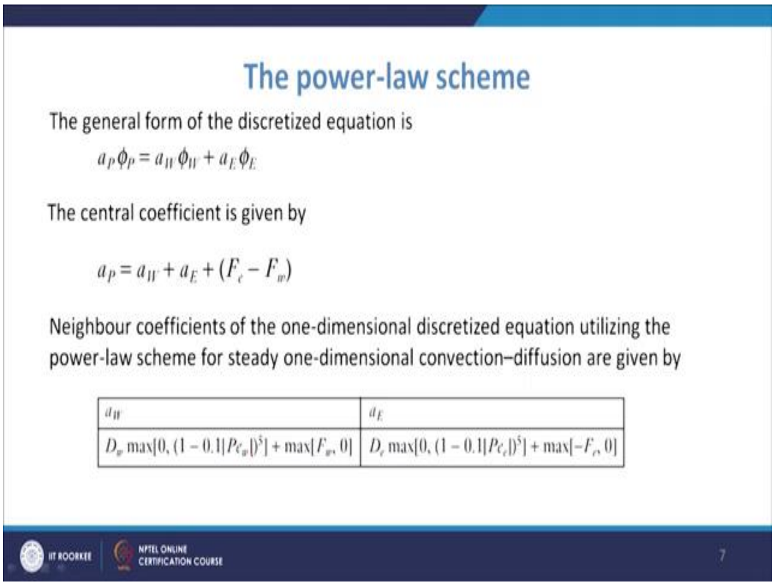
So, in that case the $q_w = F_w \varphi_w$. So, that will be in the upstream node will be the western node. And if so, it is for the Peclet number as local Peclet number at this face it is more than 2 and if it is Peclet number is less than you know less than 2 in fact so, in that case you know what you see is that the $q_w = F_w \varphi_P$.

So, that is the extreme node when we see the flow in the opposite direction. So, that way your F_P will come. Now, we can have the values this values putting into the discretized equation and then we get the formula. So, what we get is that is the central coefficient. So, a_P you know $a_P \varphi_P = a_E \varphi_E + a_w \varphi_W$ if we write.

So, in that case you can have the $a_P = a_E + a_w + (F_e - F_w)$. and for the you know hybrid differencing schemes you know for steady one dimensional convection diffusion neighboring coefficients can be written as the $max[F_w, (D_w + \frac{F_W}{2}),0]$, similarly you know $max[-F_e, (D_e + \frac{F_e}{2}),0]$ so, like that.

So, basically so, this is how in the case of a hybrid differencing scheme it will take into account both the things like a central differencing as well as upwind differencing and then accordingly it will try to you know your equations are solved and you can see the accuracy with respect to the exact solution which we can verify in our coming lectures when we try to solve and learn it through by solving a problem.

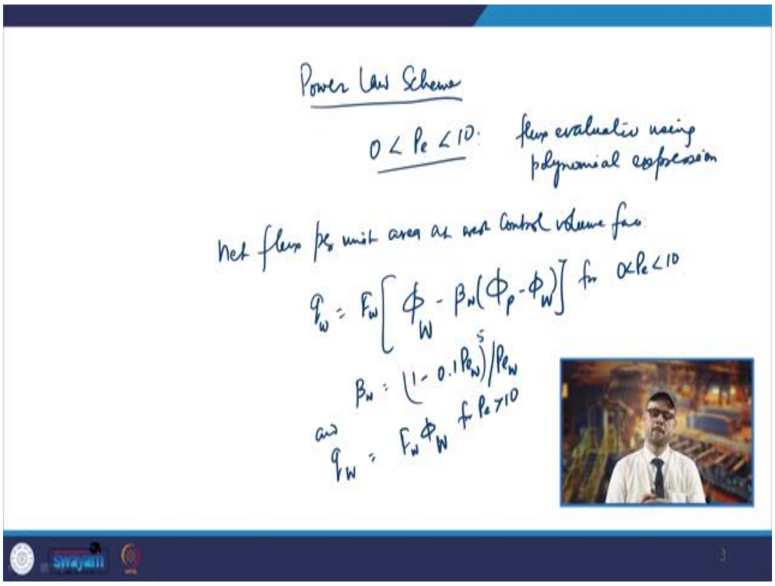
(Refer Slide Time: 21:18)



The other kind of differencing scheme which is used is the power loss schemes. So, this power loss scheme. So, it will be using certain power depending upon the you know Peclet number. So, you know in this case what you see that you see this you know Peclet number raised to the power 5.

So, in this case depending upon the cell Peclet value when it will be you know it will be exceeding 10, so, in those cases it is assumed that the diffusion is 0. So, diffusion will be set 0 and the flux will be calculated using a polynomial you know expression in this case.

(Refer Slide Time: 22:11)



So, if you go to the power law scheme so, it is basically by suggested by Patankar. So, him researcher I mean there is a book called so, very renown book by the authors author that is S. V Patankar. So, they have gone you may given the invention done the invention of this scheme. So, what it tells that if it is you know the Peclet the cell Peclet value is more than 10 in that case the diffusion is said to be 0 and when the cell Peclet value will be between 0 to 10 in that case the flux will be evaluated using a polynomial expression.

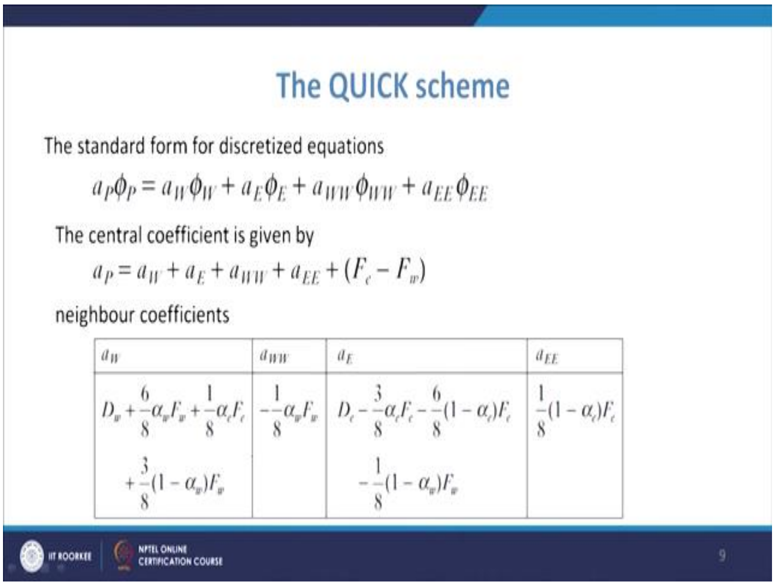
And, so in this case flux evaluated using polynomial expression and if you try to find the net you know flux per unit area so, this one at the west control volume face so, that will be given by one polynomial expression. And, $F_w[\varphi_w - \beta_w(\varphi_p - \varphi_w)]$. So, that is for the Peclet value where it is from 0 to 10 and if the you know if the Peclet number is more than 10 in that case it is considered to be purely diffusional. So, diffusion is said to be 0 so, that will be again going for the upwind differencing.

So, and in this case $\beta_w = (1 - 0.1Pe_w)^5/Pe_w$. So, this is how it will be depending and that is why it is known as a power law and when your Peclet number will be more than 10 in that case the diffusion is assumed to be seized and then it will behave as a the upwind differencing scheme.

So, in that case the q_w that will be the $F_w\varphi_w$. So, that is what is there for the you know the upwind differencing scheme where you have the upwind node is phi w. So, this is for Peclet number more than 10. So, you know this coefficient also in this case as we see in this case you have the coefficient will be $a_E + a_w + (F_e - F_w$.

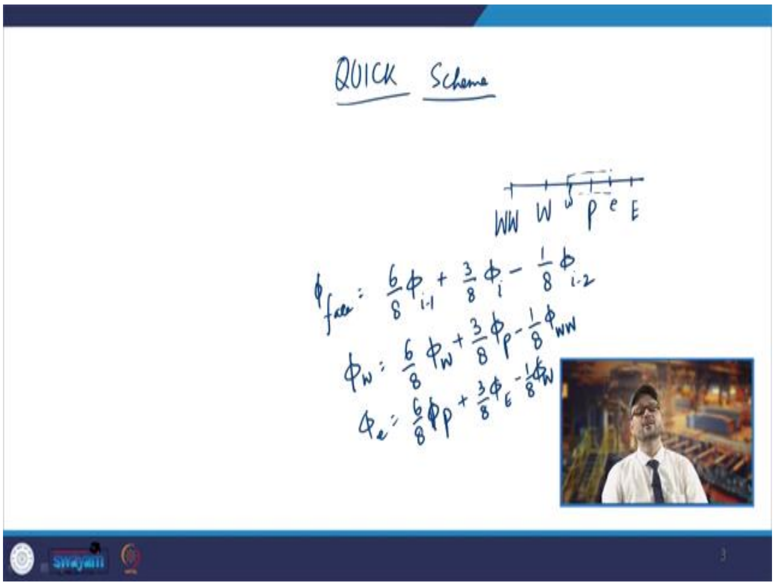
And, as we see based on this power law we have the value for the a W or a E expressed in this form for the power law you know power law is schemes which are devised by the Patankar and another you know scheme which is also used that is the higher order scheme and that is you know the quick scheme.

(Refer Slide Time: 26:17)



Now, in this scheme basically what we do is that you have the two nodes on the two sides and we also take one node in the upstream side.

(Refer Slide Time: 26:29)



So, you know if you have you know so, in the case of the quick scheme so, in this case this is based on the you know quadratic you know form so, that is quadratic upwind differencing scheme. So, in that case what we do is that you have this is your you know west face this will be west node.

So, in that case you take another you know that is WW and this is the W face and this is your P. So, you are getting you know the flux value in the form of the values at the these three nodes in the this quadratic you know upwind scheme. So, differencing upwind differencing scheme that is what is known as quick.

Now, in this case if you look at the quadratic upwind scheme if you try to understand in this case basically the $\varphi_{face} = \frac{6}{8}\varphi_{i-1} + \frac{3}{8}\varphi_i - \frac{1}{8}\varphi_{i-2}$

So, this is if your this is i and this will be i minus so, this is i. So, in fact, this is the P point this will be and you are calculating that face here and this will be I this will be i minus 1 and this will be i minus 2. So, basically you are calculating this you know 6/8 of this portion 3/8 of this portion and minus of 1/8 into this portion.

So, that way you are getting. So, if you suppose you are trying to have the phi of the W face so, in that case $\frac{6}{8}\varphi_w$. So, from here i minus 1 will be here. So, that will be phi W; then 3 by 8 i, so, it will be $\frac{3}{8}\varphi_p$ and $\frac{1}{8}\varphi_{ww}$ so, this face $i-2$ will be the WW. So, φ and WW is taken.

So, this is how you know you calculate you know the values. This is for the west face; similarly, if you have the control volume at the east face so, in that case you will the values on this side also. So, on this side you have the east face. So, your you know for this you will have p, w and e will be coming out. So, your phi e if it will be $\frac{6}{8}\varphi_P + \frac{3}{8}\varphi_E - \frac{1}{8}\varphi_W$. So, this is you know also this is a higher order type of differencing scheme which is also used you know in many cases.

And, you know in this case the neighbor coefficient will be now as you see it will be in terms of a WW or so, and a EE also depending upon where you are trying to find and accordingly you will have a W and a WW, a E and a EE how you know you find it. So, that is how it is computed in such cases.

So, this is these are the you know in just we are having these different types of schemes apart from that we have also you know other schemes like TVD schemes are there you know. So, there are many other schemes also which you may come across while dealing with these you know studies and typically in the you know softwares you will be having mention.

And, basic idea is that you must have the understanding about how these schemes you know work, what are the fundamentals of these schemes and how they are used for you know discretisation. So, that is what about the and different discretized schemes. We will discuss about the their performance by considering an example in our coming lecture.

Thank you very much.